Physics of Turbulence
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Lecture - 12
Craya-Herring Basis: Definitions

So far, what we did, we started from basic equation on Fourier space and we learnt how to compute the non-linear term and the viscous term. Viscous term was easy, but nonlinear term is tricky. It just involves a lot of algebra. So, it turns out these are basis called Craya-Herring Basis. So they are two people, Craya and Herring.

So, it was some 40 years back, this is not new basis. So, Craya is French and Herring is American and they made this basis which is quite easy; so, this what I will do today. So, what is the advantage of this basis? So, I will first describe right now in 2D and 3D both, we can construct equations in 2D as well as in 3D.

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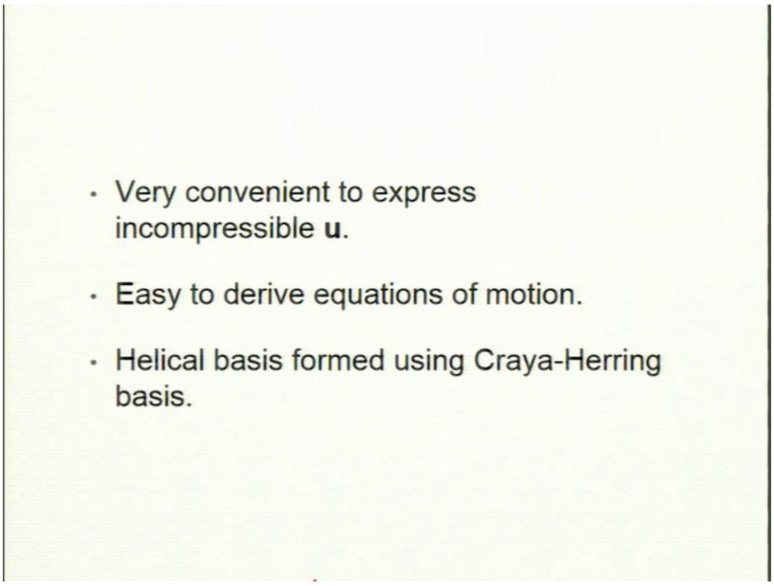

So, it is convenient to express the incompressible flows; I will show you how we can derive equations of motion easier, so algebra is reduced. And there is something called helical basis for this screw like fields that is easier expressed in helical basis which is constructed using Craya-Herring basis. So, this basis is used for making helical basis, so we will study this later ok.

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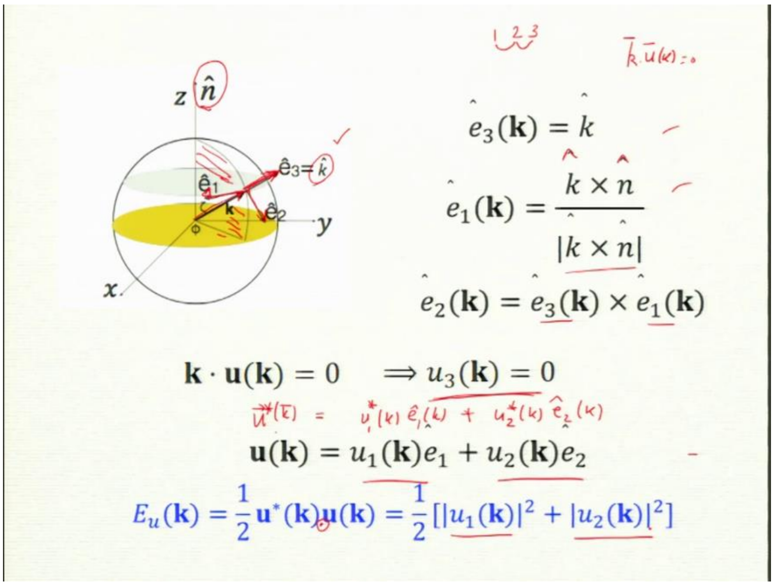

So, let's start, we will show you how to construct this basis; we will first work with 3D. This is a vector, so we have wavenumbers, so wavenumber is a vector **k**. So, this is wave vector **k**. So, please pay attention to this figure.

Now, along the direction of vector **k**, I make $\hat{e}_3(\mathbf{k})$, it will have 3 components, but this is variable. So, you will find that $\hat{x}$, $\hat{y}$, $\hat{z}$ are fixed basis, but here the basis vectors will move along with **k**. So, these are like radial polar coordinate, but is different, it is not radial polar. So, my $\hat{e}_3(\mathbf{k})$ along $\hat{k}$.

Now, we need an anisotropy direction, some directions, it need not be anisotropy direction, but we need some direction. If there is anisotropy like magnetic field or rotation, you can choose the rotation axis to be $\hat{n}$, this $\hat{n}$, ok. I need one more vector, but choice is yours. For (Refer Time: 03:05) MHD or for rotation, we can choose $\hat{n}$ along rotation axis, but it is not mandatory, you can choose $\hat{n}$ in any direction you like.

Now, given $\hat{n}$ and $\hat{k}$, I construct, $\hat{e}_1(\mathbf{k})$ like this (Refer Time: 03:24). So, these are unit vectors, cap means unit vector, so length 1, but direction along **k**. So, $\hat{k} \times \hat{n}$ divided by modulus of this, so which direction will that be in this figure (Refer Time: 03:39).

So, it is perpendicular to the plane, formed by $\hat{n}$ and $\hat{k}$, cross product. Here this plane and it is perpendicular to that plane and is along $\hat{e}_1(\mathbf{k})$, so this is the first vector. Now, what is $\hat{e}_2(\mathbf{k})$? So, it is cyclic, so 1 2 3, ok, so cyclic even everybody understands 1 2 3. So, $\hat{e}_1(\mathbf{k})\times$

$\hat{e}_2(\mathbf{k})$ is $\hat{e}_3(\mathbf{k})$, 2 3 gives you 1 and 3 1 gives you 2, so $\hat{e}_3(\mathbf{k}) \times \hat{e}_1(\mathbf{k})$. So, do cyclic 1 2 3 just go around. So, $\hat{e}_3(\mathbf{k}) \times \hat{e}_1(\mathbf{k})$ gives you $\hat{e}_2(\mathbf{k})$, it lies in the plane.

So, $\hat{e}_3(\mathbf{k}) \times \hat{e}_1(\mathbf{k})$ will be perpendicular to both $\hat{e}_3(\mathbf{k})$ and $\hat{e}_1(\mathbf{k})$ and it is in the same plane which is this plane (Refer Time: 04:36). So, velocity vector for incompressible flows, we know that is $\mathbf{k}\cdot\mathbf{u}(\mathbf{k})$ is 0. So, $\mathbf{u}(\mathbf{k})$ must be lying where, perpendicular to $\mathbf{k}$ vector, so it will lie in the plane formed by $\hat{e}_1(\mathbf{k})$ and $\hat{e}_2(\mathbf{k})$ vectors, so which is a natural basis for you, for expressing velocity vector?

(Refer Time: 05:08) $\hat{e}_1(\mathbf{k})$ and $\hat{e}_2(\mathbf{k})$. So, we don't need to worry about 3D vectors now, we just need to worry about 2D vectors. So, my velocity field is $u_1(\mathbf{k})\,\hat{e}_1 + u_2(\mathbf{k})\,\hat{e}_2$ and $u_3(\mathbf{k})$**,** the velocity has no component along $\hat{e}_3$. So, $u_3(\mathbf{k})$ is 0, just straight forward, and this is very, I mean this is straight forward, but it is very useful. So, you can use this to solve quite a bit of a problems in a less number of steps.

So, please keep in mind that we need another vector $\hat{n}$; so $\hat{k} \times \hat{n}$ gives you $\hat{e}_1(\mathbf{k})$ of course, you divide it by (Refer Time: 05:55) magnitude and $\hat{e}_2(\mathbf{k})$ is $\hat{e}_3(\mathbf{k}) \times \hat{e}_1(\mathbf{k})$. Now, given $\hat{x}$, $\hat{y}$, $\hat{z}$ or given u_x, u_y, u_z you can construct u_1, u_2 and vice versa. It is like very similar to what you do for spherical polar to Cartesian, or Cartesian to spherical polar or cylindrical. So, these are another basis vector, but we can do it that way.

Now, please remember it is in Fourier space; it is not in real space so, there is no gradient and there is no curl, but you have $\mathbf{k}\times$ so. There will be lot of cross product with **k,** dot product with **k**, but it is not in real space; it is in Fourier space. Now, we start looking at energy, so what about energy? Energy is $u^*(\mathbf{k}) \cdot u(\mathbf{k})$ (Refer Time: 06:51).

So, what will that be? By the way, please remember $\hat{e}_1(\mathbf{k})$ and $\hat{e}_2(\mathbf{k})$ are perpendicular to each other, if I take a complex conjugate of this one (Refer Time: 07:01) fully, $\mathbf{u}^*(\mathbf{k})$ vector is equal to $[u_1(\mathbf{k})\,\hat{e}_1(\mathbf{k})\,]^*$. So, $\hat{e}_1$ is the real vector right, here $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ all real vectors.

So, if I take a complex conjugate, I get same thing, but u_2 is also complex, so u_2^*, but $\hat{e}_2$ is a real vector. Now, if I take a dot product of these two, $\hat{e}_1 \cdot \hat{e}_2$; $\hat{e}_1 \cdot \hat{e}_1$ is 1; $\hat{e}_2 \cdot \hat{e}_2$ is 1, but $\hat{e}_1 \cdot \hat{e}_2$ is 0, so you can easily see that you will get these terms. So, kinetic energy is just $\frac{1}{2}|u_1(\mathbf{k})|^2 + \frac{1}{2}|u_2(\mathbf{k})|^2$.

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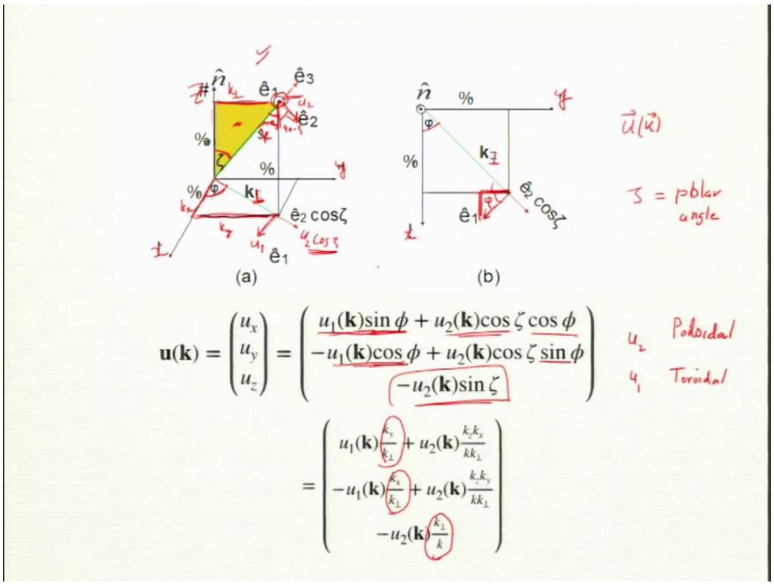

Now, let's try to understand how to convert this in real space, some of these symbols are not transported properly, so this is x y z, x y ok, so this is $\mathbf{k}_{\perp}$. So, no, that there are too many problems in this ok, so this is $\mathbf{k}$. So, $\mathbf{k}$ vector, so this 3D, this one is 3D right; see the same stuff. So, now, my plane is this yellow plane this one, now $\hat{e}_3$ is along $\hat{k}$ direction, what about $\hat{e}_1$; $\hat{e}_1$ is perpendicular to the plane towards me.

So, if I see from the top, it is going like this clockwise $\hat{e}_1$. So, please keep this in mind you look at this picture several times and you will need it quite often. So, this is $\hat{e}_1$ and $\hat{e}_2$ is $\hat{e}_3 \times \hat{e}_1$, so this is $\hat{e}_2$. Now, so what about z component of **u(k)**? So, my vector is **u(k),** velocity vector, I want to find out what is z component. So, which will give you z component, remember my **u** is only along $\hat{e}_1$ and $\hat{e}_2$, so, does u_1 has any z component? u_1 is along x and y.

u_1 is azimuthal, so it has no z component, but u_2 has z component, this is u_2 is along this. So, what is this angle? So, this angle is ζ. So, ζ is polar angle for me, so in this book we will always find ζ has polar angle.

I am reserving θ for temperature, so this is for convection, that is why we are not using θ. So, ζ and ϕ, so, ϕ is azimuthal angle. So, this angle will be (Refer Time: 10:18) $90^o - \zeta$, this angle is ζ, so this is $90^o - \zeta$. So, $u_2(\mathbf{k})\cos(90^o - \zeta)$ will give you with a minus sign, because u_z is upward, but this is downward so, that is the z component $-u_2(\mathbf{k})\sin(\zeta)$.

Now, what about x component and y component? So this $\mathbf{k}_\perp$, so my velocity vector I am projecting it down, now in the x y plane. Now, by the way there is no z component anymore, now after remove this I only left with x and y components ok. So, now, my u_1 vector will be along this ok, and my u_2 vector will be along this, but how much is coming along this? $\cos(\zeta)$; $\cos(\zeta)$ is remaining part because this part is $\sin(\zeta)$, so this part is $\cos(\zeta)$, so $\cos(\zeta)$ is there. Now, take component of these, so this u_1, so I have drawn it here. So, basically you take the component $\hat{e}_1$, u_1 is not a vector, u_1 is a number. So, this angle is ϕ, so what should be this angle? So this $90^o - \phi$, so this ϕ. So, along x direction, will be $-\hat{e}_1 \cos(\phi)$, you know, so there is y direction.

So, x component is in that direction ok, so $\sin(\phi)$, this is $\sin(\phi)$, so $u_1(\mathbf{k})\sin(\phi)$ will be this one, $u_1 \cos(\phi)$ with a minus sign is this one. Now, this is $u_2 \cos(\zeta)$. Now, I have to take a component again $\cos(\phi)$ and $\sin(\phi)$, so these are this ok. So, we need well, especially in our computer code we need to transform. So, if you solve u_1 and u_2 you will need to find u_x and u_y and u_z, so you use this transformation ok.

Now, once you have this $\cos(\phi)$, $\sin(\phi)$, you can express them in terms of k_x and k_y, k_z. So, what is $\cos(\phi)$? So, this angle is ϕ, so this vector is $\mathbf{k}_\perp$; so, $\cos(\phi)$ is this one, $\frac{k_x}{k_\perp}$ right, so this is k_x. So, $\frac{k_x}{k_\perp}$ gives you $\cos(\phi)$, this is $\sin(\phi)$, $\frac{k_y}{k_\perp}$, this one; this one is k_y, so $\frac{k}{k_\perp}$ is $\sin(\phi)$, $\sin(\zeta)$ is this $k_\perp$. So, this vector is $\mathbf{k}_\perp$, so this is $k_\perp$, $\frac{k_\perp}{k}$ will give you $\sin(\zeta)$.

So, this algebra is straight forward, but you should know how to convert this. It is very much similar to spherical polar coordinate, but they are not spherical polar coordinate, spherical polar coordinate is tangential, well there is some similarity. But yeah there is some similarity because u_2 is like $u(\theta)$ yeah actually there is some $u(\theta)$ yeah in fact they are very similar.

So, $u(\phi)$ is like u_1, so it is like spherical polar, but yeah this called Craya-Herring for fluid dynamics and it is in Fourier space. In fluid dynamics u_1 is called toroidal, so toroidal is like ϕ and u_2 is called poloidal, so polar angle poloidal ok. So, this is more commonly used word poloidal u_2, u_1 is toroidal. And there are real space equivalent, but I will not discuss that in this course ok. So, what about vorticity? You can write vorticity in terms of u_1 and u_2.

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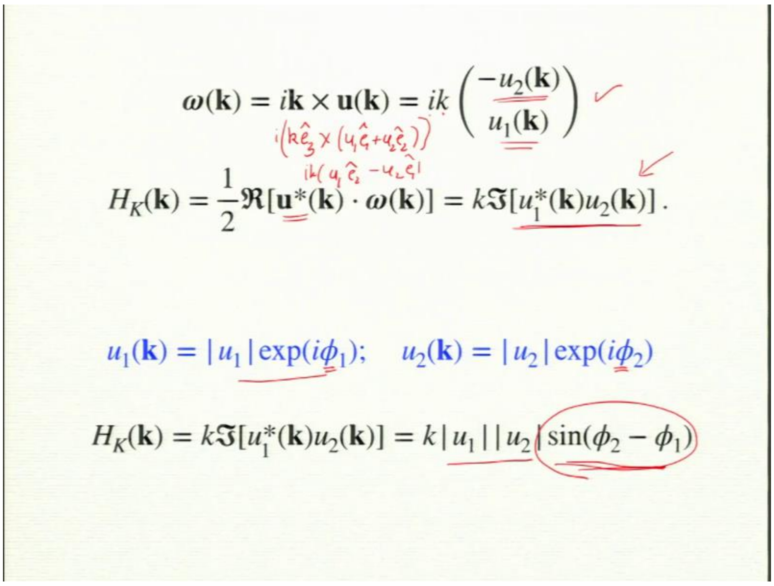

So, vorticity vector is $i\mathbf{k} \times \mathbf{u}(\mathbf{k})$, so $\mathbf{k}$ vector, so you can take a cross product. So, now, I want to write $\mathbf{k}$ as $k\hat{e}_3$, so you have to use the Craya-Herring, do not use x y z. So, $\mathbf{k}$ has only component along $\hat{e}_3$ and $\mathbf{u}(\mathbf{k})$ has component along $\hat{e}_1$ and $\hat{e}_2$; $u_1\hat{e}_1 + u_2\hat{e}_2$. So, what is $\hat{e}_3 \times \hat{e}_1$?

$+\hat{e}_2$ or $-\hat{e}_2$?

plus, cyclic know 3 1 2 so, that gives you $u_1\hat{e}_2$ and $\hat{e}_3 \times \hat{e}_2$?

$-\hat{e}_1$; this going anticyclic, so we put a minus sign, so $-u_2\hat{e}_1$, so this is what I got. So, i k, u_1 will be minus $u_2(\mathbf{k})$ and $\hat{e}_2$ component has $\hat{e}_1(\mathbf{k})$, so they are flipped; so, the minus sign.

Now, we can construct cross helicity. So, I take a dot product of this with $\mathbf{u}^*(\mathbf{k})$ and then of course, take the real part ok, so if you do the algebra which is a straight forward. So, there is complex conjugate here ok, so I will not do every step, but you will get this imaginary part of $u_1^*(\mathbf{k})u_2(\mathbf{k})$. So, what does it tell you? It tells you that both u_1 and u_2 must be nonzero and, so if both of them are real what happens? (Refer Time: 16:48)

Because both of them real, (Refer Time: 16:50) imaginary part will become 0, so both of them real, is 0 (Refer Time: 16:56).

I mean both of them complex then you take the product, but it should be if one is real other must be imaginary. And if I further is real then first one is imaginary. So, you need combination which gives you nonzero. It is very similar to what we saw that time Real part of **u** (Refer Time: 17:15) multiplied by imaginary part of **u**, so u_1 and u_2 at least one of them if you have pure real pure imaginary both of them then it is 0. And that is basically telling you that one is sin the other one is cos ok, so that is what is the thing ok, so this why circular polarization. So, both of them cannot be cos or both of them cannot be sin or combination of course.

So, u_1 is the complex number, I can write u_1 as this $|u_1| exp(i\phi_1)$. So, if I substitute it in this expression what do I get $k|u_1||u_2|\, sin(\phi_2 - \phi_1)$ ok. So, the phases; phase of u_1 and u_2 must not be in the same direction.

So, in fact, we need this $\phi_2 - \phi_1$, so any angle, so it need not be 90^o. So, if it is cos and sin then it is 90^o, but it could be any angle which is not zero for helicity and it is indeed connected with elliptical polarization.

So, if they are 90^o and circular polarization which I am not doing it in this class. I think I will skip, I will do it later. So, we will discuss about helical waves bit later. These are if 90^o then it is circular polarization, if it is not 90^o then it is called elliptical polarization. And if they are 0^o then it is planar polarization means that no helicity, ok. So, let's see equation of motion, now this is where the more difficult parts are coming. So, we can do what I did for equations of (Refer Time: 19:10) motion, so we are computing what you are computing u_x, u_y, u_z ,last time or $\mathbf{u}(\mathbf{k})$. So, I had to compute $\hat{n}(\mathbf{k})$ vector then compute pressure, so we did lot of algebra in the last class. So, let's see whether we can simplify that.

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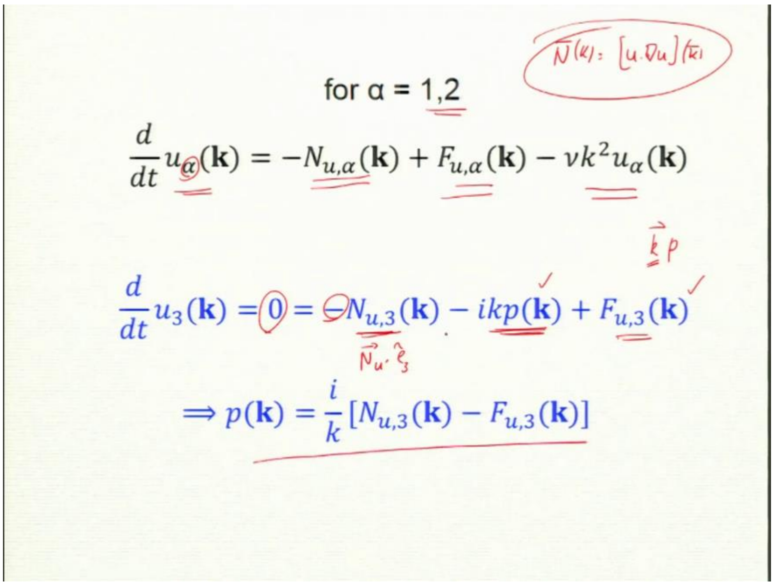

So, I have only two components for velocity field, for α is 1 and 2, $\hat{e}_1$ and $\hat{e}_2$. So, I take the component of velocity vector along $\hat{e}_1$ and $\hat{e}_2$, so this α can take values 1 and 2. So, take the component along $\hat{e}_1$ or $\hat{e}_2$ and you will get u_1 and u_2. So, this is nonlinear term, I take the component along this one, this is force and this is viscous term, what happens to the pressure? (Refer Time: 20:17).

Ok, if you put α as 3. (Refer Time: 20:20).

But, what about $\frac{du_3}{dt}$? 0. So there is no component of velocity along 3, so precisely take the component along this thing, so this I get 0. But nonlinear term can have component along 3 ok, so this is what, when you do this calculation you find that nonlinear term has component along $\hat{e}_3$ and that will be balanced by pressure and force together. So, I take the component of this equation along $\hat{e}_3$, so I have this force, external force, the pressure has full component along $\hat{e}_3$, because $k\ p(\mathbf{k})$ know, so $kp(\mathbf{k})$; so k is along $\hat{e}_3$.

So, it has that is why it is $ikp(\mathbf{k})$, and these are nonlinear term, I take the component along $\hat{e}_3$, so it is $N_{u,3}(\mathbf{k})$. So, this means $\mathbf{N}_u(\mathbf{k}) \cdot \hat{e}_3$ by the way, this is what it means and my notation please remember my $\mathbf{N}_u(\mathbf{k})$ is $(\mathbf{u} \cdot \nabla)\mathbf{u}$, Fourier transform of this, that is my notation I always. So, when I take it to the right-hand side that this minus sign is coming this minus sign.

So, remember, this is my definition of $\mathbf{N}_u(\mathbf{k})$, so is that fine? So, I can compute pressure in Craya-Herring by this formula (Refer Time: 22:01), you just invert this and you will get that. Now, let's also look at parity transformation, so this part is also tricky. So, I am going to show you the picture.

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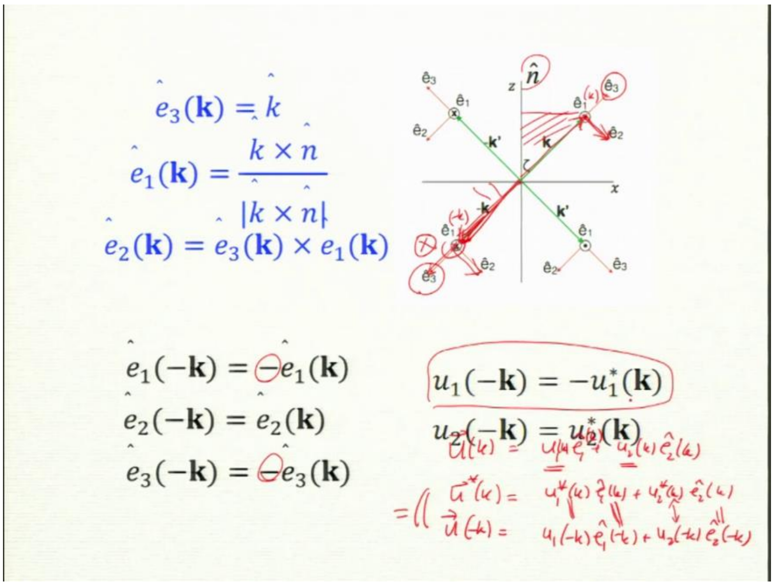

So, this is my $\mathbf{k}$, so what is parity? Parity is $-\mathbf{k}$, so you change $\mathbf{k}$ to $-\mathbf{k}$, that is called parity transformation, so $\mathbf{k}$ to $-\mathbf{k}$. So, for $\mathbf{k}$, so I think this picture is well may be you can draw it, but just focus on the slide. So, $\hat{e}_1$, so $\hat{n}$ is this way, so my plane is this. So, $\hat{e}_1$ is perpendicular to the plane for $\mathbf{k}$, towards me, I mean upward to the plane like this, $\hat{e}_2$ is downward like that. So, $\hat{e}_3 \times \hat{e}_1$ is $\hat{e}_2$ ok, so this is it; now for what happens to $-\mathbf{k}$, what is the direction of $\hat{e}_1$? (Refer Time: 23:13).

So, $\hat{k} \times \hat{n}$, so, now it is $-\hat{k}$, so this is right hand rule (Refer Time: 23:21) $-\hat{k} \times \hat{n}$. So, it is going into the plane; into this plane of paper, so that is why $\hat{e}_1$ is like this into the plane. So, $\hat{e}_1$ for $\mathbf{k}$, this is $\mathbf{k}$ and $\hat{e}_1$ for $-\mathbf{k}$ are they in the same direction?

They are not, they are in fact, they are in opposite direction. So, this is one problem with these unit vectors which are changing with $\mathbf{k}$, radial polar coordinate, that is one problem. My unit vectors are changing ok. And Cartesian is nice that way Cartesian does not change my $\hat{x}$ remains $\hat{x}$ wherever my $\mathbf{k}$ moves.

But here my $\mathbf{k}$ goes to $-\mathbf{k}$, I; my $\hat{e}_1$ changes and when I was calculating I spend one week to figure this where I had a problem and the origin was sitting here. So, it simply did not occur to that my **k,** my $\hat{e}_1$ is negative, so one has to be careful with this, what about $\hat{e}_2$? (Refer Time: 24:37).

$\hat{e}_2$ changes or does not change?

So, $\hat{e}_3$; $\hat{e}_3$ is for $-\mathbf{k}$ like this, so $\hat{e}_3 \times \hat{e}_1$, so $\hat{e}_3 \times \hat{e}_1$ will give you $\hat{e}_2$ in that direction. So, $\hat{e}_2$ does not change, $\hat{e}_2$ is in the same direction. So, my $\hat{e}_3$ has changed, $\hat{e}_3$ is opposite, my $\hat{e}_1$ has changed, but $\hat{e}_2$ does not change. If two of them changes third one cannot be, otherwise there will be a minus sign problem, so these are the transformation rules.

So, $\hat{e}_1$ is, if I go to from **k** to **-k**, I get a minus sign; $\hat{e}_2$ does not change and $\hat{e}_3$ does not change, $\hat{e}_3$ is changing, $\hat{e}_2$ does not change. And so, remember $\mathbf{u}(\mathbf{k})$ is equal to u_1**(k)** $\hat{e}_1(\mathbf{k})$+ u_2**(k)** $\hat{e}_2(\mathbf{k})$. By the way these are scalar numbers; complex number.

You should not write vector on u_1 and u_2; what about $\mathbf{u}^*(\mathbf{k})$? You just u_1^***(k)**$\hat{e}_1(\mathbf{k})$+ u_2^***(k)**$\hat{e}_2(\mathbf{k})$. What about $\mathbf{u}(-\mathbf{k})$? That is same as $\mathbf{u}^*(\mathbf{k})$ right, by reality condition is not to be broken, so these two are equal, yes or no? So, $\mathbf{u}(-\mathbf{k})$ is now going to come from here which is u_1**(-k)**$\hat{e}_1(-\mathbf{k})$+ $\boldsymbol{u_2}$**(-k)** $\hat{e}_2(-\mathbf{k})$ ok, so these two are equal?

So, what happens to the relationship between these two? (Refer Time: 26:48).

So, u_2^***(k)**, so actually I have the next slide. So, $\boldsymbol{u_2}$**(-k)** is same as u_2^***(k)** right from here, what about u_1**(-k)** of this relation? (Refer Time: 27:01).

It is the minus sign, because these are connected by minus sign, so this is this relation. In fact, this is what I spent one week to figure out this in my calculation, so this is a minus sign; so u_1**(-k)** with u_1^***(k)** ok. So, these are very useful when I am calculating things; I am calculating some equations on helicity or some field theory.

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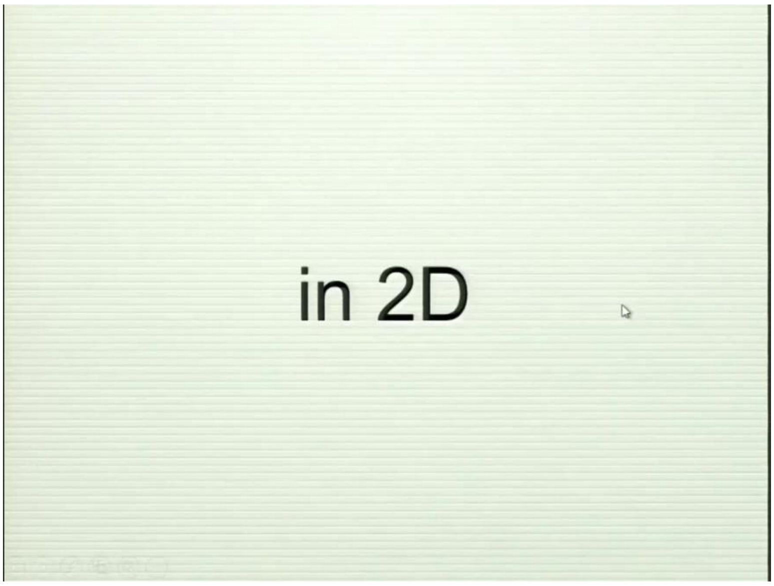

In 2D, so this is for 3D, but when I am working with 2D then I do not need both the components. (Refer Time: 27:41).

Just one component, but which one should I choose? $\hat{e}_1$, ok.

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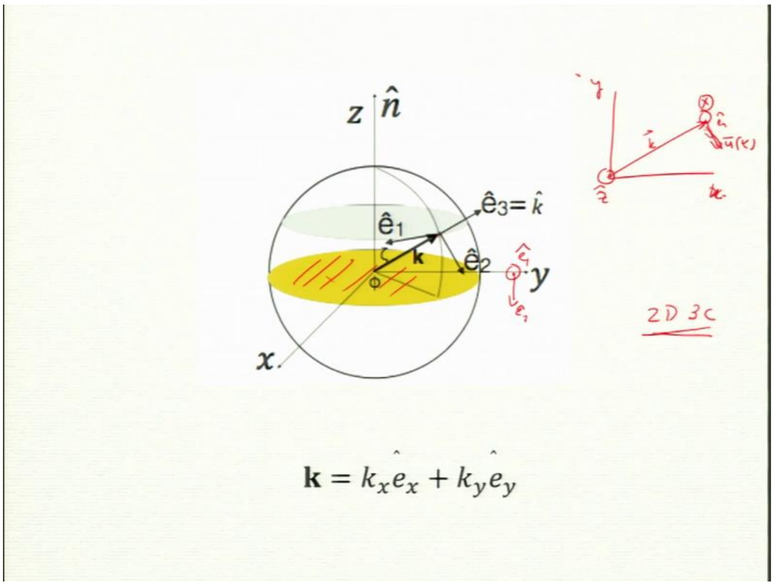

So, idea here is that, so this is a 3D. Now, I take the; I push this **k** on the equatorial plane this plane, so my $u_2(\mathbf{k})$ will be pointing downward along $-z$. So, what happens here, so

$u_2(\mathbf{k})$ will point like this and $u_1(\mathbf{k})$ will point towards me, clockwise ok, so it is in the yellow elliptic plane.

So, my $\mathbf{k}$, so is that fine, so $u_2(\mathbf{k})$ for 2D fields is not important because my plane my velocity; so you see so, this is my x and y, my $\mathbf{k}$ is like this, my velocity vector will be like this. So, it has component along only along $\hat{e}_1$, so remember z is here or $\hat{n}$, you know $\hat{z}$ or $\hat{n}$ is here, so my $\hat{e}_1$ will be in that direction, what is $\hat{e}_2$ direction?

$-\hat{k}$ ok; downward, so for 2D there is nothing along $\hat{e}_2$. But, there is a field which I want to make a mention called 2D 3-component; I did tell you about when I was doing the examples, so it has only function of x and y, but this is a component along $-z$, so that is along u_2 ok. So, for 2D-3C is only function of k_x and k_y, but it has component along $\hat{e}_2$ direction which is downward in that direction ok. But let's not worry about it, right now we will focus only on 2D, 2D-3C bit later.

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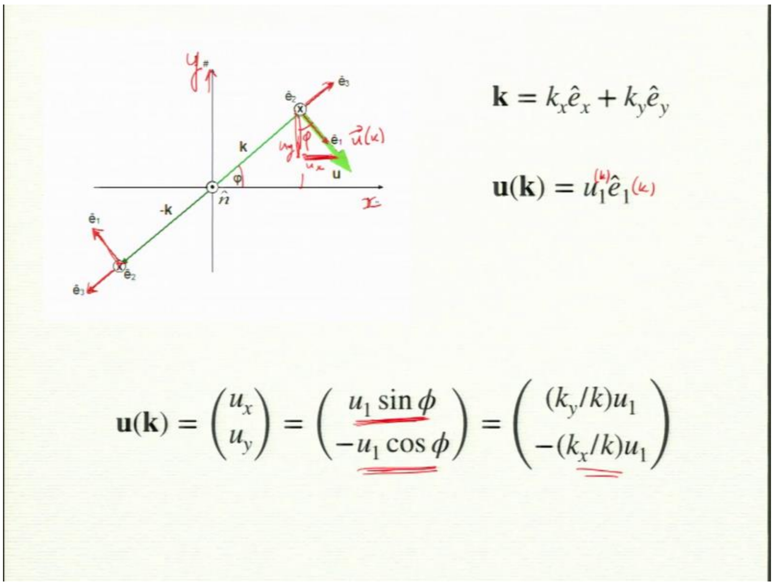

So, let us look at this picture; so now, this is in x-y plane, so this is y, it was a problem it was not able to port this picture. So, this is my vector $\mathbf{k}$ and this is my vector $\mathbf{u}(\mathbf{k})$ is along that direction ok; along $\hat{e}_1$, but what about $-\mathbf{k}$? My $\hat{e}_1$ will be in that direction, remember I said $\hat{e}_1$ is switching sign. And $\hat{e}_2$ does not switch sign, $\hat{e}_2$ is downward, we need $\hat{e}_2$; so please keep in mind, so I am going to make an example where I need $\hat{e}_2$ and $\hat{e}_3$ is of course changing sign.

So, this is under parity when you go to **k** to -**k**, so this one, so $u_1(\mathbf{k})$ is only. So, I would recommend that please write like this even though it is more writing, but specially when we have -**k** and this writing is useful which I have I skipped here too. But normally be careful you write the argument and what about x component y component? So it is straight forward. So, what is this angle? So, this angle is ϕ, so perpendicular to this is that, so this angle is ϕ, so $\mathbf{u}(\mathbf{k})$, so this is minus you know, so u_x is $-u_1$ sin ϕ (Refer Time: 31:24).

u_x is, yeah u_x in the this is u_x ok, so u_x is positive u_1 sin ϕ. So, this is my u_x and my u_y is this, which is minus because my positive y is in that direction, so minus $u_1$cos ϕ is that ok, so you can always get u_x and u_y given u_1. So, many times you will be given Cartesian vectors and you will need to convert u_1 and you need to get u_1 or given u_1 I need u_x and u_y, so especially the problems which you were doing in the last class.

So, you were having u_x and u_y right, the homework problem; now I need to solve in Craya-Herring then I need u_1, I need to convert it ok. So, this is introduction to basis function and how to get from what do they mean, how to get energy, how to get helicity ok, so we will stop.

Thank you.