Physics of Turbulence
Prof. Mahendra K. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 11
Fourier Space Representation More Examples

Now, we will work out some more examples.

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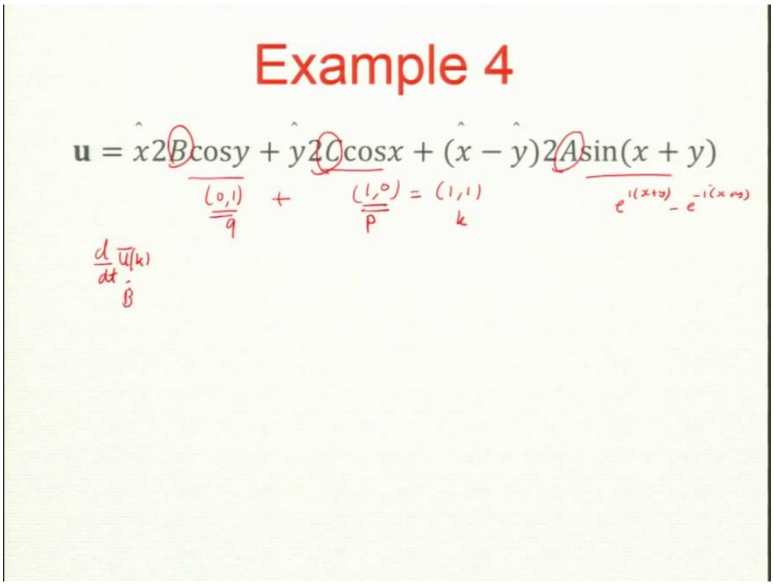
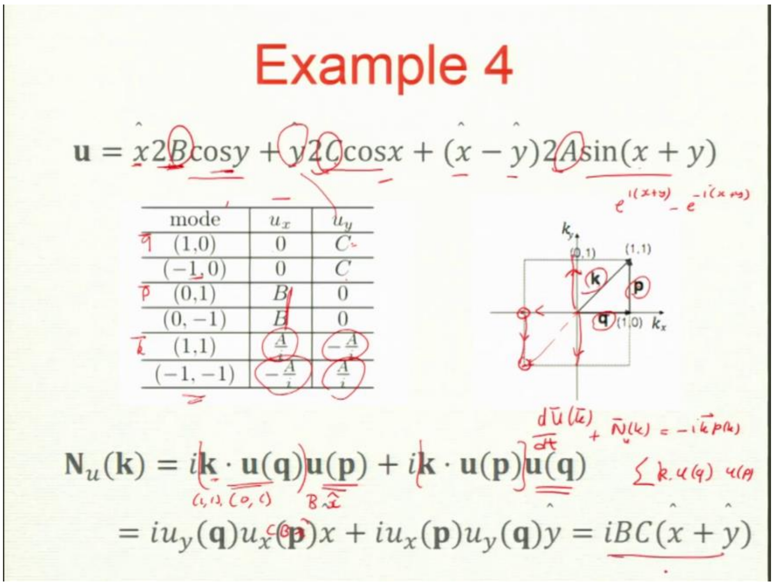


This example is slightly more complicated. See the above figure.

you see Fourier space is useful now you agree that it gives you some more handle in fact, we can visualize it if you just get used to it and see what they mean. Now this example has velocity field like this it is 2D field it is only function of x and y and they are in the plane. Now, A B C will interact. So, what is Fourier modes in this? See the above and below figures.

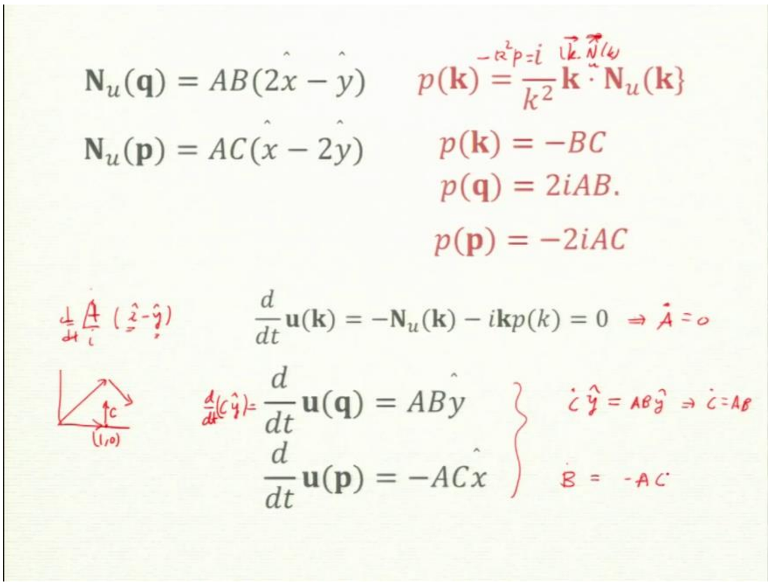
we will see non-linear interaction for this example. Now, because of non-linear interactions the B C and A these are amplitudes will change with time. So, some energy will go from A to B or something could happen right nonlinearly means, it will start doing exchange. Now the question is how to get this evolution? So, now these are real question now we need to find du(k)/dt.

So, these will give you basically I am looking for Ḃ how does B change with time? So, that is objective and the some of the midterm project I am going to give you is like this more complicated ok. So, this example please see this carefully and we will try to understand.



The modes for k = (1,0) and p = (0,1) and q = (1,1) forms a triad; k = p+q is satisfied here, and they are interacting nonlinearly. Now so, I would like to compute the non-linear terms first. I put Nu(k) in the left and that is my notation. I will not use viscosity for this. So, these are the terms which we got if you recall this plus viscous term plus force. So, force I will say there is no force and there is no viscosity. So, energy will be conserved among these interaction

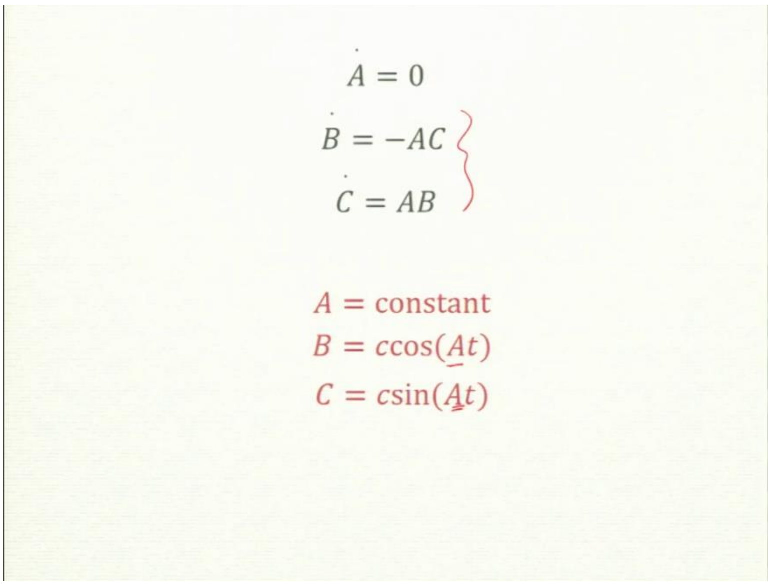
Non-linear term is a convolution. We get $\boldsymbol{N_u(k)}, \boldsymbol{N_u(p)}, \boldsymbol{N_u(q)}$ using the similar procedures as in previous examples. See the below figure.



what about pressure term? you know how to compute pressure term? I can compute pressure since I know the non-linear term.

Now once I know the pressure and non-linear term, I can get d/dt. It turns out $du(k)/dt = 0, du(q)/dt = ABy, du(p)/dt = -ACx$. So, in terms of A, b and C we get $\dot{A} = 0, \dot{B} = -AC, \dot{C} = AB$.

you are writing in a vector form and then picking up the components and this is a simple example and you can get all these stuffs.

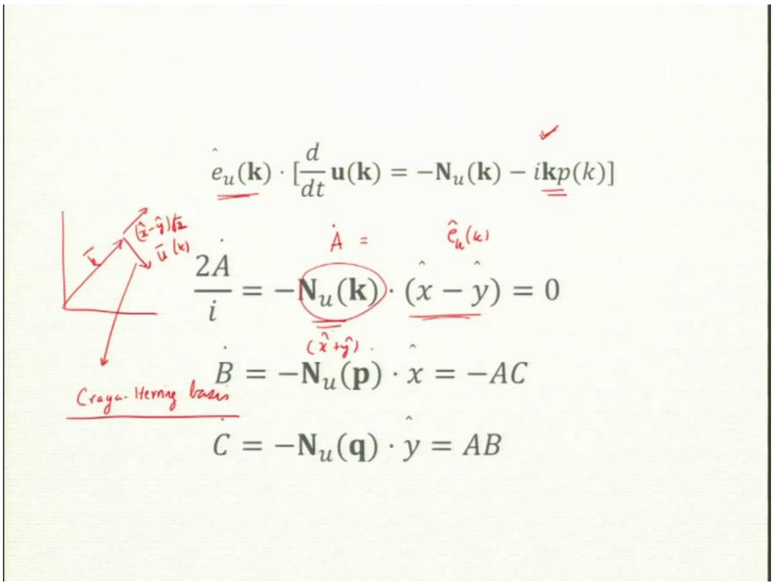


Now, this is easy solution. These equations are easy because $\dot{A} = 0$; that means, A is constant.

This is easy solution, $\dot{B} = -C, \dot{C} = B$. So, this is oscillation and solutions are this cosine. So, they oscillate with frequency.

So, they mean something. So, this means you know my modes are blinking in time. So, if you see a pattern which will be time dependent and they will be changing periodically in time, but we will understand it bit later. we will try to interpret them when you do convection hopefully, we will get enough time. So, these are how we see these patterns in non-linear physics or in flows.

just to tell you that we had a flow where the non-linear the modes were interacting with each other. There is a interaction the amplitude of the mode could change with time and we derived equation for those amplitudes and from that we can also solve it, this was easy to solve.



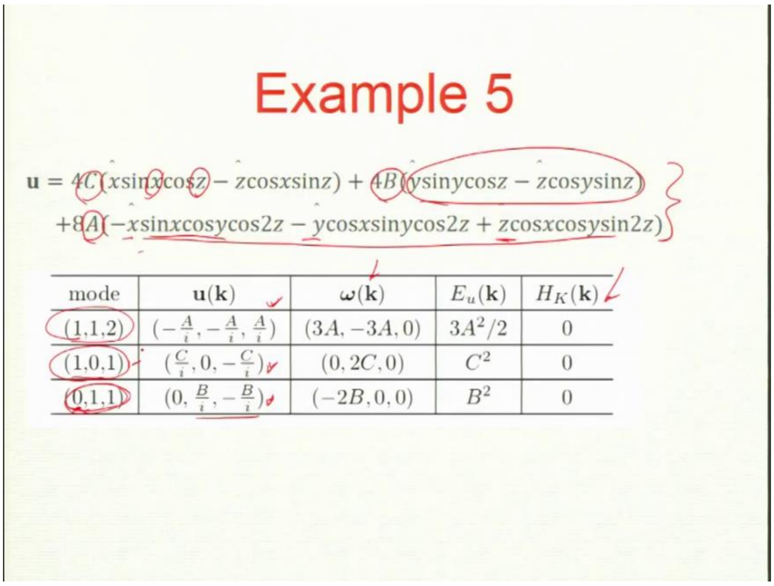
there is one more nice trick. So, before I go to the next example, which I want to just say this. See above figure. So, let us look at this $e_u(k)$. So, there is the algebra can be reduced there is nice trick which decrease your complexity by half. So, this is my k vector 1 1, now this is my equation right. So, I took Nu to the right-hand side I get this. Now I know my velocity vector is in that direction.

So, I can take some kind of direction. You take a dot product of this equation is the direction vector, I would ideally like unit vector let us make this unit vector ok. Unit vector is x hat minus y hat by 2 by square root 2 that is unit vector know. So, I dot product of this equation with unit vector. So, what happens to the pressure term, this term?

It becomes 0 because this is along the pressure gradient is along this direction. So, this term disappears, and the first term is very nice.

Now, I have computed Nu before and that was x hat plus y hat. So, that is x hat plus y hat ok. So, this is x hat to y hat we did compute before, you need to compute this ok. Now we will simplify that calculation further by some nice tricks, but dot product of this is 0. So, by choice of your basis function we can eliminate pressure term and in fact, this is called Craya-Herring basis

So, we are going to make some basis functions which will simplify our calculation and I need this at least for my work we need this stuff and you should also be able to do it when you do a project or you do homework. So, I want you to basically understand and do them for simple problems. We will do another example.

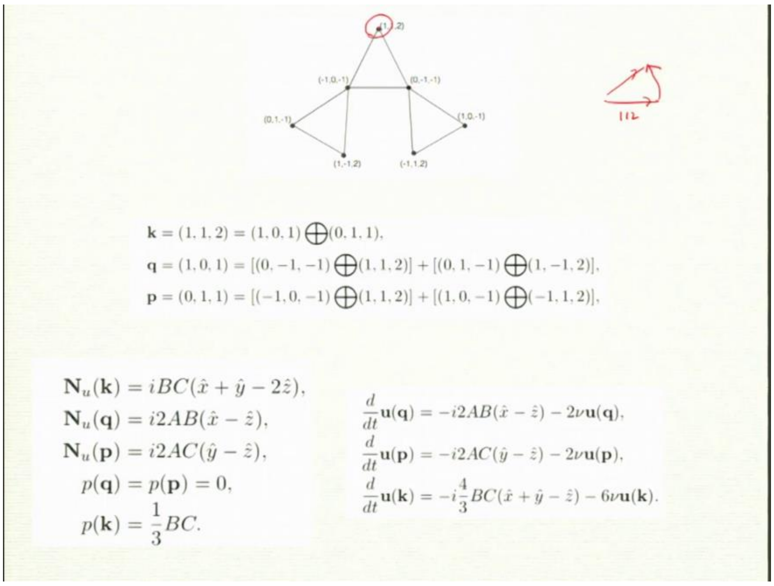


Now, this example is more complicated. See the above figure. The velocity field has again 3 unknowns A B C, but there are more terms.

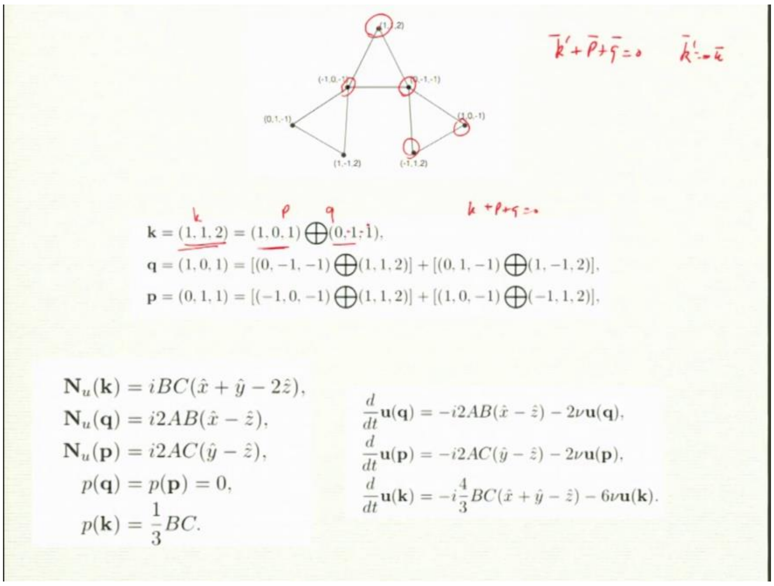
Now this is a bit of work, but once you have $\boldsymbol{u}(\boldsymbol{k})$ you can also compute $\boldsymbol{\omega}(\boldsymbol{k})$ and so on, it is you can compute the helicity, it is 0.

there are many 3D fields which are non-helical. this is now how do I compute A dot B dot C dot? Now it is a more complicated, but it can be done. Now I just want to tell you is a physical example, this is an example of 3D convection velocity field, this will be a roll along y z plane, x is not changing. So, it is the y z plane.

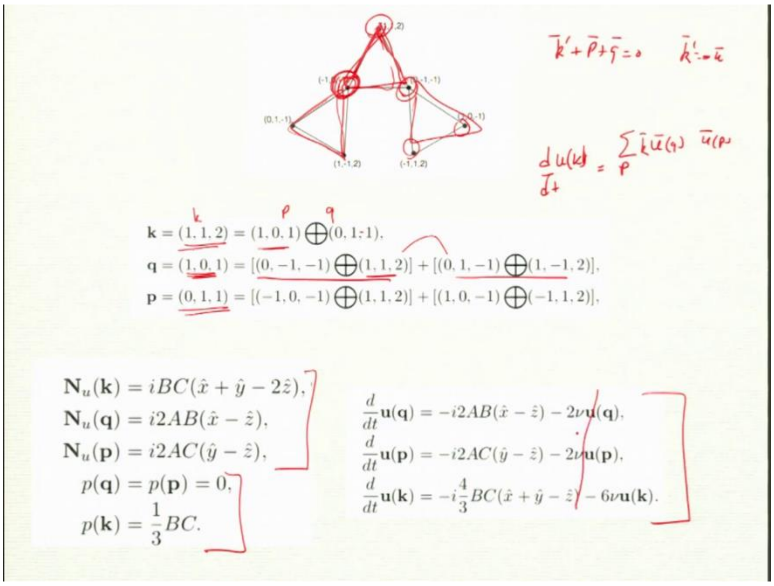
So, imagine the rolls are going in that direction. If there only 2 modes 1 0 1 and 0 1 1 they will not interact only, they happily independently doing that rotation, but as soon as we put the mode 1 1 2 then there is an interaction, then there is a person who will make them talk and they could exchange energy among them. See the below figure.



Each of these dots represent a lattice a wave number lattice so in fact, this is they represent the coordinate this is the tree diagram. So, there is no real coordinate, but this is a schematic of how they are coupled.



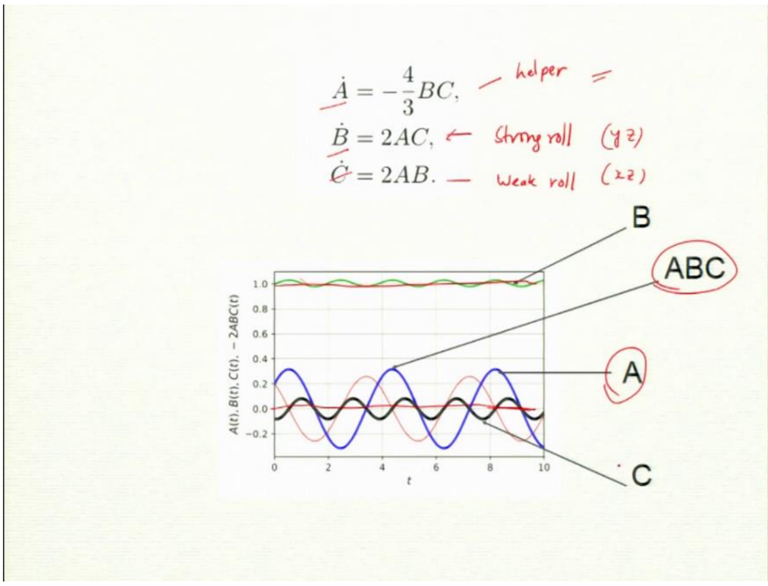
So, 1 1 2 is coupled with these two modes. If I want to write $\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q}$, then there is the only interaction there is no other interaction possible.



But 1 0 1 which is here can get energy from 1 1 2 by this channel. So, there is a channel like this for 1 0 1, but it also has another channel here. So, 1 0 1 is affected by two channels. So, one channel is here other channel is here they are independent channel. I need to add contribution from both the channels then only I get correct answer. I will not get correct answer.

So, these are the example which were I think somebody asked we have many channels and we need to account for all the channels and which one contribute more, which can more where you only after do this calculations you will know. See the above figure.

So, 0 1 1 also has 2 channels: one is this channel other one is this channel. The non-linear term will be like this, pressure is like this and the time derivative of these are like that. Now, if I turn off the viscous term, I get this equation which are not so, straight forward. See the below figure.



These are the three non-linear equations. Here, we get A B C all of them are changing in time. One interesting point which in fact, this is a physical example this happens in convection; so, B oscillating around a mean value. See these are amplitudes. So, B is green B, but is oscillating because all of them are interacting. Earlier A was not changing, B is oscillating on one mean value and A and C are oscillating around 0. This happens in convection.

So, we have a roll let us see in that direction and we have roll in that direction ok. So, the 3 D roll structure is in fact, if there is only single roll then it is 2D, its going in a plane and there is no variation along that direction, but the roll in that direction the roll in that direction and I couple them, 1 1 2 mode. one roll is strong, there is one big party which is very strong party, and the other party is weak party. So, weak party all comes and goes so in fact, it can disappear the weak party. So, I will show you may be a picture next time.

So, we see this blinking of this weak mode which appears and disappear, but the non-linear for this big roll also blinks, but around the mean value. this roll is very strong, but it also blinks, but then the week one continuous. Weak ones appear and disappears, this is a helper, C the weak roll.

So, we had this one was x z plane other one was C was x z plane and B was y z plane. So, this is y z plane. This x z plane and this is 3 D and that is helping that also oscillate helper also is not kind of does not his bank balance also going up and down now in this interaction. So, that is how they interact and these all by non-linear interactions, this is I have not discussed yet

Thank you.