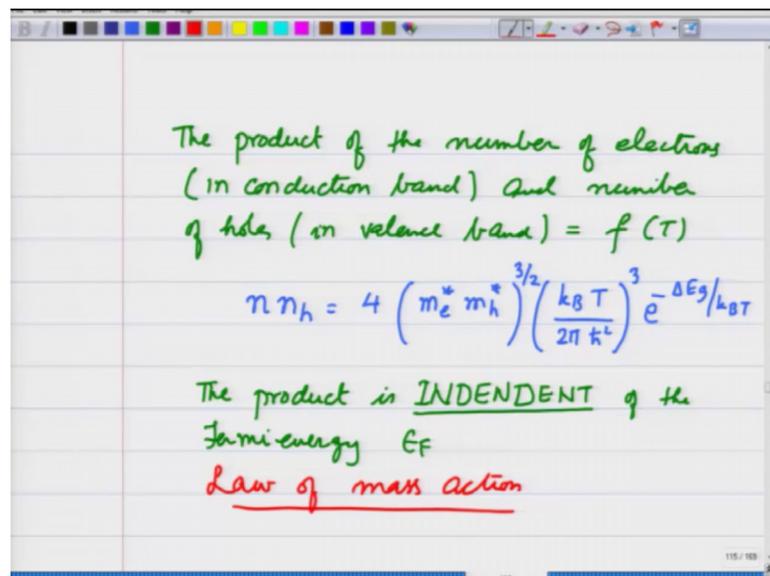


Introduction to Solid State Physics
Prof. Manoj K. Harbola
Prof. Satyajit Banerjee
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 72
Donor and acceptor energy levels in a semiconductor

In the previous lecture, I derived the formula for the concentration of electrons and holes in intrinsic semiconductors by intrinsic we mean those where there are no impurities so that the number of holes and number of electrons is the same and we also obtained.

(Refer Slide Time: 00:39)



The product of the number of electrons (in conduction band) and number of holes (in valence band) = $f(T)$

$$n n_h = 4 \left(m_e^* m_h^* \right)^{3/2} \left(\frac{k_B T}{2\pi \hbar^2} \right)^3 e^{-\Delta E_g / k_B T}$$

The product is INDEPENDENT of the Fermi-energy E_F

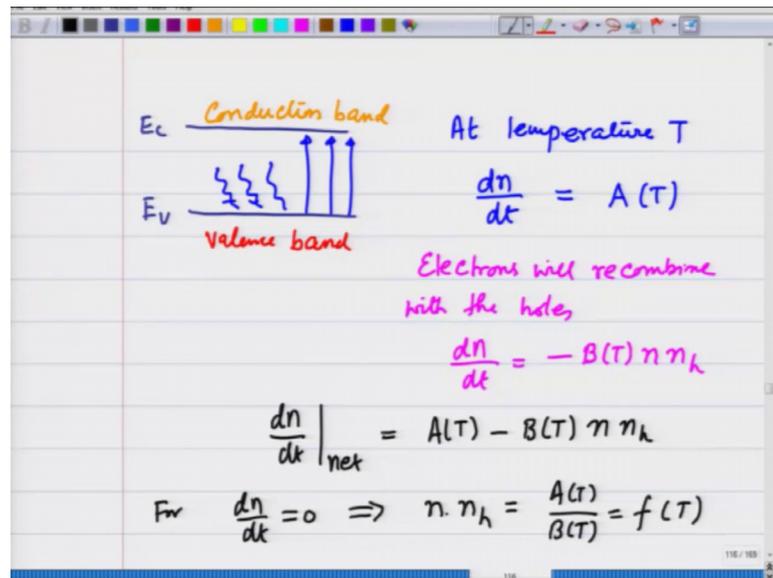
Law of mass action

And, this is the result I need from this lecture onwards that the product of the number of electrons obviously, in conduction band and number of holes in the valence band is dependent only on temperature. So, the result that we are obtained was that number of electrons n times the number of holes h is equal to $4 m_e^* m_h^* \times k_B T / 2\pi \hbar^2$ raised to 3 $e^{-\Delta E_g / k_B T}$ and there is a $3/2$ in front of m_e^* and m_h^* .

Now, notice that the product is independent of the Fermi energy E_F . This is related to something called the law of mass action through which we will now understand that this product should be dependent only on temperature. So, irrespective of where the

Fermi level is; that means, whether I dope the semiconductor whether I put some impurities by and as a consequence shift is Fermi level or do not dope it, this product is going to remain the same and this will come in handy. So, let me just get this result from the law of mass action also.

(Refer Slide Time: 03:01)



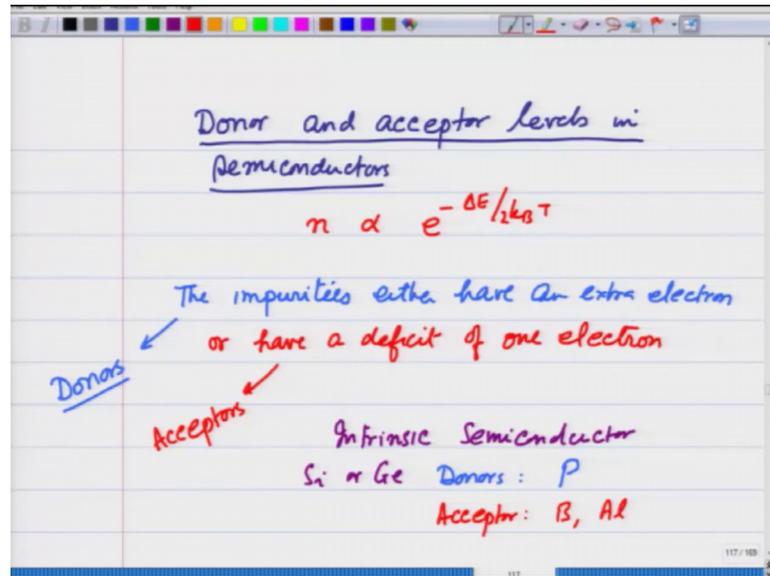
So, which says that if I have these two levels E_c E_v , the conduction band on top and the valence band at the bottom and at temperature T the rate of production of number of electrons in the conduction band would be a function of temperature alone because higher the temperature more the photons out here that will excite these electrons to the conduction band.

On the other hand, the electrons will also recombine. Electrons will recombine with the holes and therefore, there will be a reduction because of that in the number of electrons dn by dt because of that will be equal to minus to show that the number of electrons is decreasing some function of temperature times number of electrons times number of holes. If the product of number of electrons and number of holes is larger recombination rate we go up.

And, therefore, dn by dt net will be equal to $A(T)$ minus $B(T) n$ times n_h and at steady state when everything is time independent for steady state dn by dt will be 0 and this implies in this case n times n_h will be $A(T)$ over $B(T)$ which is a function of temperature alone. So, no matter where these holes and electrons are being generated from, whether

from some impurity levels whether from some valence band the product of n times n holes is going to remain unchanged and it will depend only on temperature. So, I have argued from the law of mass action also this should be so and we indeed see it.

(Refer Slide Time: 05:46)



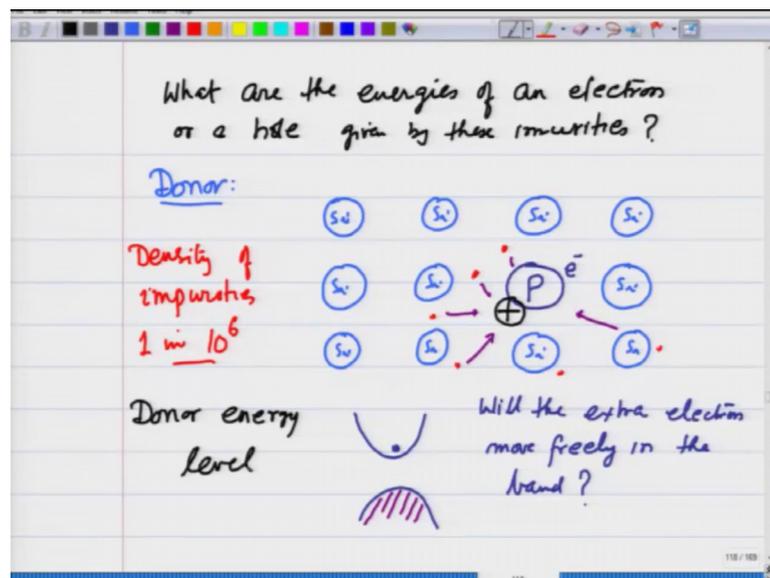
Now, what I am going to talk about in this lecture is the donor and acceptor levels in semiconductors. So, let me explain what that is. You see the usefulness of semiconductors arises from the fact that one by temperature variation the charge carriers can vary quite a lot. They actually change exponentially. The dependence of the number of electrons is proportional to e raise to minus ΔE over $k_B T$ $2 k_B T$. And, therefore, as you change the temperature things change exponentially.

So, it is very sensitive to the temperature you can use it to make thermometers you can make use it to sense the change in temperature anywhere. The other thing is that you can change the conductivity also or the second thing is that you can also change the number of charge carriers by putting some impurities in the system and those are known as donors or acceptors depending on what they do.

So, these impurities are the impurities either have an extra electron or they have a deficit of one electron depending on what they are, you know they are called either donor or acceptors. So, if you have an extra electron. So, for example, these will be called donors and these will be called acceptors and rightly so because a impurity that has an extra electron gives it to the system and that electron goes into the conduction band.

An impurity that has a deficit of one electron accepts one electron from the system and therefore, creates a hole at the top of the valence band. The typical examples for silicon based device are so, if I take intrinsic semiconductor, a silicon or germanium; then the donors would be for example, phosphorus there is 15 electrons and an acceptor would be boron or aluminum. So, they act as creator of electrons or creator of holes and therefore, I can have two kinds of semiconductors.

(Refer Slide Time: 09:05)



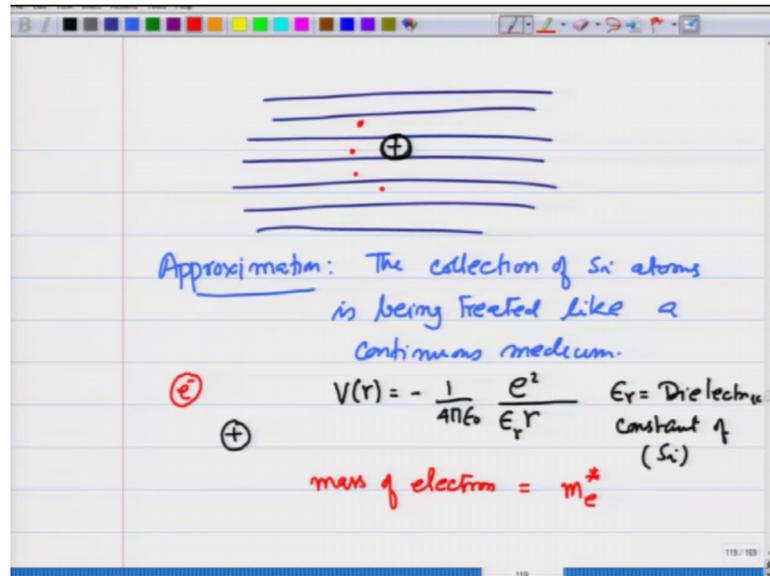
So, first thing that we want to know is: what are the energies of an electron or a hole given by these impurities. So, let us first look at the donor. So, look at this. Suppose, I have these silicon atoms and I put a donor atom here, phosphorus and typically the density of these impurities donors or acceptors are 1 in a million because you do not want to change things much, but you still want to have the control on the charge density.

So, now this has an extra electron this has an extra electron the rest of the things roughly remain the same. So, what would happen to this electron is it will go into the conduction band because the band. So, we are talking about 0 temperature. So, band valence band is completely filled and this extra electron comes here. We let move around freely in the band that is the next question, right.

Will the extra electron move freely in the band? That is the next question and the answer is no. What happens is that when this extra electron comes out, so, let me show you this by red color it is moving around, but then it is left a positive charge on this impurity ion

phosphorous and this positive charge is going to attract it. So, it is going to be bound to this although it will move around, but it will state remain bound and that binding is gives you something called the donor energy level. So, let us see how do I calculate this.

(Refer Slide Time: 12:14)



Since, the impurity is low you know I have all these silicon atoms and then in the middle I have this positive charge and an electron in this medium. I am going to make an approximation that the collection of Si atoms is being treated like a continuous medium. So, in that sense I am not taking the graininess of this into account.

So, the problem I have now is that an electron is moving around a positive charge center in a potential $V(r)$ which is $1/4\pi\epsilon_0\epsilon_r \cdot e^2/r$ the minus sign in front, but there is a ϵ_r here ϵ_r is the dielectric constant of Si or the host system and the mass of electron because it is moving in this band is it is effective mass.

So, the problem I have to calculate this energy of this electron bound to the system is that it is an electron moving around a unit positive charge with the potential being given as $e^2/4\pi\epsilon_0\epsilon_r r$ and the kinetic energy is $\frac{h^2 k^2}{2m_e^*}$.

(Refer Slide Time: 14:25)

The image shows a whiteboard with the following handwritten equations:

$$\mathcal{H} = -\frac{\hbar^2}{2m_e^*} \nabla^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{\epsilon_r r}$$

$$E_n = -\frac{13.6 \text{ eV}}{\epsilon_r^2} \times \left(\frac{m_e^*}{m_e}\right) \frac{1}{n^2}$$

$$r_n = \left(\frac{m_e \epsilon_r}{m_e^*}\right) a_0 n^2$$

Si / $\epsilon_r \sim 10$
 $m_e^* \sim 0.1 m_e$

$$E_n \sim -\frac{13.6}{1000} \sim \text{meV}$$

$$r_n \sim 100 a_0$$

} Ground state

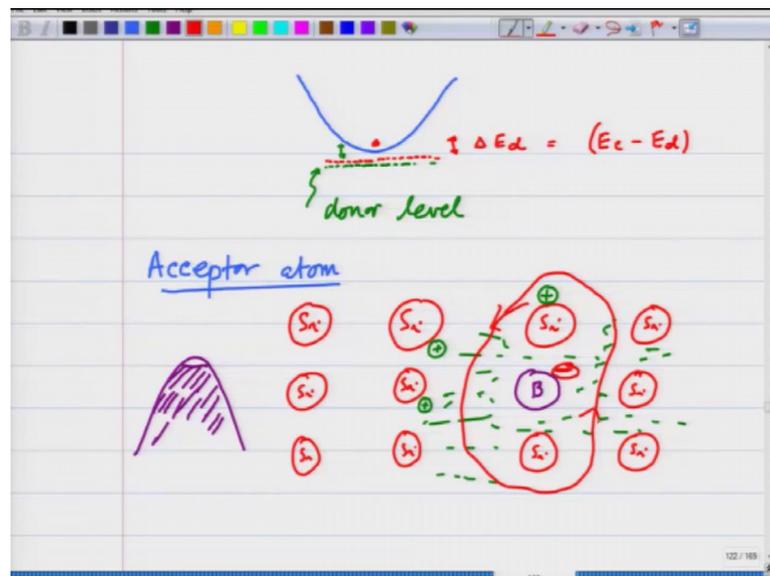
So, the Hamiltonian of this electron in this medium is minus \hbar^2 over $2m_e^*$ times the Laplacian minus 1 over $4\pi\epsilon_0\epsilon_r$ times e^2 over r . This is exactly the hydrogen atom problem, you could even solve for the energy levels using Bohr model and therefore, the energy that you are going to get E_n is going to be minus 13.6 eV and this will be scaled by this dielectric constant ϵ_r square times m_e^* divided by m_e , right because I replaced the mass of the electron by mass of the effective mass of the electron and therefore, that ratio comes in the energy.

And, energy also depends on the electronic charge as e^4 and therefore, I get a power of ϵ_r^2 in the denominator and this and an $1/n^2$. Now, radius of this electron is also going to be equal to $m_e \epsilon_r / m_e^*$ times the Bohr radius n^2 . So, you see the radius goes bigger by a factor of m_e by m_e^* times the dielectric constant, that makes sense because now the potential is weaker because of the dielectric constant mass is lighter so, electron moves in a much larger radius if mass is heavier it will be less.

So, you can see now for typically for say silicon or such materials ϵ_r is of the order of 10 and m_e^* is of the order of $0.1 m_e$ and therefore, you get E_n of the order of minus 13.6 divided by 1000 see of the order of meV and r_n is of the order of $100 a_0$ for the ground state.

So, the energy of the donor level is really small is seen of the order of $m e v$ and radius is become roughly 100 times the hydrogen atom Bohr radius. So, you can see that our continuum model that is the approximation we made that I can treat the silicon atoms distributed as a continuous medium is justified when I look at r_n because it comes out to be about 100 atomic radius. So, it will carry about a 1000 silicon atoms which is like bulk. So, in a self consistent way I have justified using the approximation for treating silicon as continuous and energy is reduced.

(Refer Slide Time: 17:43)



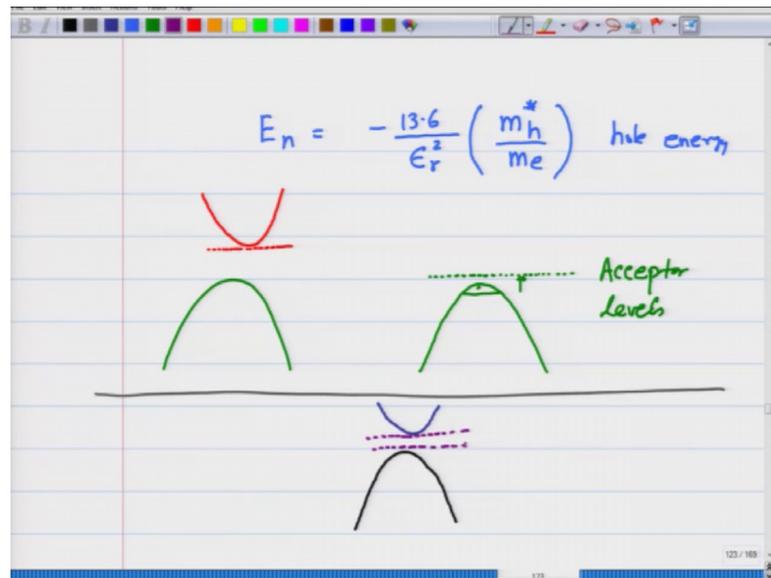
So, what is happening now, therefore, is that this electron which I have would have moved in the conduction band here freely instead of moving there it is bound to this donor atom by roughly an energy ΔE_d which is equal to E_c minus E_d level and there would be other levels lower than this $1/n^2$, but we are neglecting those this is the main level. This is known as donor level.

Similarly, if I have an acceptor atom; an acceptor atom what would happen is I have these silicon atoms and then let us say I have a boron which has 5 electrons then again I had the silicon. This has one less electron. So, therefore, when I look at this valence band, it will be less by 1 electron. So, do the rest of the electrons move around? Yes, they will move around. You know the rest of the electrons are or moving around.

But, remember our picture of the hole when rest of the electrons are moving around if they come around boron. They have left a positive charge behind on the silicon either on

this silicon or on this silicon or on this silicon and this is become a negative ion; boron would have become a negative ion. So, in terms of hole I would make a picture that this hole out here is moving around, the negative center. It is exactly like it hydrogen atom problem except now the roles of positive and negative charges are reversed.

(Refer Slide Time: 20:07)

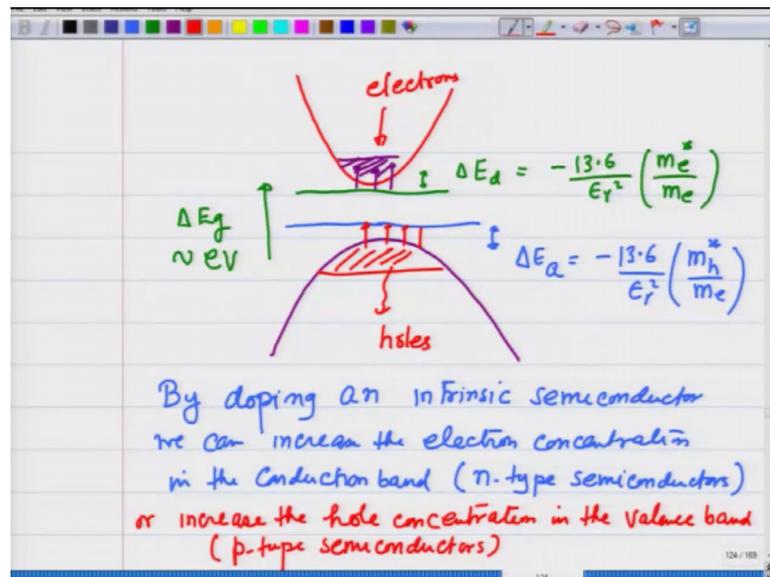


Therefore the energy of the hole would also be given as minus 13.6 over epsilon relative square which is same for both electrons and holes and I will have m_h^* over m_e instead of m_e^* over m_e for the whole energy. Again, recall our discussion from hole if this is the valence band. The whole band goes the other way minus E and what we are saying is below this are these acceptor levels.

In terms of only electron picture I would make this valence band and these are the acceptor levels. This hole is in the valence band and it requires some energy before the hole can go to the acceptor level. So, these are known as acceptor levels. We make these pictures of course, in the bands for electrons. So, therefore, they are made like this.

So, the complete picture that emerges if a semi conductor has both donor and the acceptor levels, let me draw a line here to differentiate. I am going to have a donor level here and an acceptor level here and let me make this picture better on the next slide.

(Refer Slide Time: 21:48)

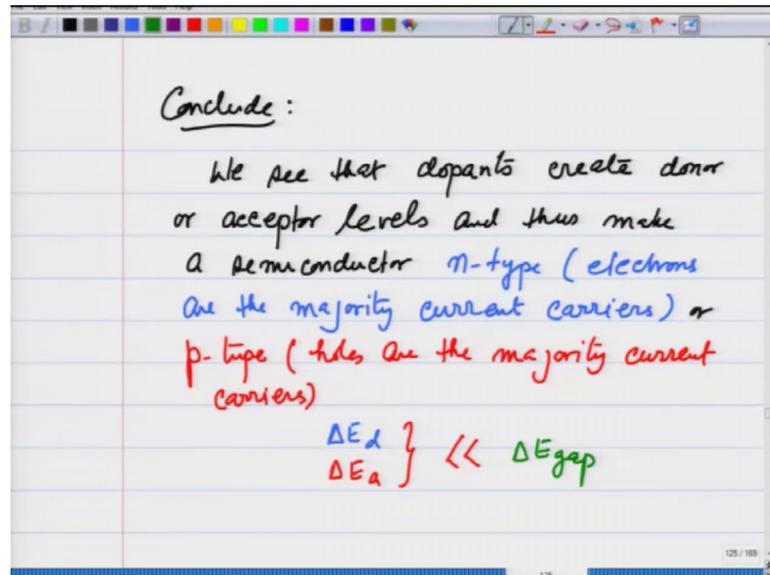


Here, I have the conduction band, here I have the valence band and I would have the acceptor level here and the donor level here. This ΔE_d is equal to minus 13.6 over epsilon relative square m_e^* over free electron mass and ΔE_a is minus 13.6 over epsilon r square m_h^* over m_e .

Now, you see instead of an electron that had to cross this barrier of ΔE_g which is of the order of electron volts now, an electron from an acceptor level or a donor level can easily cross over to the conduction band. So, these will populate the conduction band and in the acceptor level in the these electrons can go up here the acceptor level and create lot of holes here.

So, by doping and intrinsic semiconductor we can increase the electron concentration in the conduction band. If the electron concentration goes up because n times n hole is constant it depends only on the temperature the hole concentration will come down and these are known as n-type semiconductors or increase the hole concentration in the valence band what are known as p-type semiconductors and if I increase the number of holes number of electrons in the conduction band will go down. So, majority carriers in n type are going to be electrons and majority carriers in p-type semiconductors are going to be holes.

(Refer Slide Time: 24:55)



So, to conclude this lecture we see that dopants create donor or acceptor levels in a otherwise intrinsic semiconductor. So, if you put them they create these levels and therefore, make a semiconductor either n-type. So, in this electrons are the majority current carriers or p-type holes are the majority current carriers. And, this increase in the number of electrons on number of holes comes because ΔE_{donor} or $\Delta E_{\text{acceptor}}$ is much much less than ΔE_{gap} . So, it gives us a fantastic control on the material as to what kind I want to make it and how I wish to use it.

In the next lecture, we will calculate the concentrations of electrons and holes and see how they affect the properties of semiconductors when they are doped.

Thank you.