Introduction to Quantum Mechanics

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Lecture – 04
Black Body Radiation IV- Stephen's Boltzman law

Before we start our analysis on experimental facts, let us just summarize what we have learnt so far.

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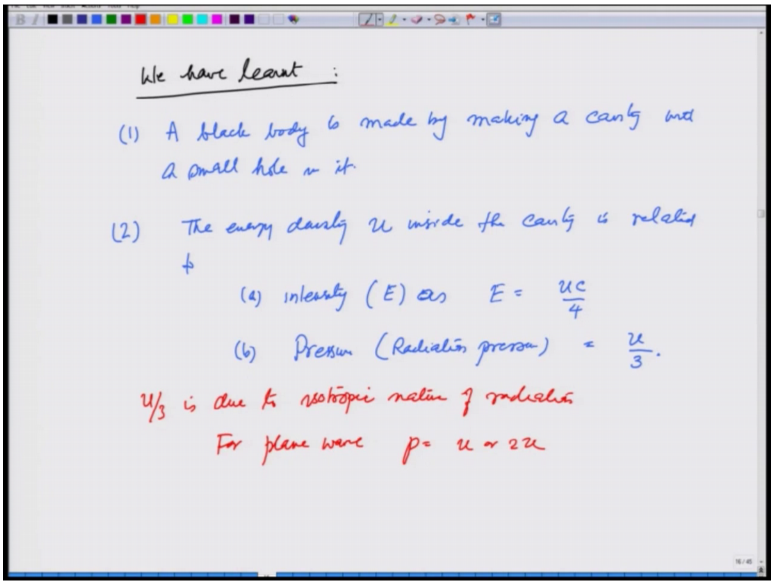

So we have learnt: 1, a black body is made by making a cavity whether small hole in it, I need this all because the radiation should be coming out. Number 2; the energy density u inside the cavity is related to a; intensity or emissivity; E as equal to as E equal u c by 4 and b; pressure which allows a call radiation pressure is equal to u by 3. This u by 3 is different from the pressure applied by plane waves.

So, let me just point out u by 3 is due to isotropic nature of radiation for plane waves pressure is u or 2 u depending on whether the radiation gets absorbed or it gets reflected, but now the answer you get is u by 3.Now, let us go further.

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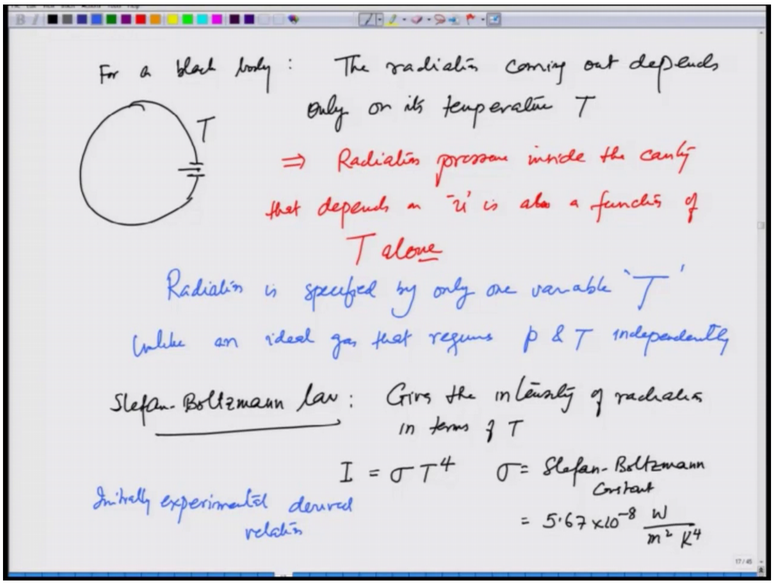

So, for a black body which I will again make a cavity with a hole, the radiation coming out depends only on its temperature, right, nothing else only on temperature T and therefore, radiation pressure inside the cavity that depends on u is also a function of T alone; unlike the gas or ideal gas where you see that you need 2 parameters to specify the state of gas, pressure volume, pressure temperature, some just 2 parameters; in radiation, it is only one parameter. So, radiation is specified by only one variable called the temperature T unlike; let say an ideal gas that requires p and T independently.

Here p and T are not independent, p depends on T alone or p depends in T. So, everything is determined by T and consequently there is something call the Stefan Boltzmann law that gives the intensity of radiation in terms of T and it says that the intensity of radiation coming out is equal to sigma T raise to 4, where sigma is known as Stefan Boltzmann constant as value is 5.67 times 10 raise to minus 8 watts per meter square; K raise to 4.

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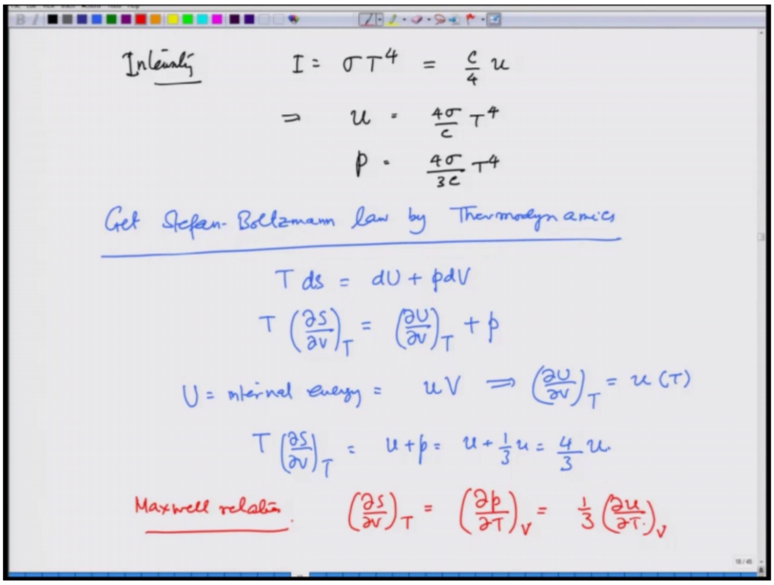

So, this is an initially experimentally derived relation. However, as you just seen that I can define pressure for radiation inside, I can define in terms of temperature. So, I can use thermodynamics also to derive this law which was done by Boltzmann and that is why it is known as Stefan Boltzmann law and let us now do that, but before that the intensity; I is given as sigma T raise to 4 and we have seen earlier that this is nothing but c by 4 u. And therefore, the radiation density inside a cavity is going to be given by 4 sigma by c T raise to 4 and pressure inside is going to be given as 4 sigma by 3 c T raise to 4. This, you already derived the formula for pressure. Now let us get Stefan Boltzmann law by thermodynamics.

How do we do that? We apply the first law which says T ds is equal to d U plus p d V, alright and therefore, I am going to have T ds by d V at constant temperature is equal to d U by d V at constant temperature plus p now u is the internal energy which is equal to the energy density times V and this implies that d U by d V at constant temperature is going to be U because U depends only on T. This is very important, U does not depend on V or anything, it depends only on T and that is the observation about black body radiation. So, you going to have T ds by d V, T is equal to U plus p which is nothing but U plus 1 third U which is 4 thirds U, right. Now there is something called the Maxwell relation in thermodynamics, we says that del s by del V at T is nothing but dp by d T at constant volume which in this case I can write as one third d U by d T at constant volume.

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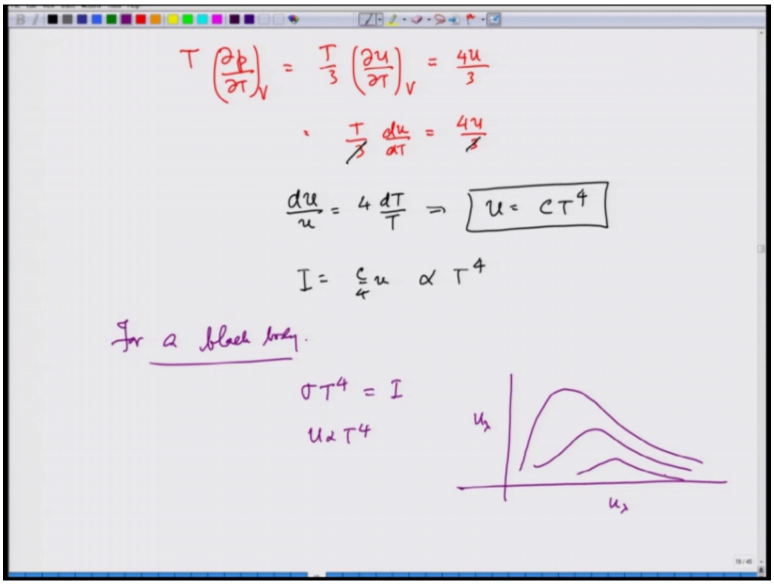

So, what we have now is that t d p by d T at constant volume is equal to T over 3 d U by d T at constant volume which is equal to u 4 u over 3. In fact, this partial derivative I can even write as 3 by 3 d u by d T because it depends only on the temperature this 4 u by 3. This 3 cancels and I get d u over u is equal to 4 d T by T implies u equals c T raise to 4 and that is energy density in I which is c is some constant, I is c by 4 u. So, this is also proportional to t raise to 4 and that is the Stefan Boltzmann law. So, what we have gotten is that for a black body sigma T raise to 4 equals I and u is related. So, u is proportional to T raise to 4, we have gotten some theoretical result relating the internal energy to the temperature.

What is more interesting; however, is how is this energy distributed on the spectral density u lambda versus lambda for different temperatures and that is what bothered scientists for a long time. The first (Refer Time: 10:36) analysis in this direction, again using thermodynamics was done by Wien and that will be our next job to do.

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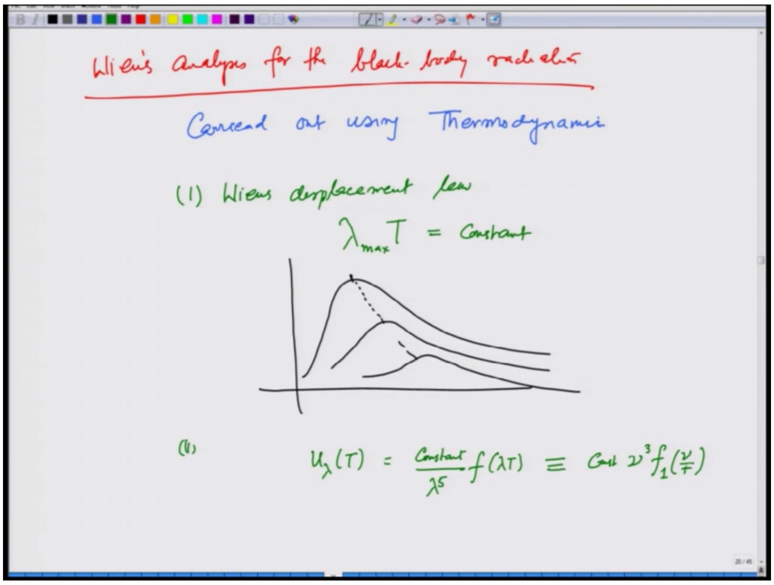

So, Wien's analysis for a black body radiation was carried out using thermodynamics and he got 2 results which are very very important and that actually open the way for the research. First result is known as Wien's displacement law which says that lambda max where the spectral density is maximum times T is a constant, you have seen that the temperature; the curves look like this. So, this maximum shift in such a way as temperature goes down. So, that it goes to higher and higher lambda in such a manner that lambda max time T becomes a constant.

And a second result is that u lambda as a function of temperature varies as some constant divided by lambda raise to 5 times a function of the product lambda and T. So, lambda and T appear in this function as a product, they do not appear in any other form which is also similar to a constant times nu the frequency cube some other function, let us call it f 1 nu over T.

So, these are the two results that we have obtained through thermodynamics, and their derivation is going to be subject of our lecture in the next one.