Introduction to Quantum Mechanics
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Lecture - 01
Introduction to the correspondence principle

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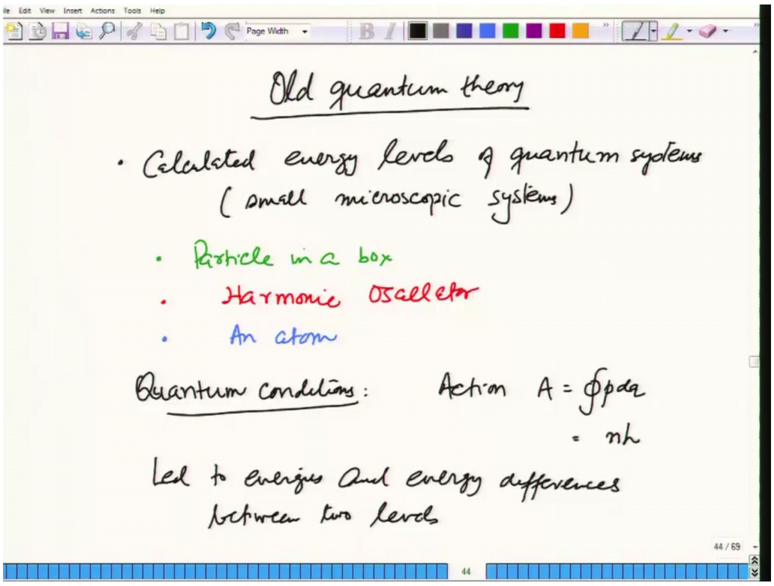

Let me just review what we have done so far and what we have done is the old quantum theory in which you calculate it energy levels of quantum systems and when I say quantum systems, I mean small microscopic systems in particular, what I did was particle in a box I also did harmonic oscillator and I also did an atom.

And what we have used here are some thing called the quantum conditions that tell you that the action a which is defined over a period pdq, I am just using dq for generalized coordinate is n h and that gave you the energies or energy differences. So, this let to energies and energy differences between 2 levels and once you know the energy difference between the 2 levels.

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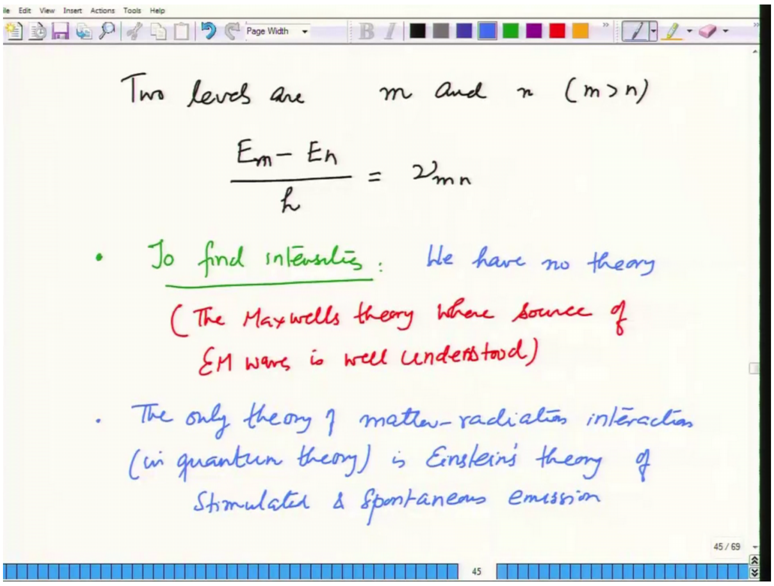

So, suppose two levels are m and n, let us say m is greater than n, then E m minus E n over h gave you the frequency m n. So, we could calculate the frequencies of radiation, the other thing we did was to calculate to find intensities of these radiations, we have no recipe, no theory. So far, the only thing that we have is this is in contrast with is the Maxwell's theory where source of E m waves is well understood.

The only thing that we have is Einstein's theory of a and b coefficient. So, the only theory of matter radiation interaction in quantum theory is Einstein's theory of stimulated and spontaneous emission in this really all we know is there exists a coefficient a there exists a coefficient b. However, we do not know how to calculate it.

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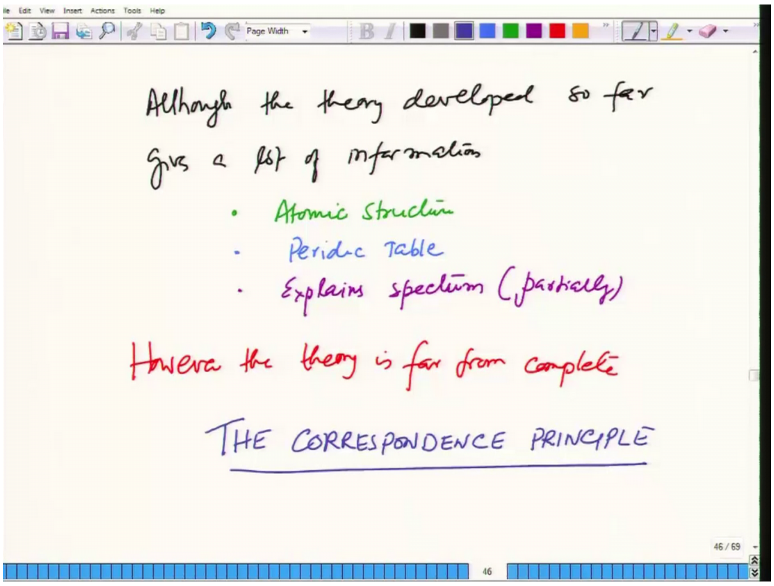

So, although the theory developed so far gives a lot of information and what all have we learned so far, it gives you atomic structure, it gives you periodic table explains spectrum partially because it gives you only the position, right and many other things still the theory is not complete; however, the theory is far from complete. So, we do not really know; how to calculate intensities, what is the structure inside, what is the quantum structure and to understand all that one had to make connections with other theories particularly classical theory which is well known and that is where the correspondence principle came in.

So, what we are going to discuss today is the correspondence principle that made a connection of what happens at quantum level and how quantum theory goes over to classical theory and the way it helped, it is a really important principle because the way it helps is that once we knew the classical results and how quantum theory should approach the classical result, one could sort of backtrack and extrapolate to the quantum regime what would happen there. So, it is a very important principle and I am going to spend this lecture and the next lecture discussing this and also see how one could look at the structure of classical theory and from that guess what should happen in quantum theory.

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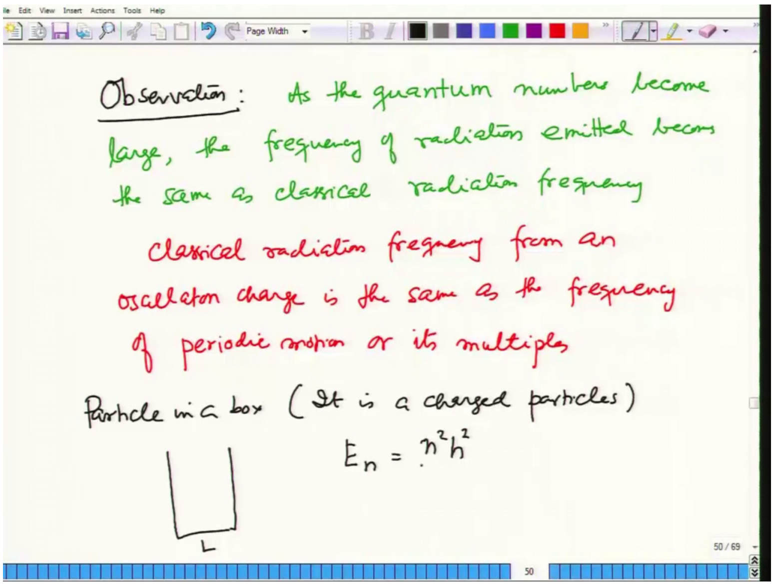

So, to understand correspondence principle let us start with an observation and this observation is going to be as the quantum numbers become large, the frequency of radiation emitted becomes the same as classical radiation frequency and let me also remind you that the classical radiation frequency from an oscillating charge is the same as the frequency of periodic motion notice that I am using word periodic motion because oscillation does not mean a simple harmonic oscillation, it could be any periodic motion or its multiples, what are written about classical result is known from classical theory. So, this is well known and what the observation was that as quantum numbers become large the frequency of radiation emitted due to quantum process that is by jumping of an electron from one level to the other becomes the same as the frequency in classical motion let us see that. So, I will take the example of particle.

In a box and; obviously, we are going to assume it is a charged particle because only charged particles radiate when accelerating. So, that is beside the point, but particle in a box of length L has energy levels E n is equal to h square n square divided by 2 m L square that is the energy.

Now, let us see what happens when this particle makes a jump from an upper level to a lower level.

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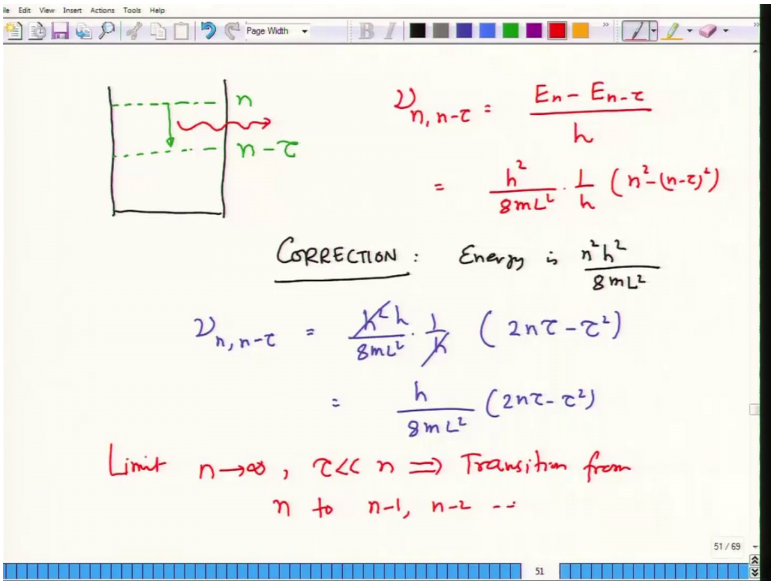

So, I am taking this particle in a box at an upper level n lower level; lower level is n minus tau, they both are integers and when this jump takes place, radiation comes out and from Bohr's theory, I know that this frequency nu when it is jumping from nth level to n minus tau th level is going to be E n minus E n minus tau divided by h which is going to be h square over 8 m L square 1 over h n square minus n minus tau square, a correction; the energy has h square over 8 m L square times n square. So, the earlier slide, I had in 2 m L square.

So, let us proceed then and therefore, nu n n minus tau is equal to h square over 8 m L square one over h 2 n tau minus tau square, right. This h cancels with one of the hs and I get h over 8 m L square times 2 n tau minus tau square. Now let us take the limit of n tending to infinity that is n very large and tau much much much less than n.

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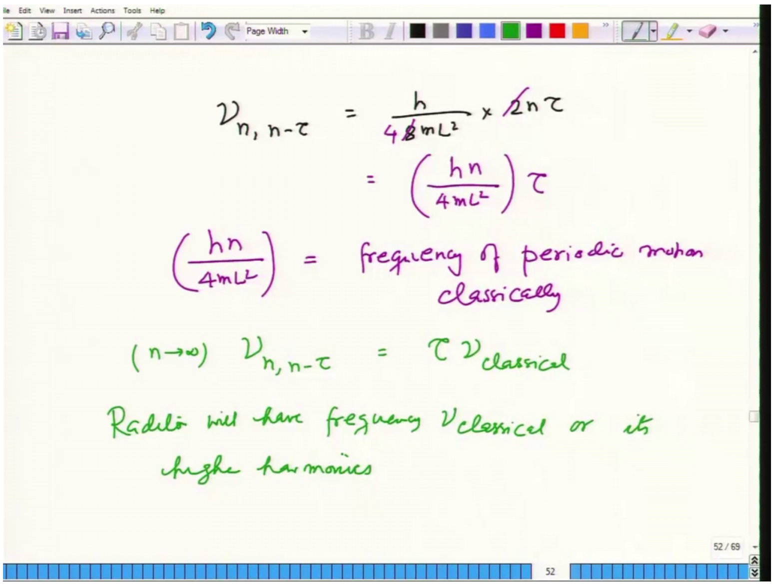

So, transition this implies transition from nth level 2 n minus 1 n minus 2 and so on, alright and then I have nu n to n minus tau is equal to h over 8 m L square times 2 n tau because I can neglect tau square and this comes out to be this 2 cancels, it gives you 4 h over 4 m L square times n time tau.

Now, I am going to show that h n over 4 m L square is the frequency of periodic motion classically. So, the radiation which is coming out in the limit of n tending to infinity is equal to tau times nu classical. So, the radiation will have frequency nu classical or its higher harmonics; let us now see that this is true.

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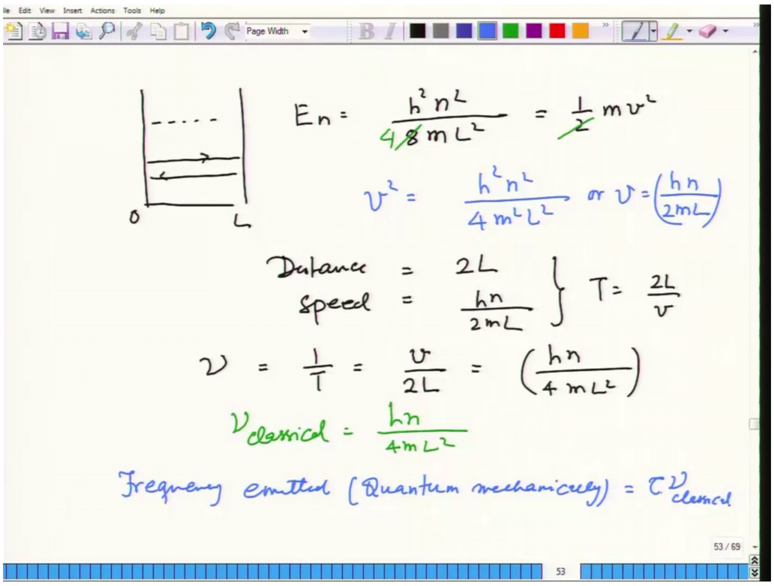

So, for this particle in a box where E n is h square n square over 8 m L square is purely the kinetic energy of m v square and if I cancels 2 with this it gives me 4. So, therefore, I get v square is equal to h square n square over 4 m square L square or v the speed equals h and over 2 m L.

The time period for 1 complete motion is going to be when it goes to the right and comes back 0 and L therefore, the distance for a time period is 2 L speed is h n over 2 m L frequency nu is going to be one over T which is nothing but v over 2 L. This is this gives me T equals 2 L over v. So, this is v over 2 L which comes out to be h n over 4 m L square this is a classical frequency. So, I can say that the classical frequency nu classical is h n over 4 m L square and if you see it what happened earlier. So, frequency emitted quantum mechanically is nothing but tau a tau is integer nu classical.

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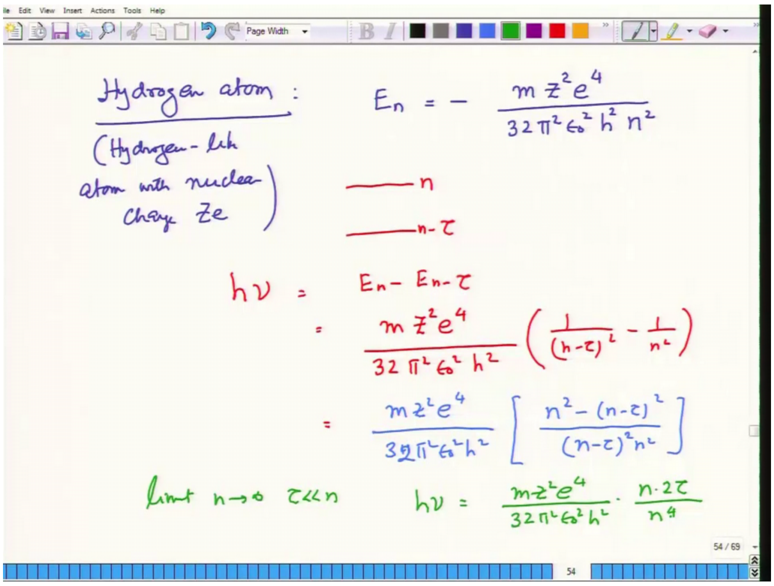

So, it is as if when the quantum system is radiating in this large n limit, it is radiating either the classical frequency or its harmonics we can understand this result and I will comment on it little later through Fourier series let me give you another famous example it is a hydrogen atom for a hydrogen atom, the energy of the nth level is given as minus m z square e raise to 4 over 32 pi square epsilon 0 square h square n square, there is the energy of a hydrogen atom and since I am taking z. So, let me call this hydrogen like atom with nuclear charge z e.

So, when in a hydrogen atom the transition is taking place from n to n minus tau then the energy radiated h nu comes out to be E n minus E n minus tau which will be m where m is the mass of the electron z square is going to be m z square e raise to 4 divided by 32 pi square epsilon 0 square h square times 1 over n minus tau square minus 1 over n square and this comes out to be m z square e raise to 4 divided by 32 pi square epsilon 0 square h square inside, I am going to get n square minus n minus tau square divided by n minus tau square n square.

And let me now take limit n tending to infinity and tau much much much less than n, then you will see that h nu comes out to be m z square e raise to 4 divided by 32 pi square epsilon 0 square h square and you get n times 2 tau divided by n raise to 4 which is nothing but.

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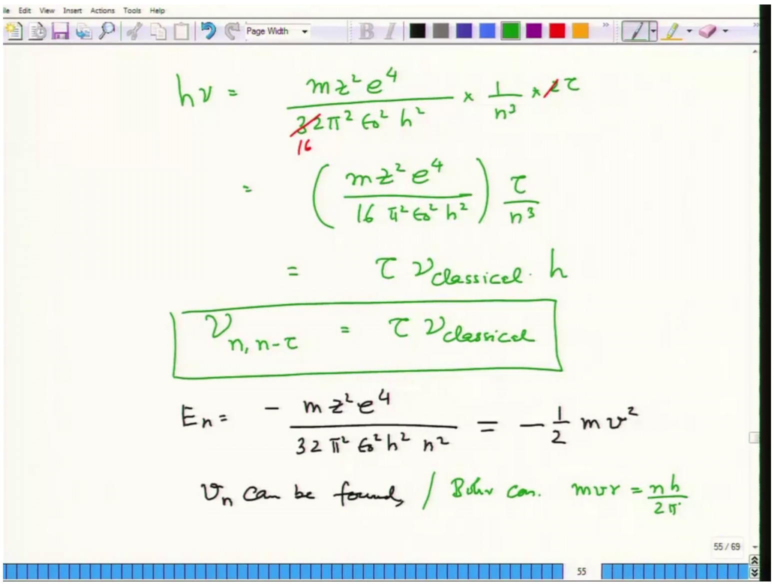

So, h nu comes out to be m z square e raise to 4 divided by 32 pi square epsilon 0 square h square times 1 over n cube times 2 tau which is m z square e raise to 4 over 32 pi square epsilon 0 square h square or 32 will cancel, let me cancel this, this becomes 16. So, let me write this as 16 pi square tau over n cubed and I am claiming that this is again tau times nu classical times h. So, that nu when it is making a transition from n 1 to n minus tau is nothing but tau times nu classical.

I leave it as an exercise for you to show that this is nu classical and this is how you proceed E n is given to be minus m z square e raise to 4 over 32 pi square epsilon 0 square h square n square and this is also equal to minus 1 half m v square from this, you can find what this speed of the electron in the nth orbit is. So, v n can be found once you have v n you also know Bohr condition gives you m v r equals n h over 2 pi. So, you can find implies this implies that r n can be found and v n by r n divided by 2 pi gives you nu classical. So, that what come.

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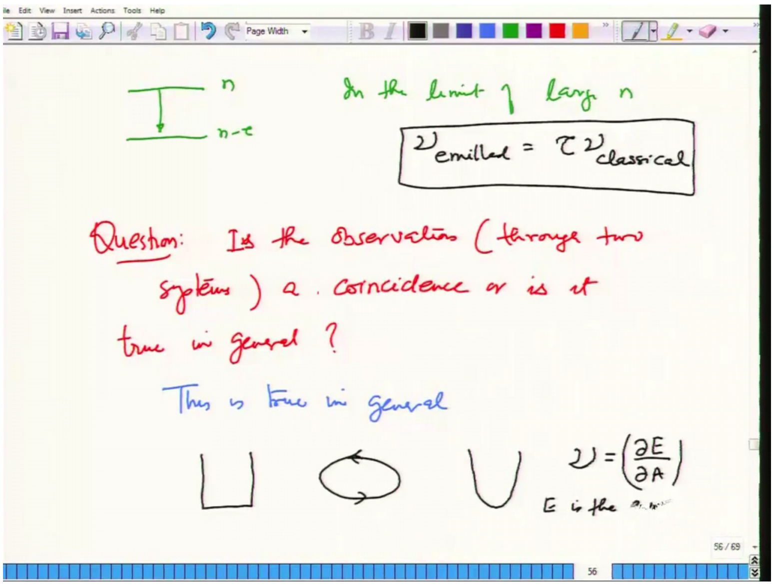

So, what we have learned through these 2 examples is that if there is a transition from nth level to n minus tau th tau is also an integer then in the limit of large n, right, the nu emission emitted is equal to a harmonic of nu classical. So, this is the observation is this observation a coincidence or does it have something deeper. So, let me raise this question; question is the observation is through only 2 systems a coincidence or is it true in general that is the question and the answer is this is true in general and this comes from the classical theory which says suppose there is a system of you know I have considered 2 systems.

Here I have considered the system of particle in a box, I have considered an atom in which the electron is moving around in a circle, I will also consider a system harmonic oscillator and what I will show you is that in these systems the frequency of oscillation is going to be partial e over partial a where e is the energy and a is the action let me elaborate on that a bit.

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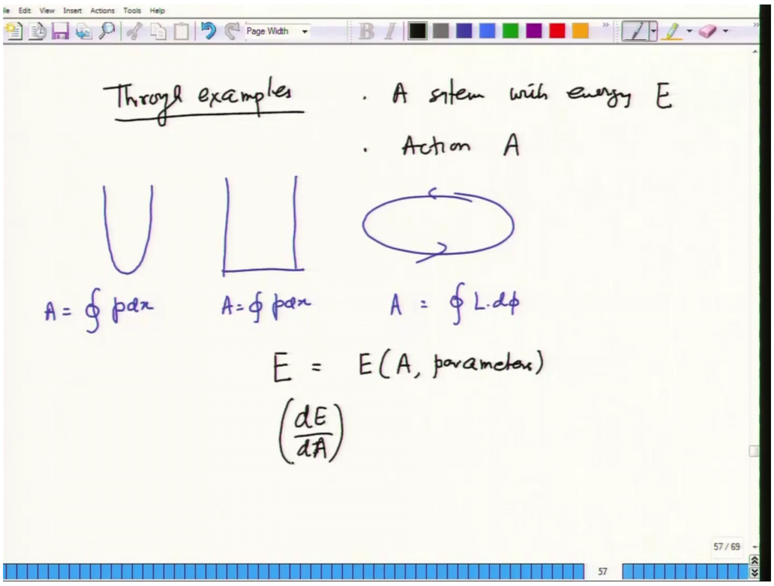

So, I am claiming that I will show it only through examples; more general proof requires little more understanding of classical mechanics which at this time is slightly beyond this course, I will do only through examples that if I take a system with energy E and action A; let me remind you what these are. So, for example, it is a particle moving around in harmonic oscillator action is given by pdx in this particle in a box, again action will be given by integral pdx and for this particle going around in a circle, the action remember was given by the angular momentum d phi.

If you just recall from previous few lectures and action, therefore, is a constant is a number is a number given for this orbit and E the energy for this orbit can be written as a function of this action and some other parameters of the system; obviously, you see that if I change the action value, I will change the orbit. And therefore, the energy is also going to change and what is claimed is that for a particular orbit is going to be the frequency of that orbit classical.