Introduction to Quantum Mechanics
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Lecture - 05
Interaction of atoms with radiation: Einstein's A and B coefficients

So far in the lectures what we have discussed is structure of a system whether it is atomic structure or we discussed the energy levels of a harmonic oscillator.

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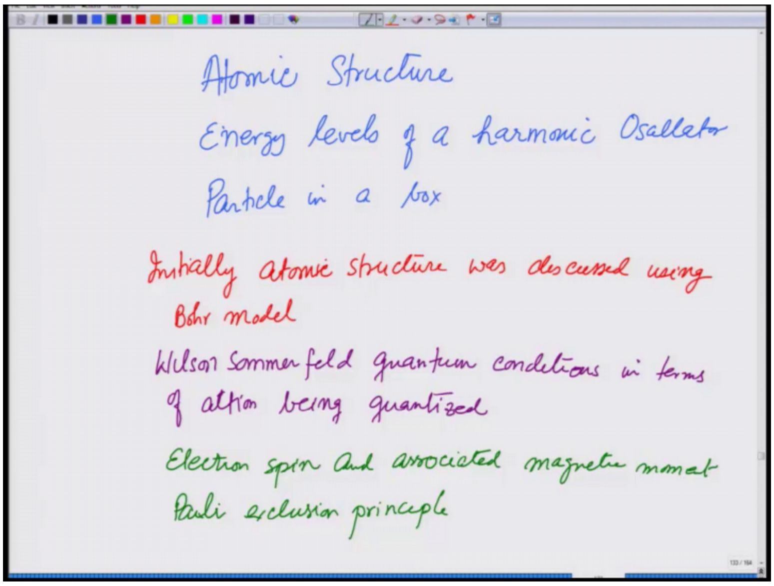

We also discussed particle in a box and initially this was done using Bohr model, initially atomic structure was discussed using Bohr model and it taught us some things it led us towards applying quantum conditions to angular momentum and things like those, it taught us to apply quantum conditions in terms of angular momentum and then this was generalized to Wilson Sommerfeld quantum conditions in terms of action being quantized. And by analyzing atomic structure and the related spectrum we also introduce the idea of electron spin and associated magnetic moment I pointed out at that time that the magnetic moment of spin is twice as much for given angular momentum as the orbital magnetic moment.

And what you learned in the all this and finally, we also said something called the Pauli Exclusion Principle exist that says is that no 2 electrons can have all the quantum numbers the same.

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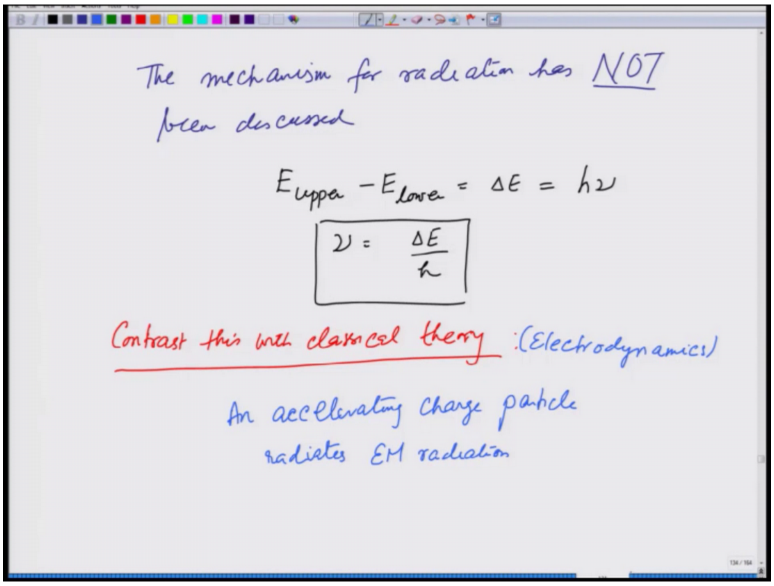

Notice in all this the mechanism for radiation has not been discussed. All that has been said in this is when an electron makes a jump from an upper level let us say an E upper to E lower it gives out an energy delta E which comes out in the form of a photon and therefore, the frequency of the out coming light is delta E over h that is all that has been said. It has not been said how they radiate what makes them radiate and so on, contrast that with classical theory contrast this with classical theory and the radiation mechanism and classical theory is given an electromagnetic theory or electrodynamics where it is shown that an accelerating charged particle radiates electromagnetic radiation.

So, anytime you have a charged particle at accelerate it gives out radiation, but what happens in quantum mechanics what should be the mechanism and that was not clear and the first step in this direction was taken by Einstein.

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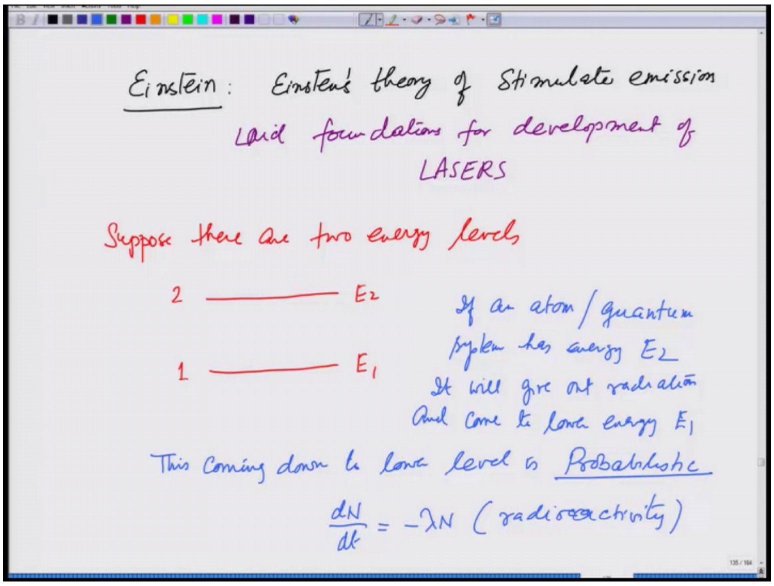

And is now famous Einstein’s theory of stimulated emission this also laid foundations for development of lasers. So, next 2 lectures are going to be devoted to this mechanism of radiation and place introduction to lasers. So, let us see what happens, what Einstein did what Einstein said was suppose there are 2 energy levels let us call them with energy level 1 level 2 with energy E 1 and E 2 right now just consider 2 levels. Then if an atom is in the upper state if an atom or a quantum system has energy E 2 it will give out radiation and come to lower energy E 1, this is what we see classically also something that is excited something it that has more energy gives out energy and comes to lower energy.

So, that is not a big thing, what is big is that this coming down to lower level is probabilistic it is statistical, it has a probability of coming down. So, Einstein introduced the idea of probability in this radiation, let us say it is random. If you recall from your 12th grade you must have been introduced to the phenomena of radioactivity where we say that a nucleus decays in a probabilistic manner where you see that rate of decay is minus lambda N this is from radioactivity radio activity in a similar manner although unrelated what Einstein said is take these 2 energy levels E 1 and E 2 then an atom with energy E 2 will radiate with probability a 2 to 1 delta t in time delta t.

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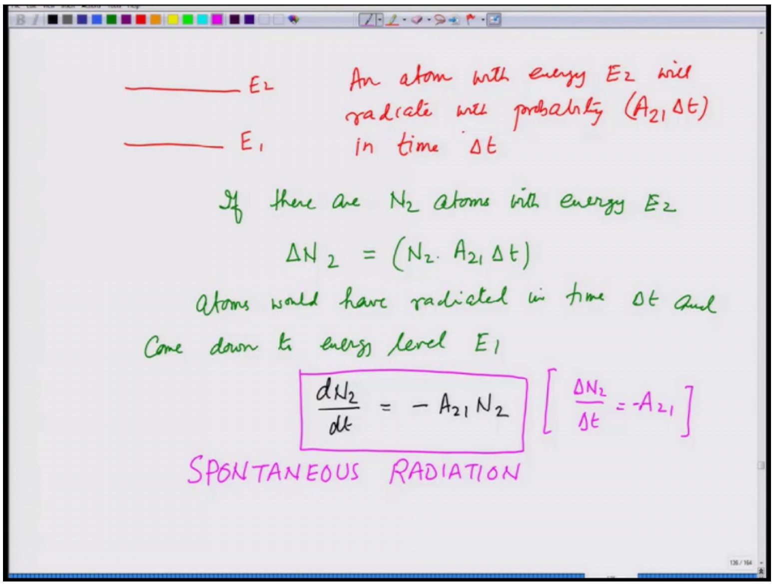

So, the probability of the atom radiating is going to be A 21 delta t and therefore, if there are N 2 atoms with energy E 2 delta N 2 equals N 2 times A 21 delta t atoms would have radiated in time delta t and come down to energy level E 1.

And therefore, you can write that dN 2 by dt is equal to minus A 21 N 2. How I got this is like this I divided delta N 2 by delta t and it came out to be A 21, a minus sign because delta N 2 by delta t is negative and 2 is decreasing and this is known as spontaneous radiation. Nobody told the atom to do it, did it a spontaneously spontaneous means by itself it just though the probability per unit time that group dk to lower energy level and it did. So, this is what would happen. So, all the atoms that have been excited to an upper level would radiate and come down to lower energy level and the rate would be like this. So, if you were to plot it if only this thing happen.

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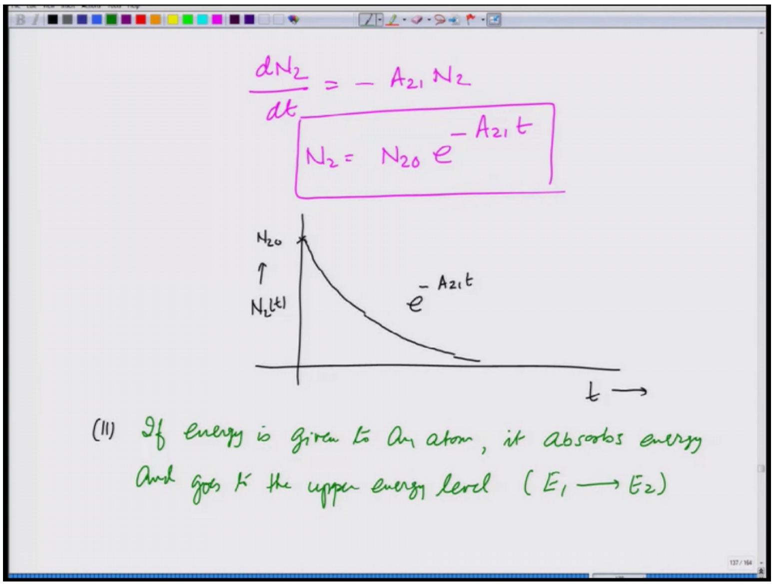

So, I have dN 2 over dt equals minus A 21 N 2 and a solution is going to be N 2 equals N 2 at 0 time E raise to minus A 21 t.

So, if I add atoms only in the upper level at time t equals to 0 this is N 2 0 they would the number would decrease with time exponentially like this this is time and this is N 2 the decay constant is this curve is like E raise to minus A 21 t. At the same time I also know what happens if I pump in energy right. So, let us take the other thing what happens. If energy is given to an atom it absorbs energy and goes to the upper energy level. So, it goes from E 1 to E 2 energy, for this also Einstein said that this is a probabilistic process in which the number of atoms, if that is N 1 going to opera state would be proportional to the energy corresponding to that particular frequency times the coefficient B 12 delta t this will be the probability times N 1 it is exactly what we said earlier.

So, this u nu T times B 2 has the same dimensions as A 21. So, dimensions of A 21 are the same as u nu T B 12 where u nu T is the spectral energy density at frequency nu equals E 2 minus E 1 over h and the atoms are at temperature T.

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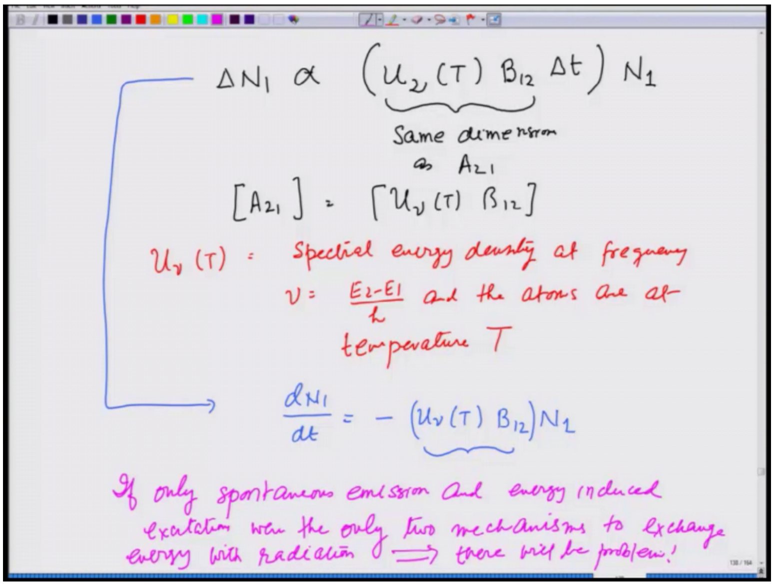

So, at temperature T there exists some energy density and that makes these atoms excite and they go up. So, therefore, I would write this equation in differential form it will become dN 1 over dt would be equal to minus u nu T B 12 times N 1 and this is what I said has the same dimensions as a that you can check because dN 2 dt was equal to A times N 2. If these were the only 2 mechanisms there comes a problem let me show that. So, let me state this first.

If only spontaneous emission and energy induced excitation were the only 2 mechanisms to exchange energy with radiation there will be problem and what is the problem let me state that first.

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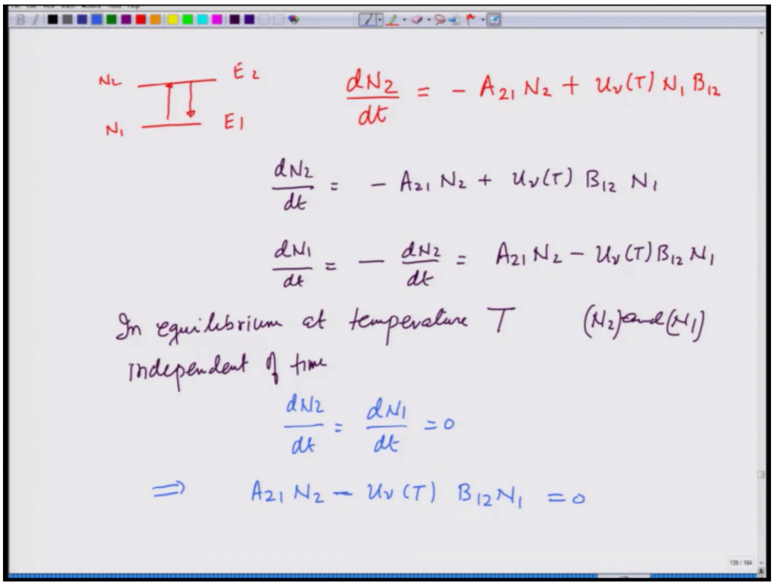

So, again take these 2 energy levels E 2 and E 1 the number initially is N 1 here and N 2 here these are absorbing and going up the upper atoms are coming down. So, if I were to check the rate at which the population of atoms at the upper level is changing this would be equal to minus A 21 N 2 at the same time it is gaining atoms which are coming from ground state to excited state and that will be u nu T N 1 times B 12. So, what do we have we have dN 2 over dt is equal to minus A 21 N 2 plus u nu T B 12 and 1 and dN 1 dt will be minus of dN 2 dt which will be equal to A 21 N 2 minus u nu T B 12 N 1 because N 1 is losing number of atoms which are getting excited and gaining atoms which are coming from the excited state or (Refer Time: 17:14).

Now, in equilibrium at temperature T, so we have this box in which all these atoms are there and they are at equilibrium and temperature T we have N 2 and N 1 independent of time; that means, they do not change because if it is in equilibrium and; that means, what we have is dN 2 over dt is equal to dN 1 over dt is equal to 0 and that gives me this implies A 21 N 2 minus u nu T B 12 N 1 is equal to 0.

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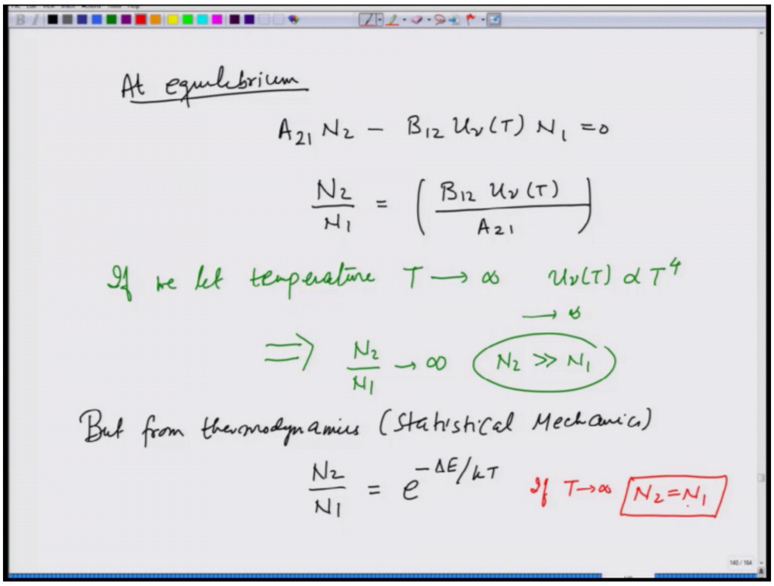

So, what we are writing is that equilibrium we have A 21 N 2 minus B 12 u nu T N 1 is equal to 0 and therefore, N 2 over N 1 is equal to B 12 u nu T over A 21. So, this is the result if these are the only 2 mechanisms.

Now, let us see if we let temperature T go to infinity basically become very large then I know that u nu T is proportional T raise to four and this is also go to infinity and this would imply N 2 over N 1 will go to a very large number N 2 would be much much much greater than N 1. So, you will have large number of atoms sitting in the upper sate, but thermodynamically, but from thermodynamics or more precisely from the statistically mechanics Boltzmann result is that N 2 over N 1 should be equal to E raise to minus delta E over a T. So, if t goes to infinity N 2 goes to N 1 they become equal. So, maximum number of atoms that you can have in the upper level is equal to the number of atoms in the ground level and that is so they will be divided equally and you will have total number of atoms divided by 2 in the 2 levels.

On the other hand look at this result as temperature goes to very large value the number of atoms in the upper state become very very large and that is not acceptable there is a contradiction.

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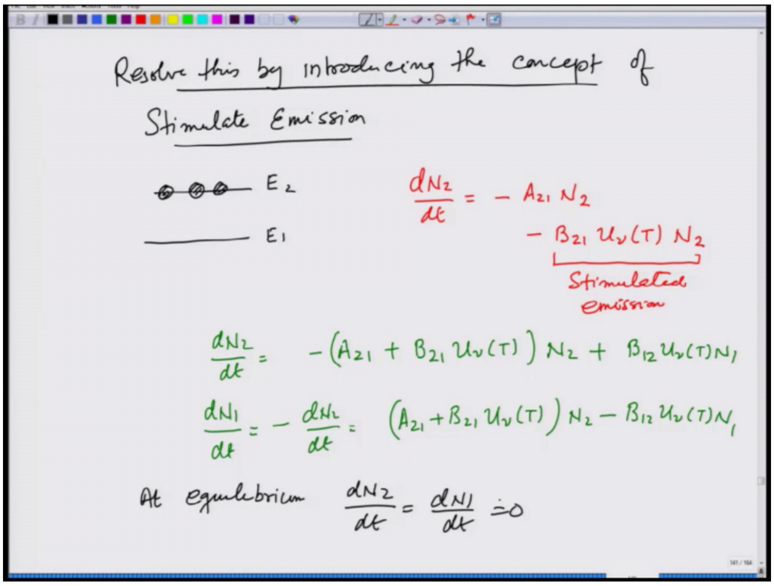

We resolve this, resolve this by introducing the concept of stimulated emission and what that means, is that this radiation which is there if it becomes more and more it will also stimulate atoms to come down from a exited level 2 ground level. So, suppose there are atoms in E 2 and E 1 and there is this atom N 2 sitting here then what you have going to have earlier we said the dN 2 by dt is minus A 21 N 2. What we are also going to say in addition there is a possibility that if there is a radiation of a spectral density u nu T this will also make atoms jump from upper level to lower level.

So, presence of radiation makes more and more items also come down and this is known as stimulated emission. So, what will happen as temperature goes up u nu T becomes larger not only the number of atoms that are going from ground state or lower level to upper level increases. So, does the number of atoms that come down from the upper level to lower level and therefore, we may achieve properly equilibrium. So, what we have going to now have the equations as is dN 2 by dt they decay because of the spontaneous a machine and also due to stimulated emission N 2 plus anywhere there are atoms coming from the lower level and that will be B 12 u nu T N 1 and dN 1 by dt will be minus dN 2 by dt and that will A 21 plus B 2 1 u nu T N 2 minus B 12 u nu T N 1. So, we have introduced the idea of stimulated emission and let us see if this leads to proper equilibrium.

So, again we are going to say that at equilibrium dN 2 by dt is equal to dN 1 by dt is going to be 0.

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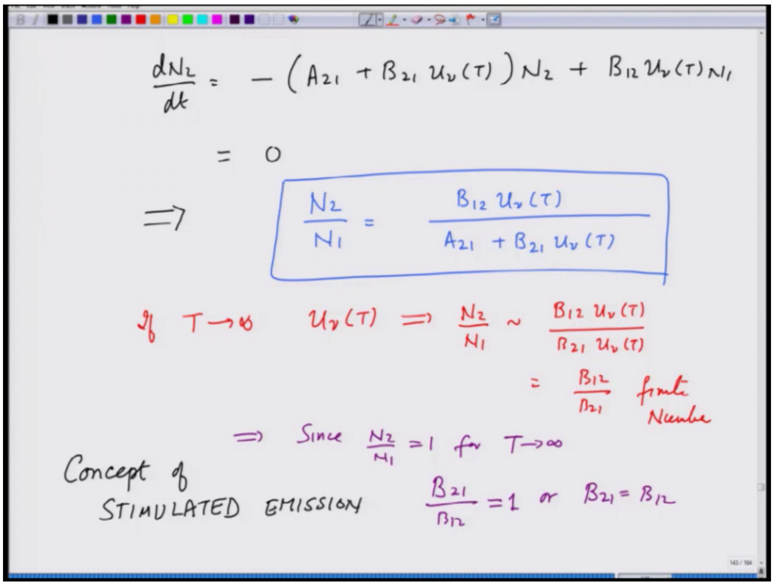

And let us see what is that give me. So, dN 2 over dt is nothing, but minus A 21 plus B 2 1 u nu T N 2 plus B 12 u nu T N 1 and this equals 0 implies that N 2 over N 1 is equal to B 1 u nu T divided by A 21 plus B 2 1 u nu T. Let us see what happens if the temperature goes to a very large value. So, if t goes to infinity, so does u nu T and therefore, this implies N 2 over N 1 can be written as B 12 u nu T divided by B 21 u nu T which is equal to B 12 over B 2 1 which is the finite number. Not only that it also indicates since N 2 over and 1 goes to 1 for T going to infinity that B 2 1 over B 12 is also equal to 1 or B 2 1 equals B 12.

So, not only the idea of a stimulated emission reserves the statistical result that N 2 over N 1 is E raise to minus delta over u over k T it also gives you a new mechanism through which this equilibrium is maintained and that is these stimulated emission. Radiation falling on atoms some of which are exited and some of which are in the ground state lower energy state can not only take them to the upper state it can also make the atoms in the upper state come down. So, this is the concept of stimulated emission. You stimulate you got the atoms to come down to (Refer Time: 26:21).

Now let us work on this further now you will see that same result B 21 equals B 12 would also arise for the concentrations.

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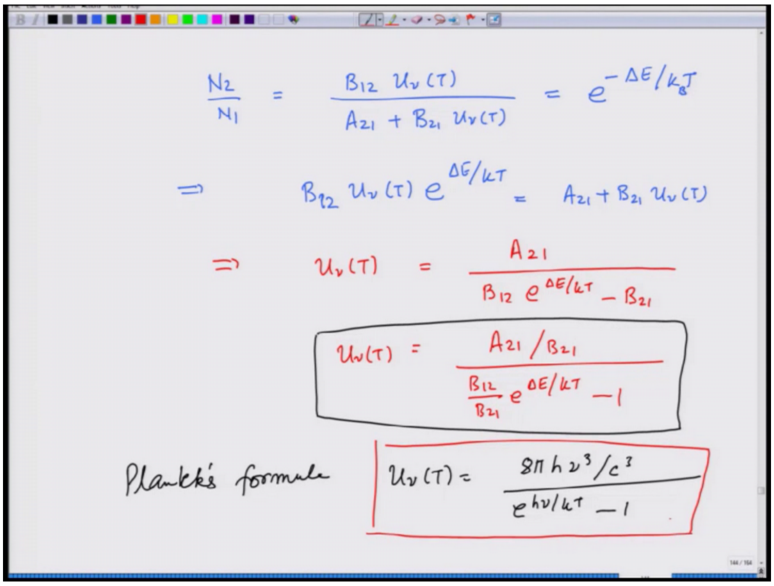

So, now we have N 2 over N 1 which is B 12 u nu T over A 21 plus B 2 1 u nu T and this should be equal to E raise to minus delta E over k B t where k B is the Boltzmann constant and this gives me B 12 u nu T E raise to delta E over k T is equal to A 21 plus B 2 1 u nu T and you take B 2 1 and u nu T term on the same side and this gives you u nu T is equal to A 21 divided by B 12 E raise to delta E over k T minus B 21 which can be further written as A 21 divided by B 21 divided by B 12 over B 2 1 E raise to delta E over k T minus 1.

So, keep this result in mind that the coefficient for stimulated emission and spontaneous emission as such that they give u nu T in this form.

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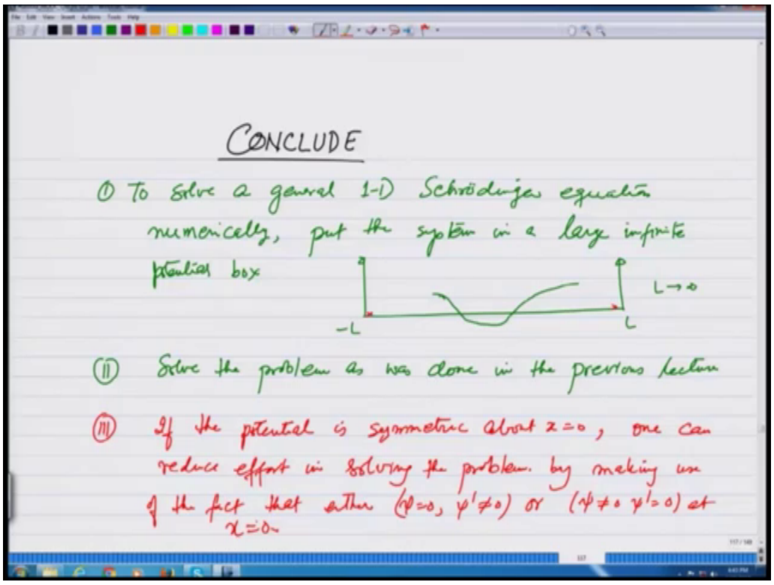

Now from the Planck's formula we know that u nu T is equal to 8 pi h nu cube over c cubed divided by E raise to h nu over k T minus 1 this is the Planck's formula. So, let me also box this and you compare the two from the considerations of atom radiation interaction we get u nu T is equal to A 21 divided by B 12 over B 2 1 E raise to delta E over k T minus 1 and Planck's formula gives you u nu T is equal to 8 pi h nu cube over c cube 1 over E raise to h nu over k T minus 1. And this immediately implies that A 21 over B 2 1 is equal to 8 pi h nu cube over c cubed and B 12 equals B 21 and delta E u equals h nu.

So, not only we have introduced the idea of stimulated and spontaneous emissions you have also got the Bohr condition the delta E is equal to h nu when the radiation comes out it comes out in the form of a photon with frequency nu equals delta E over h; so all that is recovered, so this was the great leap towards understanding how radiation takes place. In this Einstein also introduce the idea of probabilistic nature of radiation that is in classical theory electron revolves around a nucleus or whenever it accelerates it just gives out radiation in a continuous spanner, when we consider the particle or quantum nature and electron in an upper state has a finite nonzero probability of making a jump or the atom has a non zero probability of making a jump this process is statistical and described by probability. Let us now explore this a little further.

So, hopefully by now you are getting familiar and comfortable with the idea of spontaneous emission there coefficient 8 to 1 with the stimulated emission and the corresponding coefficient B 21 and B 21 is equal to B 12 and the energy level difference is related to the frequency of radiation is through h nu.

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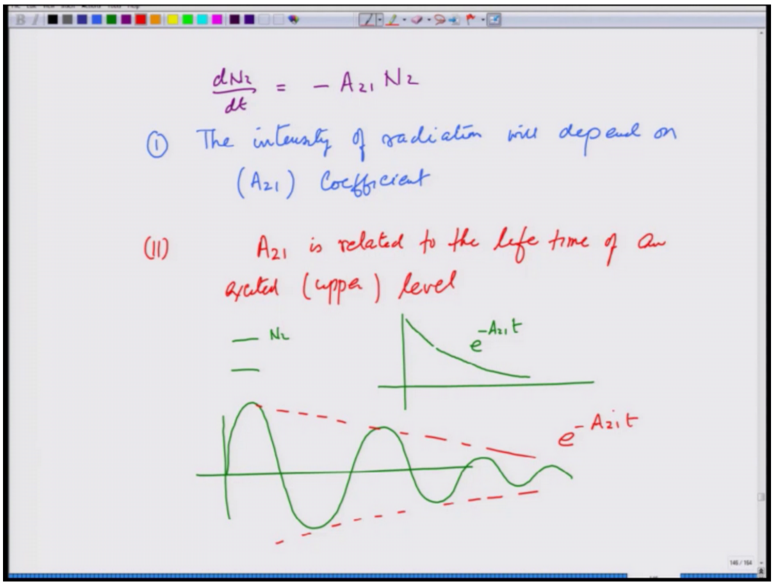

So, let us see now what we have introduced is that dN 2 by dt an atomine upper state has this rate of coming down and when it comes down it gives out photons. So, one thing we understand from here is number one that the intensity of radiation will depend on A 21 coefficient because if you leave and atom in the upper level it will make more and more atoms will make transition to lower levels if A 21 is larger. So, intensity would be larger if A 21 is small then the intensity coming would be smaller.

Number two, A 21 is related to the life time of an excited or upper level let us understand that how. Is if you consider a lot of atoms N 2 in the upper level when the radiate the radiation initially is going to be strong and slowly it will come down exponentially. Slowly it comes down if you consider only 1 atom it has a probability of decaying per unit time which is equal to 1. Now the intensity comes down if you look at the radiation coming out it would be large amplitude in the beginning and slowly decays. So, let me make this curve is red curve is going to be like E raise to minus A 21 t and now we will we are going to connect it with them harmonic oscillator.

Suppose the radiation is coming from it an oscillator and electron oscillating back and forth when it radiates will radiate the energy will go down and slowly it is amplitude will go down.

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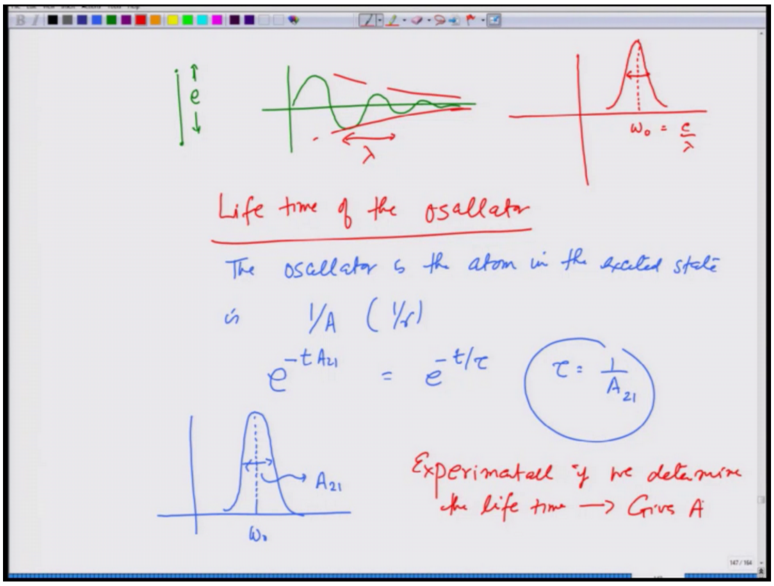

And from Fourier theory I know that whenever there is a profile like this the corresponding amplitude and you will urge you to go back and read them harmonic oscillator is nonzero for frequencies around the main frequency omega 0 which is related to this lambda as C over lambda, but this is non 0 near that omega 0 also with some spread right. So, and since the intensity is going down like this, so lifetime of the oscillator and in the in our case the oscillator is the atom in the excited state is 1 over A or 1 over gamma the gamma is damping coefficient.

So, this because I can write this as E raise to A 21 as also E raise to minus t over tau where tau is over A 21 and that gives you the lifetime. And this is also related to which I re-plot here that if you look at the profile of light coming out it will have maximum intensity or maximum amplitude at omega 0, but would also have a width and this width is going to be related to A 21 or the coefficient of decay. So, this width gives me the value of A 21 it is also related to lifetime and experimentally if we determine the lifetime this gives A. So, let us check that.

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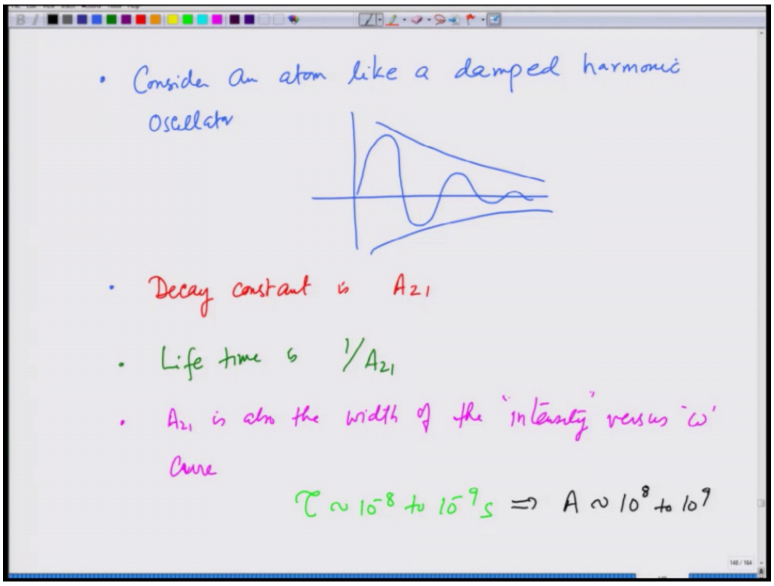

So, what we have just said is that consider an atom like a damped harmonic oscillator in its statistical sense. So, an atoms actually give out this kind of radiation is goes down then the decay constant is A 21 and third therefore, lifetime is 1 over A 21 and forth A 21 is also the width of the intensity versus frequency curve.

So, these I can determine what A 21 would be. So, let me give you now the order the lifetime of an excited status of the order of 10 raise to minus 8 to 10 raise to minus 9 seconds and therefore, this implies a between 2 levels is of the order of 10 raise to 8 to 10 raise to 9.

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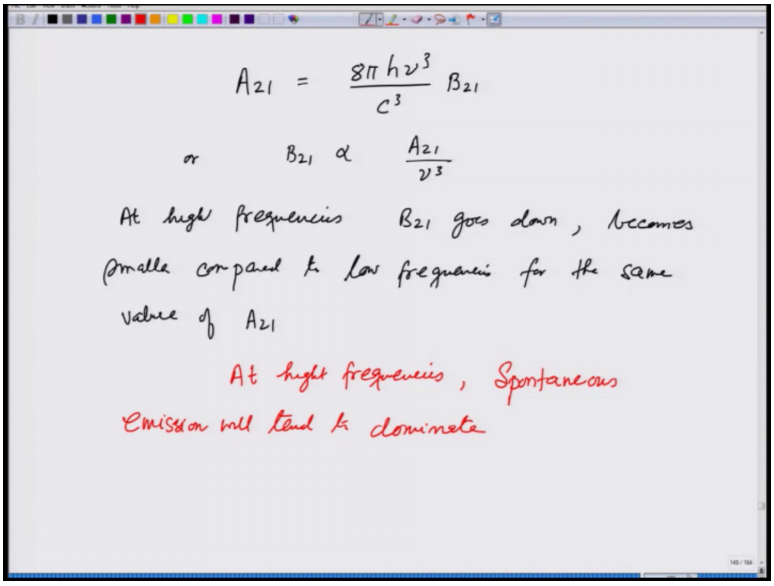

The second thing that you notice about this, these coefficients is that A 21 is equal to 8 pi h nu cubed over C cubed B 2 1 or B 2 1 is proportional to A 21 over nu cubed. So, at high frequencies B 21 goes down; that means, becomes smaller compared to low frequencies for same value of A 21. So, at high frequencies even if you are to shine light on systems the spontaneous emission will tend to dominate.

So, we conclude this lecture by summarizing whatever we have learnt 1 the atom radiation interaction is governed by probability or statistical laws.

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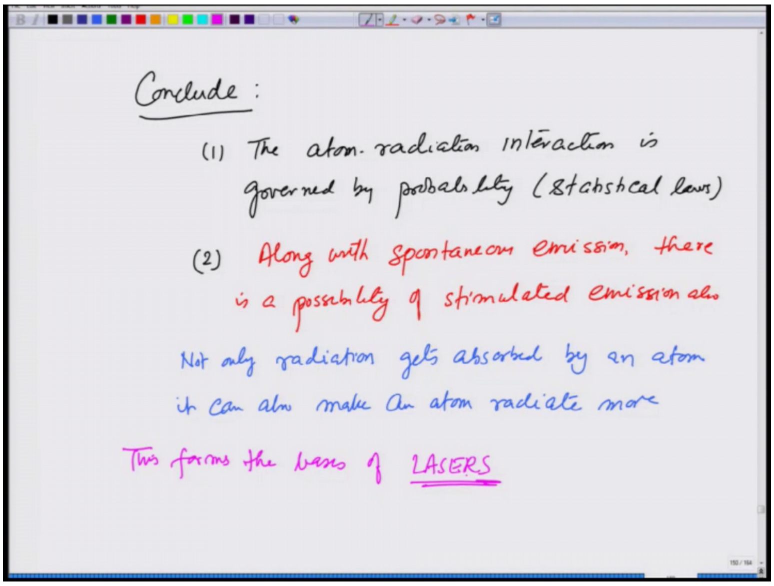

Two along with spontaneous emission there is a possibility of stimulated emission also. So, by shining light on and exited atom or an atom and upper state you can make it radiate. So, not only variation let us absorbed by an atom or a quantum system it can also make an atom radiate more, because if you stimulates it to radiate more radiation would come out and this forms the basis. This forms the basis of lasers which we will cover in the next lecture.