Nuclear Physics Fundamentals and Application

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Lecture - 38
Nuclear fusion Contd..

So, in the last lecture we talked about thermonuclear fusion and we discussed that the most important parameter is reaction rate. That is how many fusion reactions take place per unit volume per unit time? If a particular plasma concentration is confined in a certain volume at certain temperature and I gave you an expression and promised that I will justify it in this lecture.

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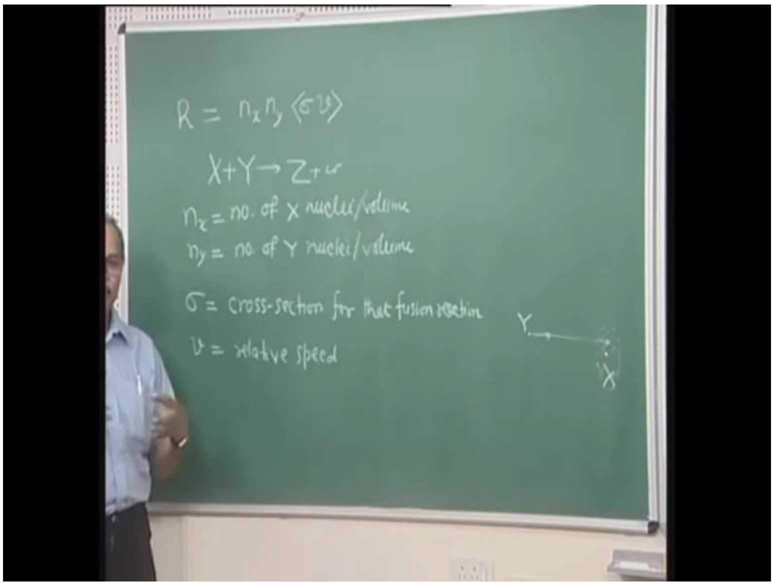

And the expression was that reaction rate would be something like the concentration of the 2 interacting species and then average value of sigma v. So, this is reaction rate n x n y x and y are the 2 nuclear species which are fusing which are doing that fusion reaction. So, x plus y is going to z and possibly some other particle gamma particle or a small nucleon or something.

So, this x and y and this n x n y are the concentrations that means number of nuclei, number of x nuclei per unit volume and similarly, this n y is number of y nuclei per unit volume sigma is the cross section for that particular reaction. Roughly speaking sigma you can take as an area presented by x nucleus. And if this y nucleus hits that area the fusion reaction takes place that is how we define cross section? We remember cross section is in units of centimeter square. It is an area unit. So, if you have nucleus x and another nucleus y here and if they are coming towards each other you can take x relative to x y is coming and then this x mentally you think that this is an area.

This area surrounding this x nucleus and if this y hits somewhere in this area the reaction will take place so that, is how the cross section is geometrically conceived so that, is this sigma and v is the relative speed. The 2 nuclei x and y are approaching each other and the relative speed the speed of y with respect to x or of x with respect to y that is this v. So, first let me see how it is proportional to v. Sigma is of course, is directly related to the probability of that fusion reaction. So, if sigma is large the rate will be large, if sigma is small the rate will be small. And for all this fusion reaction this cross section is very small so that, sigma is there v let us look at this v first consider a nuclear say scattering. Scattering a nuclear reaction says scattering.

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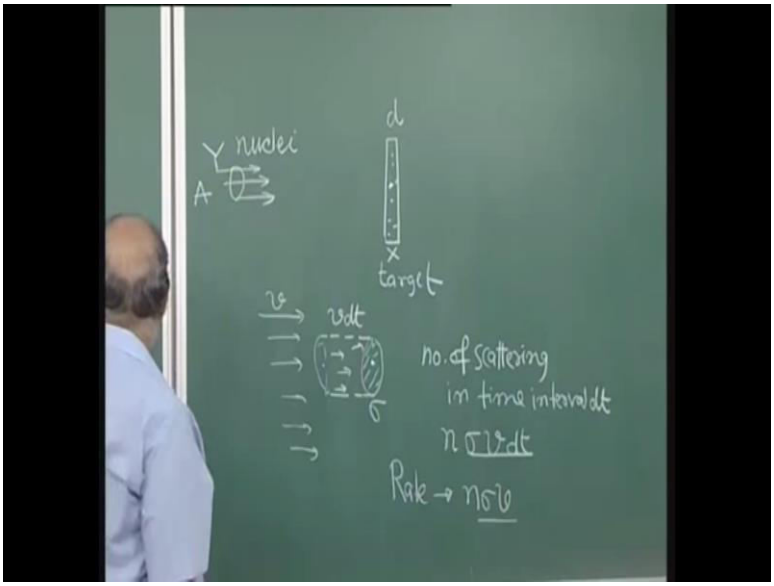

So, if you have the target normally in a scattering geometry you have a target fixed and then a beam of this projectile nuclei or ions they come. So, these are nuclei let us call them Y nuclei and in this target you have x nuclei these are x nuclei. This thickness you take as d the beam diameter you take as some capital A. So, how many scattering will take place per unit time? So, for that how we make the calculation? We consider a particular nucleus x here a particular nucleus x here. So, what which nuclei will scatter from this nucleus? So, the cross-section comes into picture if the cross-section is sigma then you can you make an area here.

This area is sigma around this x nucleus, this x nucleus is writing here zoomed version because the cross-section areas are very small as compared to any target in the laboratory so that, sigma. Now this beam is coming. So, if the velocity here is v if all these nuclei in the beam they are coming with velocity v and I want how many of these y nuclei are hitting this area in some time interval d t? So, What I can do? I can construct a cylinder here mentally construct a cylinder here with 1 phase as sigma other phase parallel to it and length of the cylinder v into d t. So, at time t at any instant whatever y nuclei are here, whatever y nuclei are inside this volume they will hit this area. Anything behind this will not be able to reach this sigma in next time interval d t and anything inside it will suddenly hit this because this length is v d t nuclei moves a distance of v d t. So, the nuclei which are here on this phase they will also reach this sigma and all the nuclei ahead of them they will certainly reach this sigma and anything behind will not.

So, how many nuclei are there in this volume? So, if the concentration is n concentration of y nuclei in the beam, so per unit volume of the beam if there are small n y nuclei. Then the number of this scattering or reaction in time interval d t, that will be small n into sigma into v into d t sigma into v into d t. This is the volume sigma is this area v d t is this length. So, sigma into v into d t is this volume n is the number of y nuclei at any instant or unit volume in the beam. So, this is the number of nuclei at a particular instant in this volume and hence in time d t these many scatterings will take place or whatever nuclear reaction will take place and therefore, the rate the reaction rate will be proportional to the concentration and sigma and v so that, sigma v essentially comes from this.

Now in the thermo nuclear fusion the case is slightly different from this. We do not have a solid target? It is a volume in which you have both the x and y nuclei in gaseous form. In plasma form you have electrons you have this positive nuclei and they are all moving in random directions and random with random speeds. So, not like a beam coming in a particular well defined direction but, essentially this essential idea remains there and finally, you get that sigma v term there also.

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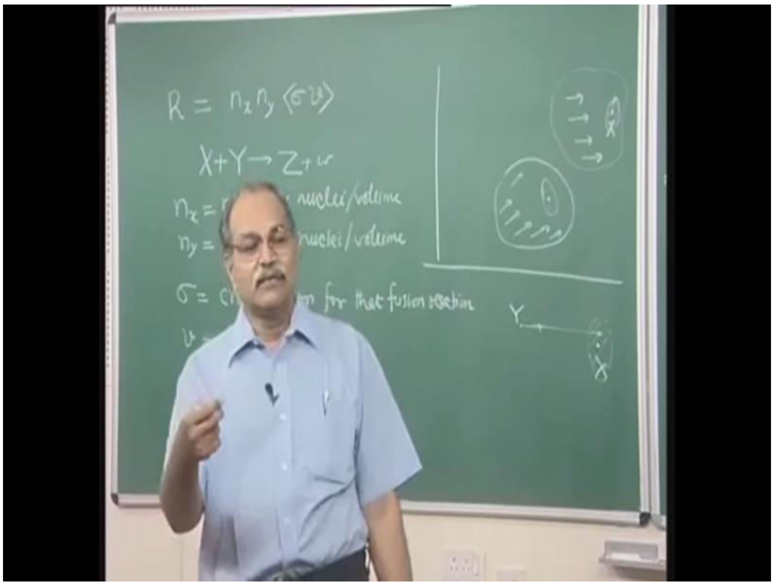

What I can do here is once again in the gap. Now, in the thermonuclear plasma here the volume in that volume you have a gap in which these x nuclei are there y nuclei are there electrons are there or all moving but, still you can consider x nuclei here and relative to this you can look at the velocity of the y nuclei.

So, if I use the relative velocity in place of v and then look at the group of nuclei y nuclei which are coming towards this nucleus from a particular direction. So, consider those nuclei which are coming towards x in this direction. So, this situation is very close to that situation the directions are different that is but, if the nuclei are coming from some other directions. So, for this direction will construct a sigma and then see that how many are going through this? If you have some other kind of the group of molecules nuclei you consider this the same nucleus x here you construct sigma this way. The sigma the cross section as such is only the magnitude of that area presented by the nucleus. So, if the projectile nuclears are coming from the direction area to be constructed this way. The projectile nuclears are coming from top the area sigma is to be constructed this way.

So, for each group 1 can do this, so that is that is how they resemble. Another difference would be that in scattering experiment the same nucleus x can scatter the many of the incoming particles but, in fusion reaction if 1 fusion has taken place that particular nucleus is consumed it is only the other x which can come. So, all those things will be there but, that expression we will there.

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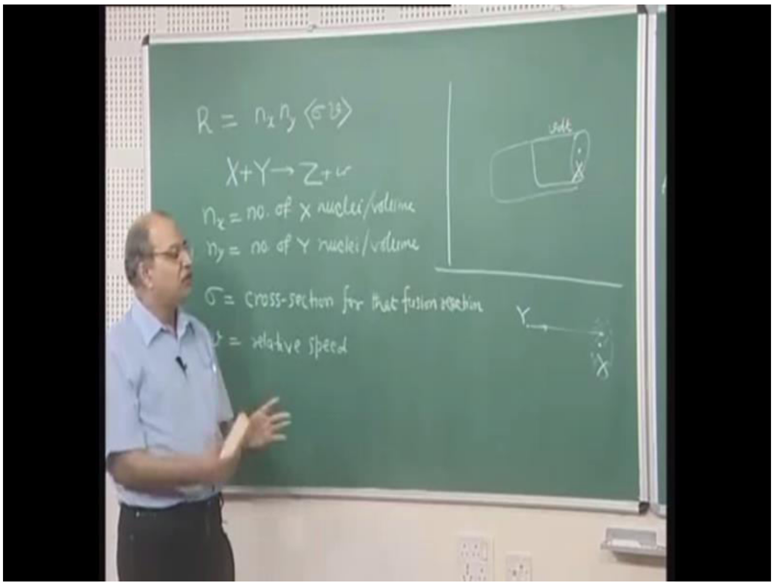

You can think of a particle x here and then you can ask ok. I construct this sigma. I construct this sigma into v d t volume. So, this v d t is length. Now generally the concentrations are low, sigma is extremely small. So, there is hardly 1 nucleus y here in a typical case. So, the reaction rate will be proportional to the probability that you have a y nucleus here and that probability will be proportional to this v because if you take larger volume for same d t larger volume the probability of finding a particle y here will be larger. So, the probability next d t there will be a fusion with this x again proportional to this v and of course, proportional to sigma and therefore, it is sigma into v that comes into picture so that, is how the sigma v comes here and of course, they concentrations of x nuclei and why nuclei it has to be proportional to that. In fact this gives you the number of pairs of x y which can fuse per unit volume. So, it is this. Now, what is this average business?

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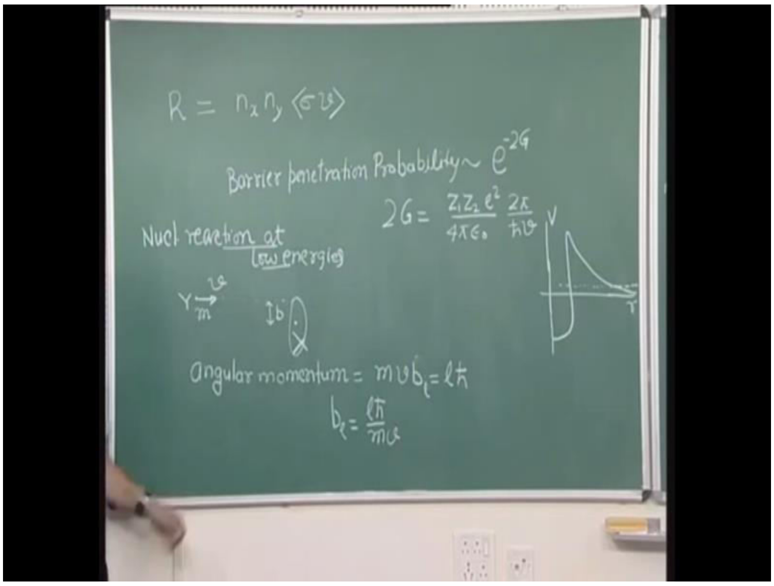

Why I am putting these angular brackets here? These angular brackets denote that we have to average over different velocities because in this gaseous plasma the nuclei are moving with random speeds. As we discussed earlier also you can assume Maxwell-Boltzmann distribution. So, you will have different values of relative speed v sigma will be same for the same kind of fusion. And the in the similar at the same temperature and velocity and all that but, then the particles will be moving with different velocities and hence and average is needed over these different velocities.

What velocity are these? These are relative velocities of y nuclei with respect to x nuclei or of x nuclei with respect to y nuclear. And I have put sigma also inside the angular brackets. This is because sigma also depends on velocity and the dependence of sigma dependence of cross section of fusion on velocity of relative velocity of these 2 particles which are time to fuse with each other has 2 components 1 is of course, we had discussed from the very beginning that this fusion reactions. They have to go through a barrier penetration process because when 2 nuclei are fusing you have 2 positively charged particles coming very close to each other.

So, when they are coming close to closer to each other then the coulomb potential energy is increasing and 1 when they come into this nuclear range so that, the nuclei are now ready to fuse. They are just penetrating each other then the nuclear potential will take place. So, you have a potential barrier and the probability that this barrier will be penetrated depends on that relative velocity. And that expression is barrier penetration probability that expression is like e to the power minus 2 G where 2 G is written for Z 1Z 2 e square over 4 phi epsilon naught and then 2 phi by h cross v. So, the relative speed comes here the barrier is remember it is something of this sort the coulomb type of potential and then here it nuclear thing takes place and potential is like this. So, this is v and this is r separation and the energy that is the kinetic energy of the particles before the fusion when they are widely separated, so that kinetic energy and this is the barrier that they have to penetrate and then come into this nuclear range.

So, if the relative speed is high that means the kinetic energy of these initial kinetic energy of the particles is high and this dotted line will go up and therefore, the probability of penetration way increase so that, is how this penetration probability depends on v. And the cross section which is again some kind of probability of fission so that, is directly proportional to this barrier penetration probability. If the barrier penetration probability is high then the fusion probability is also high, the cross section is also high.

So, sigma will has s factor of e to the power minus 2 G because of this barrier penetration coming in but, this is not the only velocity dependence. One thing is that this is an approximate expression and this is valid only if this available energy is much smaller than the height of the barrier all of this range is very high. That means if the barrier penetration probability itself is very low. Then you use this expression and that is the case in any thermonuclear reactor 1 tries to design or that is there in the starts.

Now, another dependence is apart from this barrier penetration and coulomb potential and all that for low energies for any nuclear reaction there you have a velocity dependence and that velocity dependency is proportional to 1 by v square. So, I will justified that also, suppose you have a nucleus x here and nucleus Y here and this Y is going with velocity v mass of this y is let us say m. Now we are talking of nuclear reactions in general at low energies. So, again I will be using lots of classical lots of semi classical geometrical pictures and so on. So, suppose it is coming like this and this impact parameter this distance. This is the line of motion and here is that nucleus towards which it is coming. You can take the relative velocity and this distance is impact parameter we call it b small b. So, the angular momentum is m into v into b this is b, this is angular momentum. Classically this b can be very continuously and therefore, angular momentum can take any value. But, if I bringing some quantum mechanical picture the angler momentum has to be quantized in any direction it is some integral multiple of h cross.

So, if I construct a circle here around this excel of radius b these nuclei can come from any side of the nucleus if they are hitting here impact parameter b then they can fall anywhere on the circle. So, all nuclei Y which are falling on this circle will have this angular momentum and if I say that it is integral multiple h cross say l into h cross. If I call this as let us put b l and call it l into h cross, then this b l should be equal to l h cross by m v. The nuclei Y which are coming with the same velocity from the same direction but, with a different impact parameter corresponding to the next unit of angular momentum l plus 1 times h cross.

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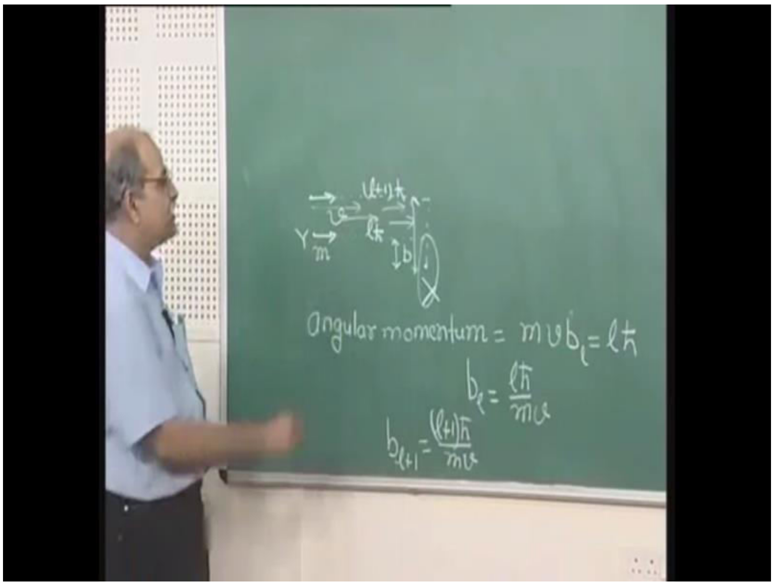

For them this impact parameter will be b l plus 1 that will be l plus 1 times h cross by m v. What happens? So, these are the nuclei which are coming like this. So, this corresponds to let us say l h cross this corresponds to l plus 1 h cross for them this b here will be given by this. What about the nuclei which are coming with distances which is between this b l and b l plus 1? So, let us say that all these nuclei will be clubbed with this v l. So, any nucleus which is giving you angler momentum less than l plus 1 h cross but, more than l h cross. So, let us club that all with l h cross so that, we are quantizing this classical continuous quantity. So, all these nuclei will be taken as corresponding to angular momentum and l h cross then from l plus 1 to l plus 2. Here all these nuclei will be considered to have angular momentum l plus 1 h cross and so on.

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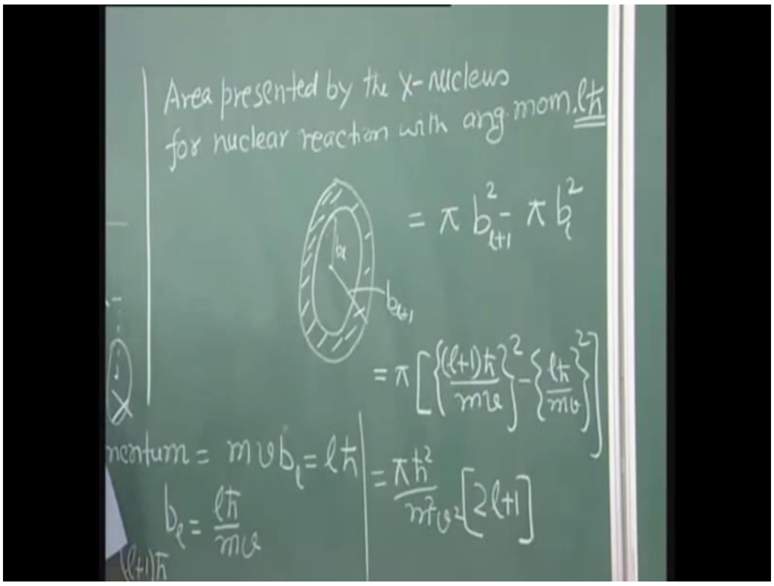

Now, let us calculate number or area prevented by the x nucleus for nuclear reaction with angular momentum l h cross. How much is the area presented here? So, if you have this nucleus x here and this is b l and draw a circle and then b l plus 1. Draw another circle this area any y nucleus falling on this area will have impact parameter more than b l less than the b l plus 1. And they all those y nuclei will be considered to have angular momentum l h cross. So, the area around this x which admits this angular momentum l h cross is this shaded area between b l and b l plus 1 and that area is pi times b l plus 1 square minus pi times b l square, and that is equal to pi b l plus 1 a square is here. So, l plus 1 h cross by l plus 1 h cross by m v mass times that relative velocity this mass is also reduced mass. So, this whole square and minus l h cross by m v square and we can further simplify it pi here h across square m square v square here. So, I have taken h crosses square, m square, v square out it is l plus 1 square minus l square which will be 2 l plus. So, it is 2 l plus 1. So, the area presented by this x nucleus for nuclear reaction which angular momentum l h cross is this so starting from l equal to 0.

How much is the area presented? For l equal to 0 for l equal to 1 for l equal 2 and so on. Up to some maximum l max what is that maximum l maximum l is given by the nuclear range these 2 nuclei are coming and their interacting through nuclear forces. So, you have a nuclei range for that capital R. If the distance the lowest distance between the particles is more than that then there are no nuclear interactions. So, the maximum impact parameters which are allowed for these 2 particles. So, come within a nuclear range and therefore, interact through nuclear potentials is that capital R.

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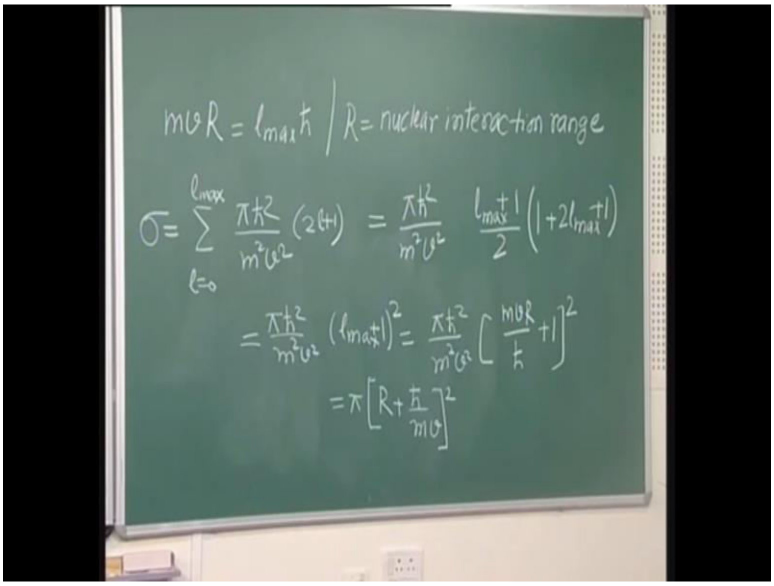

So, m into v into this should be equal to l max times h cross. Once the impact parameter has become capital R that is the maximum you can take if impact parameter is more than this capital R. Then they are never in nuclear range and no nuclear reaction will take place. So, the maximum value of this l max is given by this equation m into v into capital R is equal to l maximum 2 h cross this capital R here is the nuclear range nuclear interaction range. So, up to this l max if I consider starting from 0. What I get is the total area presented by this nucleus or nuclear reaction and that is the total cross section for this reaction.

So, the cross section is equal to summation l going from 0 to l max and then this quantity phi cross square by m square v square 2 l plus 1. That the total area presented by the nucleus for nuclear reaction with different values of l starting from l equal to 0 to l max and this quantity is phi h crosses square m square v square and this is just an arithmetic progression arithmetic series 2 l plus 1 l equal to 0. So, it starts with 1 1 plus 3 plus 5 plus 7 and so on sum of all odd numbers. And how many terms are there? 0 to l max so, l max plus 1 terms are there. So, summation will be l max plus one-second total number of terms by 2 and then first term plus last term. First term will be just 1 and the last term will be 2 l max plus 1 using the equation for summation of a arithmetic service. So, 2 you can cancel and it will be l max plus 1 square.

So, this is equal to phi h crosses square m square and then l max plus 1 square and that is phi h crosses square by m square v square l max is here. Where is l max is given by this. So, l max is m v capital R by h cross so it is l max is m v capital R by h cross this is l max and plus 1 square. If you take h crosses square by m square v square inside this bracket it will be phi and if I take it inside it is h cross by m v h cross a by m v. So, this will cancel this will be r and plus h cross by m v whole square. Now estimate the 2 terms capital R and h cross by m v for a typical case, where the plasma is going through this thermonuclear fusion.

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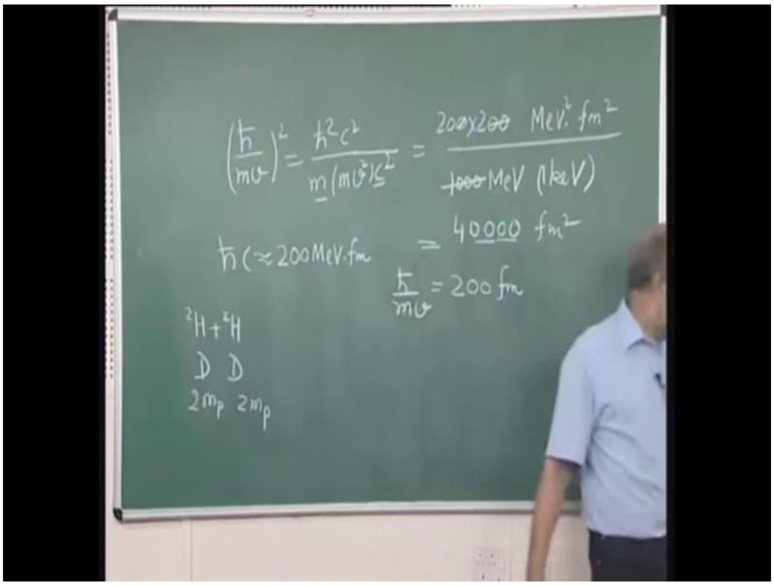

So, h cross by m v h cross by m v you can take square. So, square of this will be h cross square m square v square m v square. Let me write this way and multiply by c square here and c square here. Now h cross c roughly 200 mega electron volt femtometer. So, this is 200 into 200 M e V square f m square divided by mass into c square m c square. So, if take typically let us say d d reaction deuteron reaction 2 h 2 h reaction. This is deuteron so is called d d reaction. So, the mass here is 2 times mass of proton approximately and here 2 times mass of proton and therefore, the reduced mass will be just m p mass of proton. So, mass proton into c square is 939 mega electron volts take

1000 mega electron volts. So, m c square is 1000 M e V then m into v square, this v is those relative speed. So, m v square half m v square is the kinetic energy and that kinetic energy depends on temperature three-second k T for the 10 to the power 7 kelvin or so, for protons it will be like 1 k e v 1 kilo electron volt and typically people try to work at few kilo electron volts. Because one has to raise the temperature to at 10 power 7 10 power 8 range.

So, I take this m v square as 1 k e v kilo electron volt. So, this is k e v 1 k e v. So, this will be 2 0's here 1 0's here that will be cancel with this M e V goes with M e V 1 M e V here and that M e V by k e v is 10 to the power 3. So, it is 40, 40 into 10 to the power 3. So, 40 into 10 to the power 3 f m square femtometer square. So, this is 200 femtometer just an estimate typical estimates. So, 200 femtometer here could be 100 femtometer or 50 femtometer or 60 femtometer or 150 femtometer. That is h cross by m v this is h cross by m v. Al right this I had calculated h cross by m v square I have taken square root here. So, h cross by m v is this.

And R capital R is the range of nuclear interaction which is typically says 2 femtometers range of interaction of nuclei so, typically 2 femtometer. Here, it is 200 femtometers. So, this term dominates over that capital R and therefore, you can write sigma for low energies. This m v is low this velocity here I have taken it 1 k e v.

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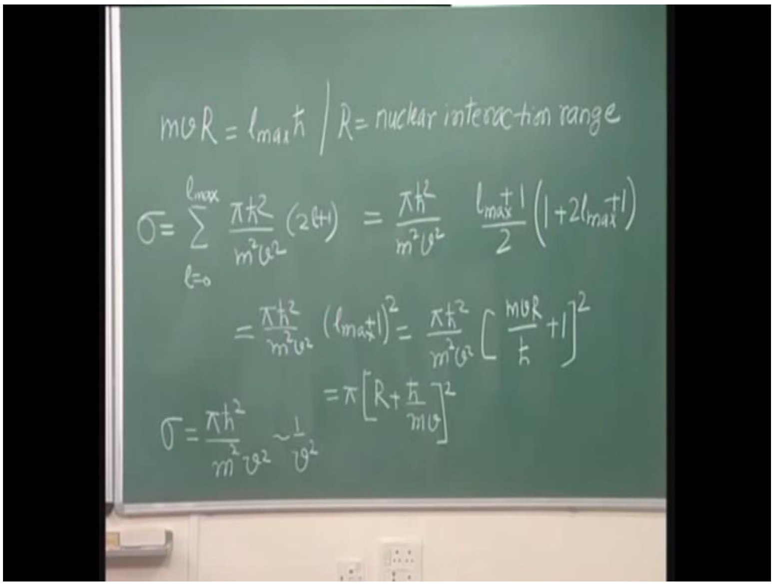

So, for low energies this r can be neglected and then this will be phi h crosses square over m square and v square. So, you can see that it is proportional to 1 by v square. So, the cross section because of this nuclear interaction goes as this at low energies and this is much larger than this phi into capital R square which classically you can think of that. If the range of interactions is capital R then it is presenting an area phi into capital R square anything falling beyond that does not interact with this anything falling inside interacts and therefore, the cross section should be phi into capital R square this line of argument is also but, what happens at low energies?

Is that the y nucleus which is coming with this low energy is not very small point particle it has its own De Broglie Wavelength and that De Broglie wavelength is large. So, even though the center that y nucleus the line I draw could be beyond capital R but, because of the spread in De Broglie wave it will interact with the nucleus and the cross section will become larger. So, that is the physical picture.

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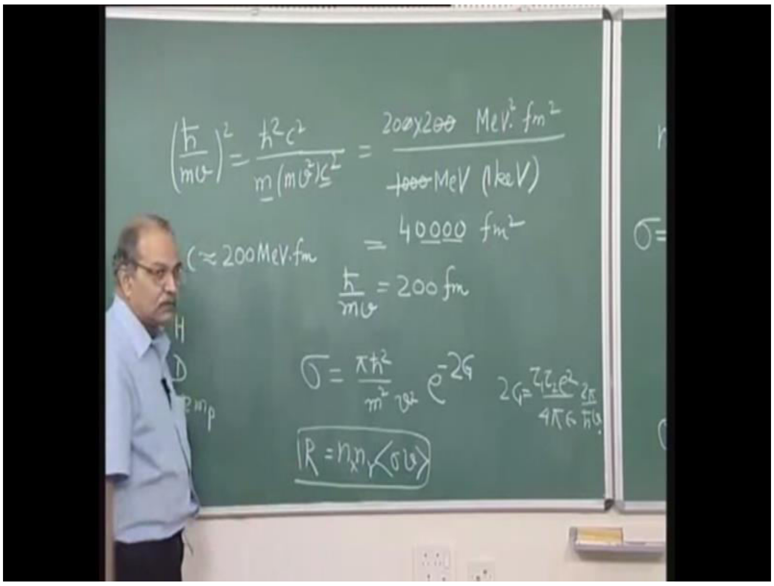

So, the cross section therefore, as 2 factors one is this 1 by v square say phi h crosses square by m square v square do not go on the exact expression. The important thing that it is 1 by v square something constant multiplied by 1 by v square and then e to the power minus 2 G and G itself contains velocity remember 2 G is Z 1 Z 2 e square by 4 pi epsilon naught and then 2 pi over h cross v. So, that is how sigma depends on velocity and therefore, we when we average this expression for reaction rate sigma also goes inside that angular bracket sigma v x n n y that is reaction rate right. So, how does one calculate this average of sigma v?

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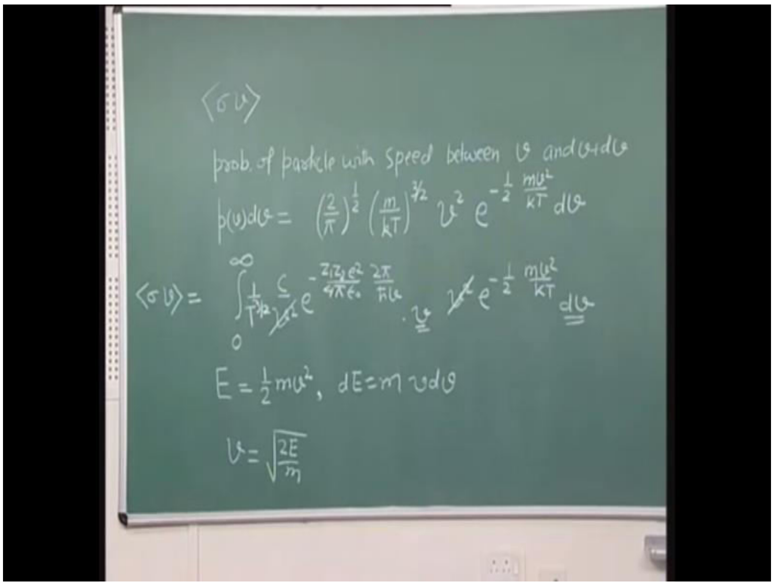

This average of sigma v remembered velocity v. The velocity v has a maxwellian distribution and that is why this averaging is needed and that distribution is probability or fraction of particles with speed between v and v plus d v and call it p v d v. We can take it as a fraction of the total particles which is having this velocity or here fraction of the total pairs which are having this relative velocity between v and v plus d v. So, this is given by that Maxwell- Boltzmann distribution 2 by phi half m by k T three-second then v square e power minus half m v square by k T and d v.

So, sigma v average value of sigma v will be integration v going from 0 to infinity and then sigma into v and sigma. We have seen it is some constant divided by v square and then e to the power minus that Z 1 Z2 e square by 4 pi epsilon naught 2 pi over h across v. That is sigma is this expression and then into v sigma into v and into this fraction p v d v. That is all this constant I am interested in temperature. So, all this constant can go here but, that 1 by t to the power three-second that I would like to keep. So, 1 by t to the power three-second that I would like to keep here all other things go into this constant and then you have this v square here and e power minus half m v square by k T and d v this v square can be canceled with this v square and this v remains in numerator.

So, you can define or you can work with this E equal to half m v square. Al right this E is that kinetic energy thermal kinetic energy. So, E is half m v square for different pairs you have different values of relative speed v and therefore, different values of E here we are averaging over v here we will be averaging over E. Say if I use this then in terms of this if it d E will be equal to m times v d v. So, this v d v that we can write as d E by m d E is 2 cancels here m v d v. So, for v d we will put this d E by m and this expression we can write in terms of e here it is already half m v square. So, it will be just capital E and this expression that this v has to be converted and let me do that.

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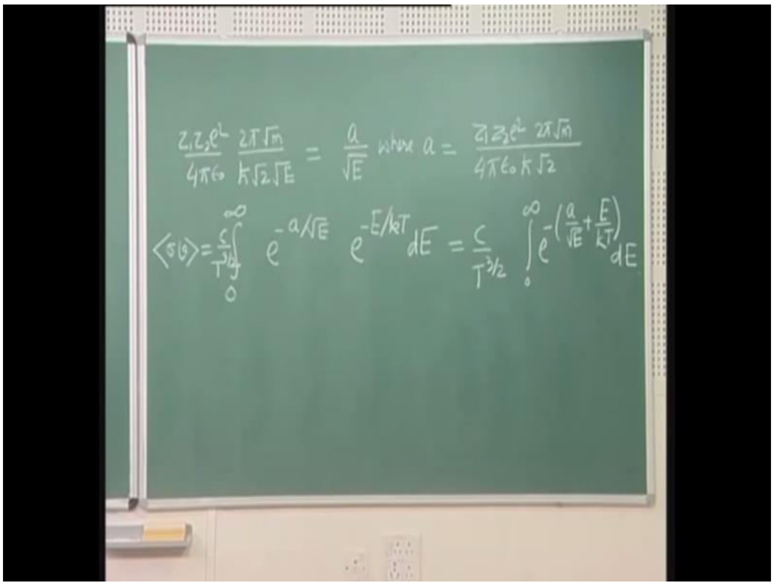

This is Z 1 Z 2 e square by 4 pi epsilon naught and then 2 pi over h cross times v and v is from here. What is v square root of 2 times E over for n? So, this v will be putting here in place of v. We write square root of 2 square root of E and square root of m. And this whole thing I can write as a divided by square root of E where a is written for everything except this root E that is Z 1 Z 2 e square by 4 pi epsilon naught h cross 1 square root of 2 here and then 2 pi here and square root of m here.

This whole thing is written as a then it is a by square root of this sigma v is equal to integration. The limits for capital E are also 0 to infinity v relative velocity v that goes from 0 to infinity and E is half m v square. So, the limits also go for e also goes 0 to infinity. So, 0 to infinity then you have 1 by t to the power 3 by c. You can write this as you can write it outside also c divided by t three-second temperature it same for pairs.

So, this then E to the power minus a by square root of E that is here e to the power minus this whole thing is a to the power a divided by square root of e this whole thing is e to the power minus capital E by k T and then v d v is thus d E by m. So, d E here m I am not putting gets absorbed into this c here. So, this the express c by d three-second and 0 to infinity E to the power minus a divided by square root of E and plus E divided by capital T k T d E.

Now, there is catch the barrier penetration probability I am using as E to the power minus 2 capitals G and this expression I said is approximate. A good approximation only if the kinetic energy of these particles initial kinetic energy of these particles is small as compared to the Barrier height. Barrier height is typical in mega electron volts and the kinetic energy. The average kinetic energy I said is typically between 1 and 10 kilo electron volts and therefore, that expression was used for barrier penetration probability but, now I am taking care of that Maxwellian I am taking care of that Maxwellian distribution and therefore, I am integrating from 0 to infinity capital E going from 0 to infinity, the initial kinetic energy going from 0 to infinity. So, we are taking care of all that large speed pairs also even though the average kinetic energy remains few kilo electron volts.

So, in this full integration range capital E going from 0 to infinity should I use this expression for barrier penetration probability because for large values of capital E which are involved here this expression will not be valid but, it does work why we will see on our excel sheet on the computer.

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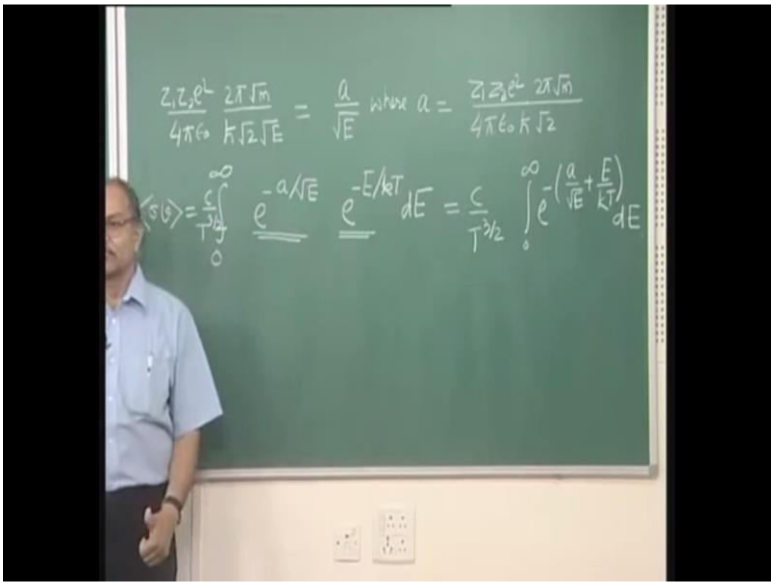

What I will be doing? I will show you separately how this factor changes as e goes from 0 to infinity and how this factor changes as e goes from 0 to infinity and how the product changes as e goes from 0 to infinity. So, let us go to the excel sheet.

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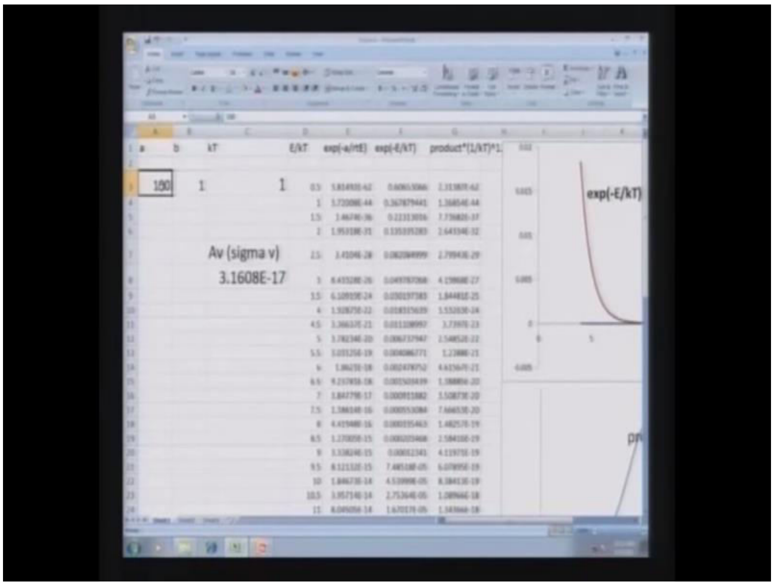

So, here is an excel sheet I hope you are able to see this small a is here and the numbers I have calculated and put some reasonable feasible number. This is small a you remember it comes from the expression e to the power minus small a by square root of e that small a is here, k T is here.

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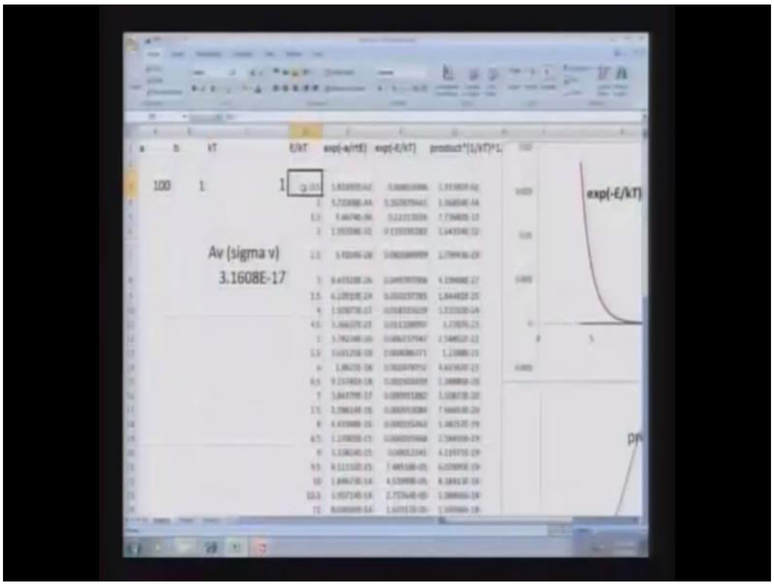

And then e is variable capital E the energies variable that goes from 0 to infinity. So, e by k T I am taking 0.51, 1.5 and so on.

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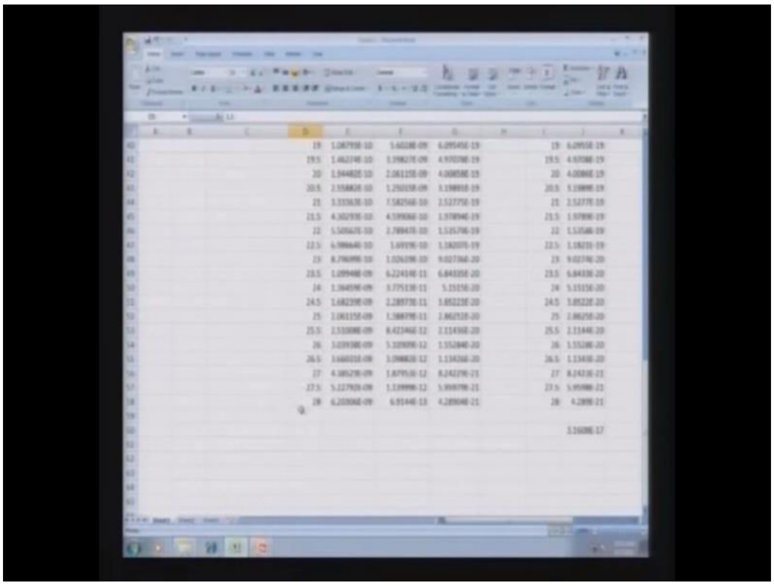

And it goes to large values up to 28 I have taken, 28 times k T upto their I have gone. So, a is there k T is there E is there.

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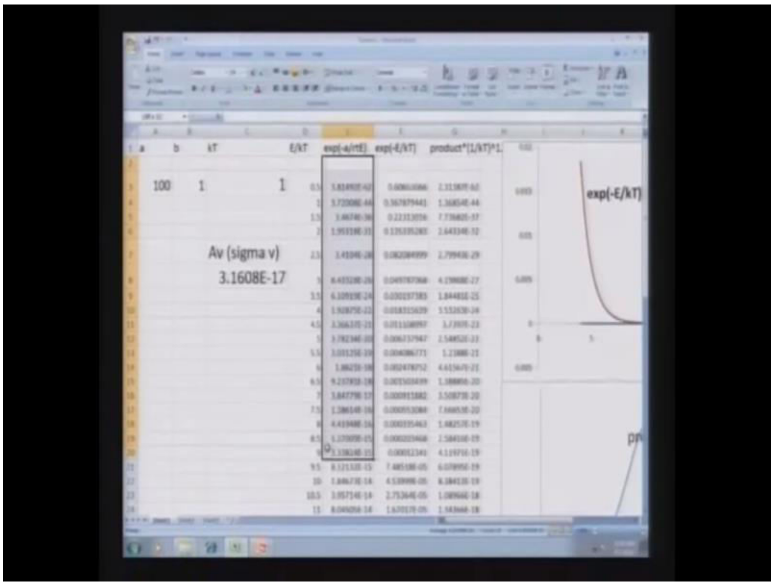

Then, I am calculating here this e to the power minus a by square root of capital E you can see here r t e it is written r t is for square root. So, this column this whole column is for E to the power minus a by square root of e; the first factor in our expression.

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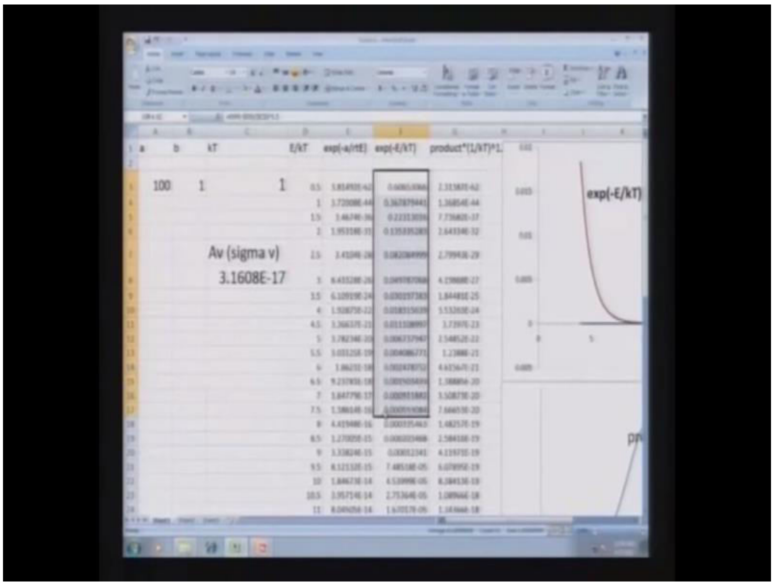

And then, we have the second factor e to the power minus capital E by k T so that, is this column.

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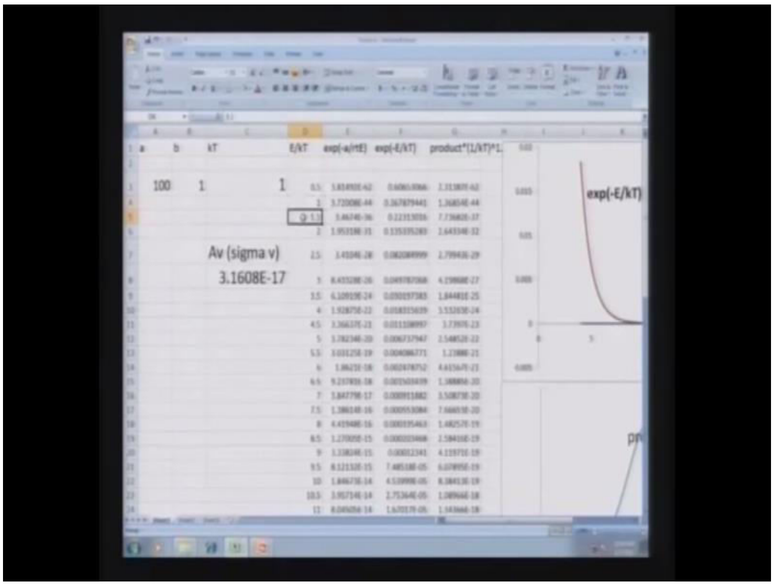

So, we have calculated for each energy these are the different energies I am considering at any interval of 0.5 k T. So, at each of these energy we are calculating this e to the power minus a by square root of e, e to the power minus e by k T.

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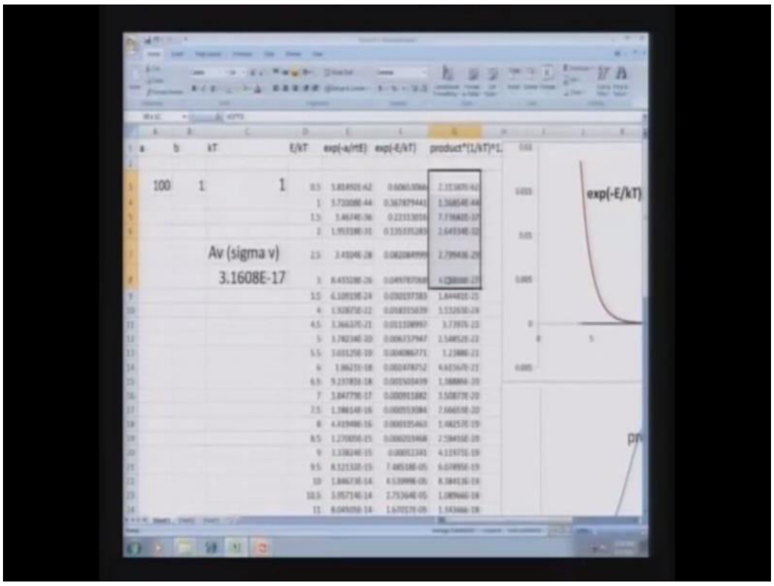

And then, the product; product of the 2 terms are in this column and then we are plotting this product of the 2 factors as a function of this capital E in units of k T.

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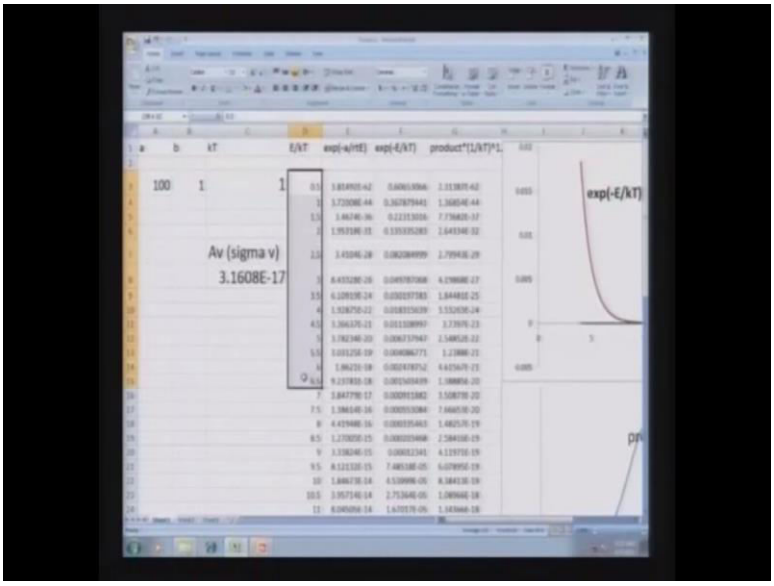

So, essentially what we plotting is e by k T on the x axis.

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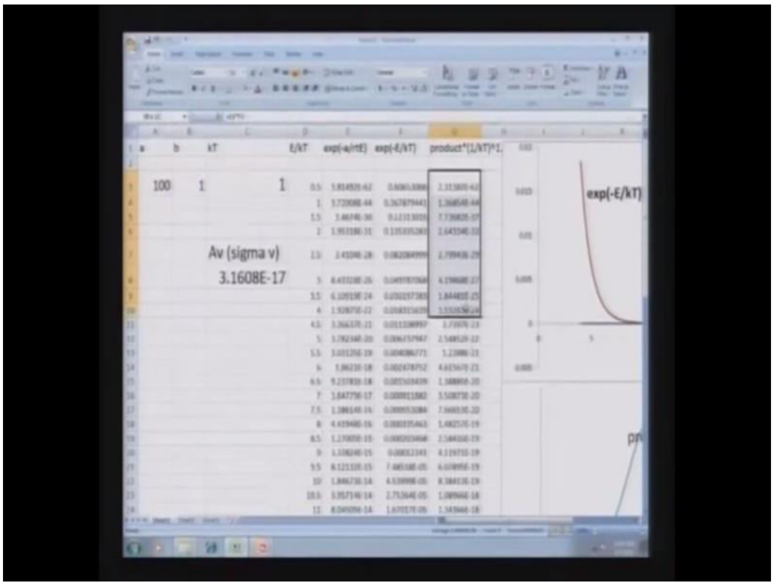

And correspondingly the integrant here, the thing which we have to integrate that we are plotting.

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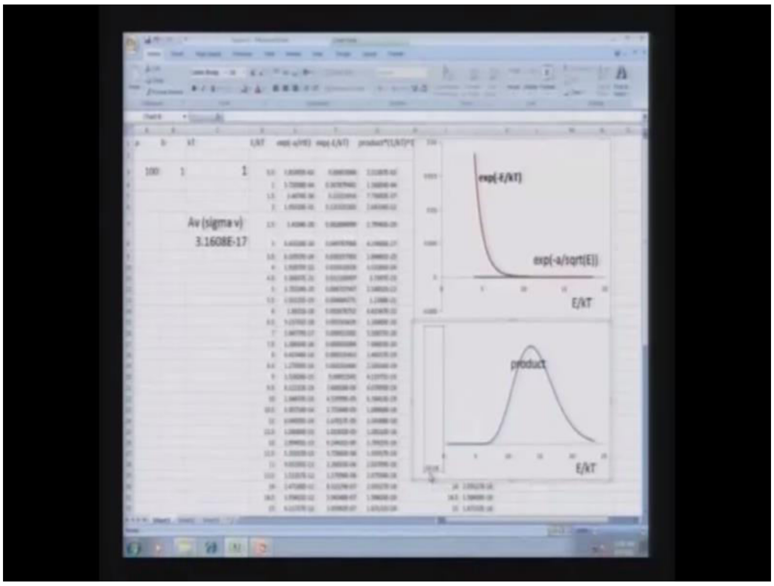

So, if you look at the plot, the plot is already here this is you can see that red line here and a blue line here hardly visible. This red line is for e to the power minus capital E by k T, it decreases as this way exponentially decreases and then this blue line which is almost quensident with the x axis because at the scales which are given on y axis 0.01, 0.015 on this scale the second factor is very small and it is not visible here. The lower graph here is the product.

So, we are showing you as a function of e by k T 2 factors separately on the upper graph and then the product on the lower graph. So, what you see is this product goes through a maximum and then again it decreases and becomes very small later on. The scale here you may not be able to see the scale. Here is 5 into 10 to the power minus 19. So, for this lower curve do not get misguided by the large height that is being shown here this is 10 to the power minus 19 scale and here it is 10 to the power minus 2 scale. So, as compare to the scale here y axis scale here device scale here is 16 orders less but, in that it goes through a maximum and then it becomes very small. So, let me change the kinetic energy k T little bit temperature little bit and see what happens from 1 lets go to 5 or 4.

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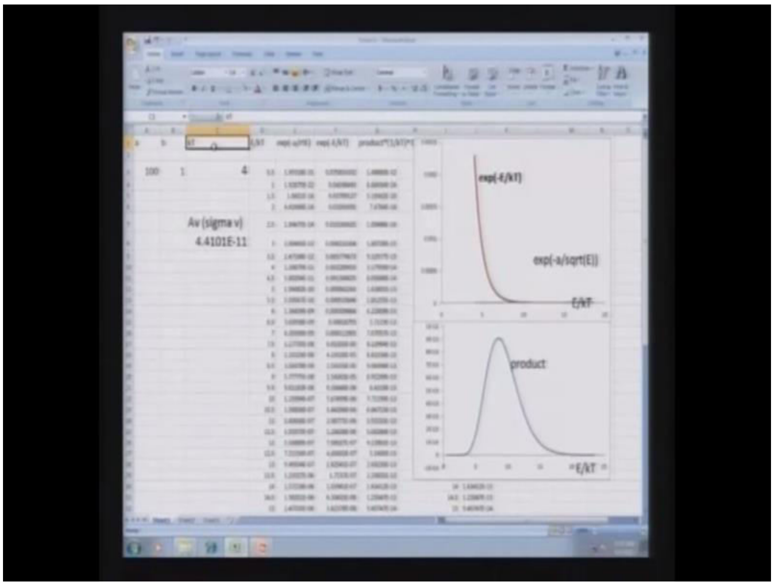

So, I am putting 4 here. Now k T is 4 and see what happens not much visible. So, let us go 6. What happens?

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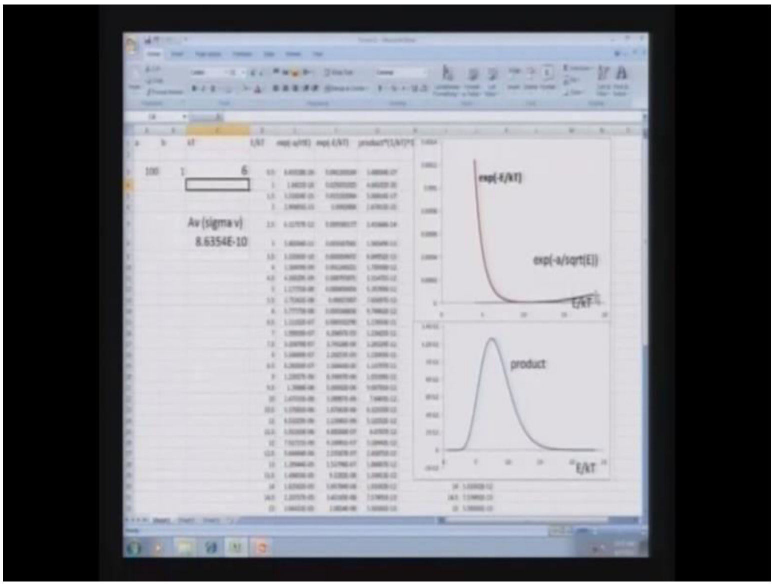

Now, you can see this blue line coming up on this curve here upper curve you this blue line is coming up at 6 k T is equal to 6 kilo electron volt. The factor the barrier penetration factor e to the power minus a by square root of capital E that is started showing up on this scale.

So, the red curve drops where as the blue curve rises as you go for higher and higher value of e. And then the product comes from somewhere this area where the 2 have some appreciable value beyond that the red line goes to almost 0 and therefore, the product is almost 0 here and before that the red line is there but, then the blue line is almost at 0 level and so the product is almost 0 here. So, if you look at the lower curve and here of course, the scale is all now it is 10 to the power minus 12. So, when I increase this k T from 1 k e v to 6 k e v all this product has increased from minus 19 order to minus 12 order. So, the total area has increased here.

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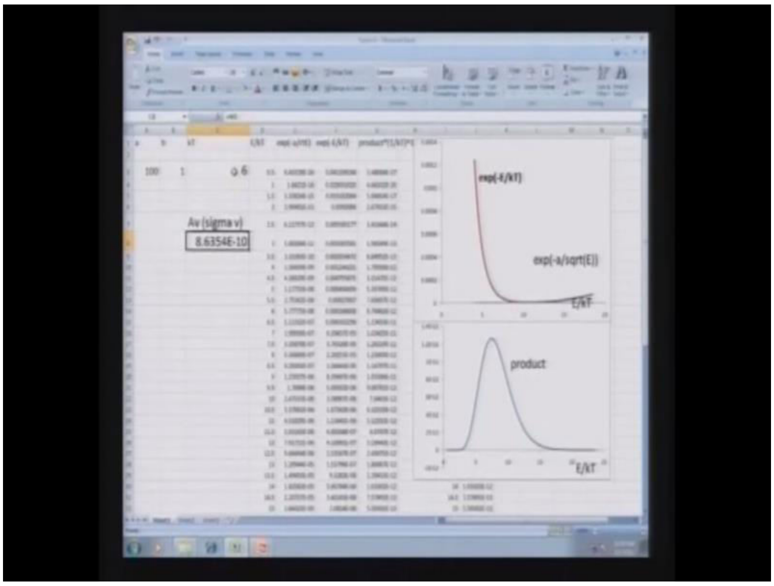

The total area is also shown here. The total area is obtained by just adding all these product terms and property multiplying by some factors. So, the area number in number numerical value is given here.

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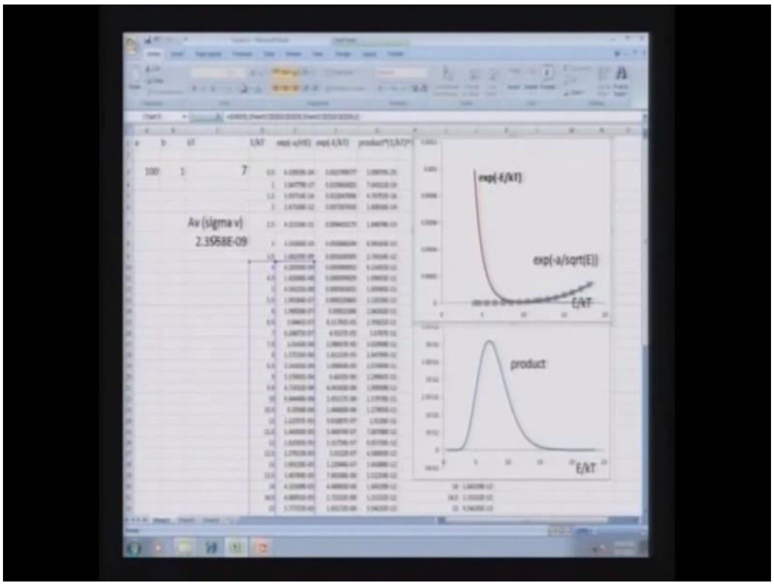

If you go slightly beyond say 7 k e v see what happens. So, you can see that barrier penetration probability here as increased considerably and the product here is also increase it was 10 power minus 10 here. Now it is 10 power minus 9 so that, increase this scale is increased. So, important part is that the integrant it really does not go to up to infinity that integrant becomes quite small once you have say in this particular diagram 15 k e v or e e by k T equal to 15. More than 15 the product goes to very small values this is how the thing goes. Now I will just show one more thing, suppose I increase kinetic energy and see what happens look at the kinetic energy here k T here and the value of this integral the whole integral sigma v integral.

So, let me zoom it slightly more and then I will change this value of k T and look at this is the integral.

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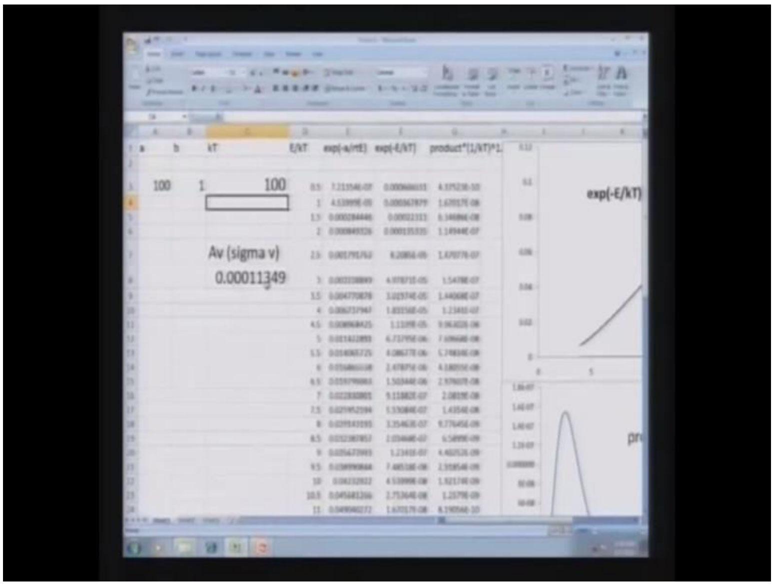

So, look at this from 7 I am going to 100 the number was e 10 to the power minus 9 and see what happens a it becomes much 10 to the power minus 4 or so increased.

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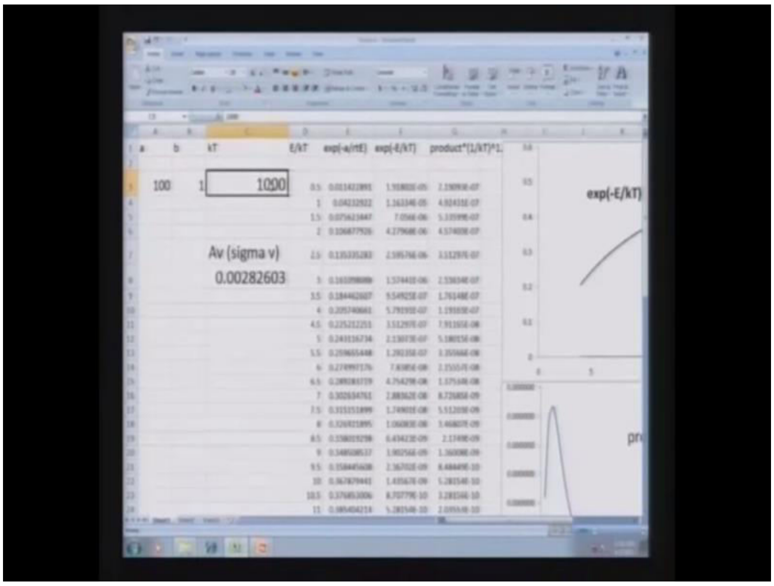

Go beyond 1000. Further increase 0.02 see the value here this 0.002.

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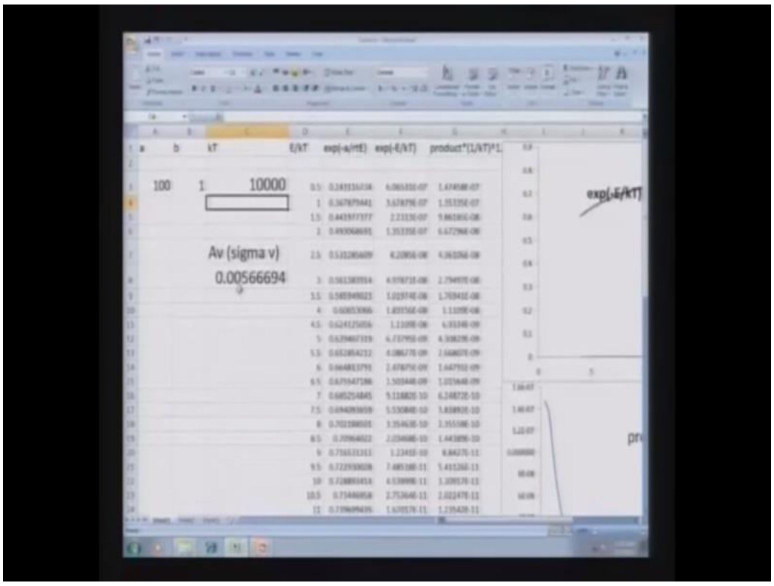

Further increase 10,000 from 0.002, 8.0056 increasing.

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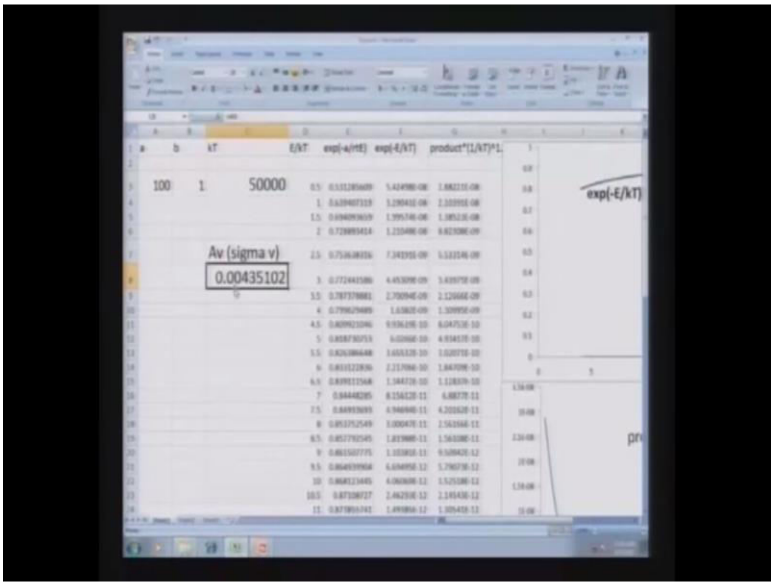

Go for 50,000. This is 0.0056 at present and it is 0.0043. So, it has decreased. Go for 1, 00,000.

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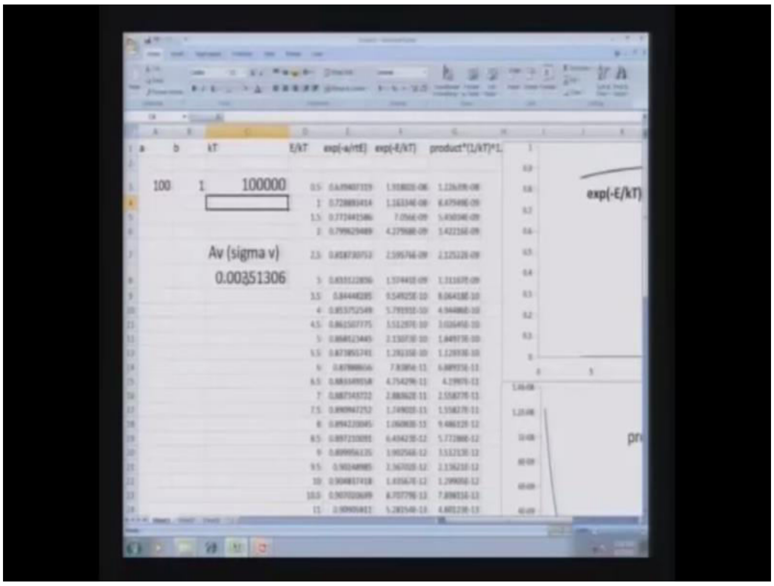

From 0.004 it has become 0.003. So, this integration if you go to very very high temperatures then this integration the value of sigma v will again start decreasing initially it increases at 1 k e v.

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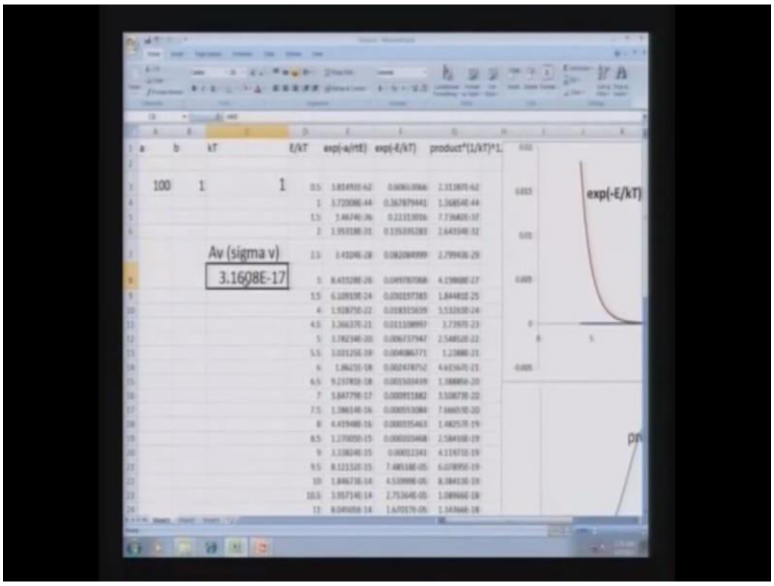

Now, it is 1 k e v it is 10 power minus 17.

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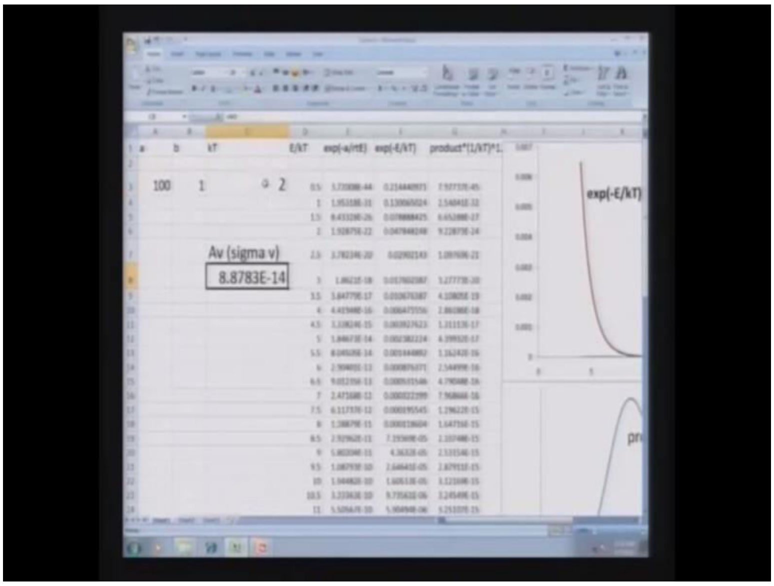

And from 1 if I go to 2 it is 10 to the power minus 14.

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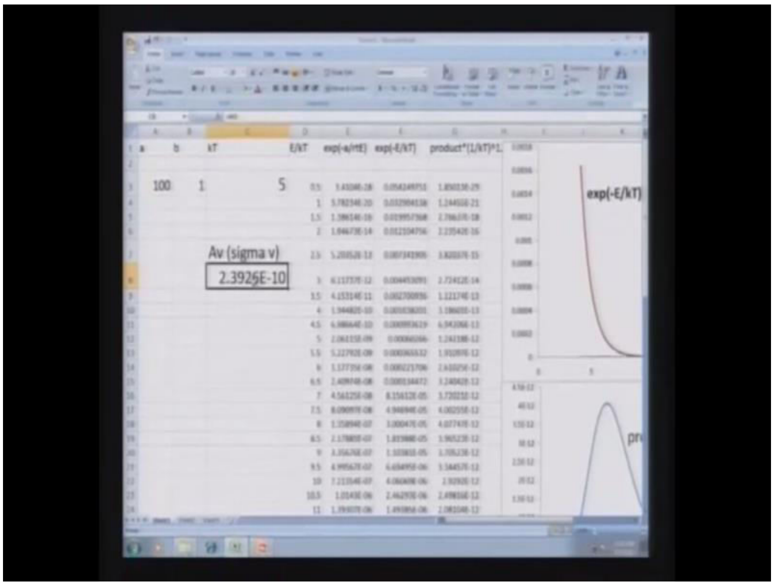

From 2 if I go to 5 it is 10 power minus 10.

So, initially this sigma v increases with temperature but, if you increase the temperature beyond a certain limit it goes to very very high values, so going to the board now other 2, 3 minutes.

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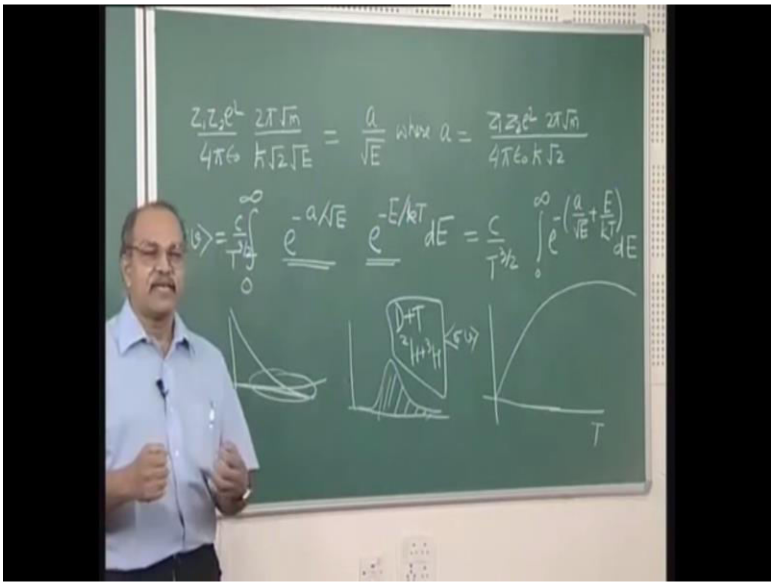

So, this product we have seen this exponential and this exponential and 1 exponentials just drops like this is e to the power minus e by k T and this rises somewhere like this. So, it is this region which contributes and the product goes like it goes through a maximum and it is this area it is this integration that gives you sigma v which depends on temperature initially this sigma v here temperature initially it increases and then if you increase the temperature very high values it will start taking it downward. So, using these kinds of things one can calculate the reaction rates of course, the concentrations are needed.

So, one can calculate the reaction rates for a given concentration. What is the rate for different nucleus precious? So, it turns out that for low z nuclei which we are interested in if we have design a thermonuclear reactor. So, for low z nuclei fusion which for coulomb barrier has to be low this deuteron plus triton 2 h plus 3 h this has the best D plus T. This is 2 H and plus this is 3 H this particular reaction has the best possibilities. So, we will talk more about it in the next lecture. What kind of considerations are there to design a thermonuclear fusion reactor? Why it is not yet commercially available? Why scientists are taking decades and decades of research? And still are not coming up with a fissile design which can give us power from nuclear fusion, all these topics in the next lecture.