Lecture - 14
Scattering of nucleons

(Refer Slide Time: 00:20)
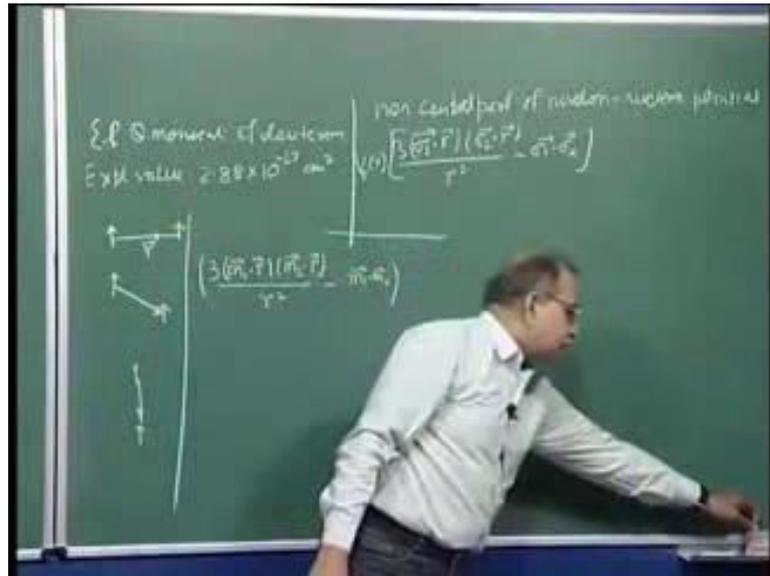
So, quadrapole momentum we were talking electrical quadrapole moment of deuteron and the measured values, experimental value was 2 point how much?

Student: ((Refer Time: 00:38))

Into what?

Student: centimeter square.

Centimeter Square. Though it is small when you compare with quadrapole moment of many other nuclei, but still it is significant, and it tells that the charge distribution is not spherically symmetric. And that means the this l equal to 0, alone is not the very correct description. So, it is mixed with l equal to 2; the same conclusion that we drew from the magnetic moment, right. So, if the angular moment orbital, angular moment are getting mixed up in the energy eigen state; that means, the Hamiltonian is or the potential is not central potential. This nuclear interaction should have a non central term. The other

familiar situation where this non central force comes in is when you consider interaction of 2 magnetic dipoles.

If we have 2 magnetic dipoles, one dipole here and another dipole here, so this is let us say separation vector is r, the dipoles are oriented like this. So, these dipoles exert forces on each other; so there would be some potential corresponding potential energy, you can write. Then the same dipoles you can arrange in other fashions, like this or like this. So, the force between these 2 dipoles, and force between these 2 dipoles, and force between these 2 dipoles are different. So, this separation vector are as it changes its orientation, you can fix up some z access and x y theta phi r theta phi. So, the force is depending on theta phi; depending on angels. If the dipoles are placed this way, so the r vector is this way; force is different, same separation, same magnitude of separation, same number of centimeters, but if you put the, if you just orient it in different fashion, so that r vector is now this, same separation. So, in r theta phi r remains same, theta phi changes; but the force changes. So, this is one kind of non central force.

And, if you write the magnetic energy correspond to this dipole dipole interaction, that turns out to be 3 time m 1 dot r and then m 2 dot r divided by r square; this is unit vector r here in fact and then minus m 1 dot m 2, some constants something like this. If you taking clue from here, it will write the non central part of the nuclear potential of nucleon nucleon potential as 3 times sigma 1 dot r sigma 2 dot r divided by r square and minus sigma 1 dot sigma 2. And this whole thing has to be multiplied by some r dependent part.

So, this type; there are many other term which one has to be bring on to really explain all the experimental data. The if you try to write the potential between nucleon it is a long expression, right. You had seen a central part, spin in dependent central part; that was the basic square well potential that considered. On that spin dependent part p sigma and now the non central part and then there are more terms also; it is a long procedure ok. So, this much we learn from what we call deuteron bound state. Now, the next phase is you can also learn about the interaction from the scattering states. That is why it is not bound but it interacts and goes. So, how that interacts and that will result in different distribution of scattered particles and from there you can derive information about the interaction.
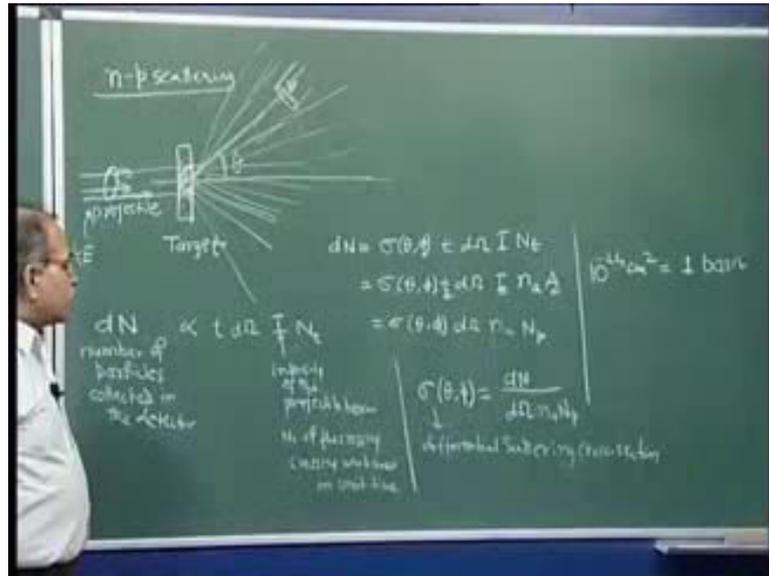
So, in a scattering, the geometry is something like that you have some kind of material which you call target and then you send beam that you call projectile. Normally, a parallel beam or projectile particles are sent; and they hit the target material and energy is controllable so you give some kinetic energy, fixed kinetic energy of these projectile particles. Then from this target, in the target you have scattering centers because in any experiment you will not put one proton or one molecule or one atom, all right. When you are doing the experiment in lab, you have to put some foil or some plate or some gas something; that will have many centers.

What we are trying to work out is n-p scattering. So, here you will have target containing hydrogen and the nuclei, the protons will be the scattering centers, that will be fixed here in the target and neutrons will come from one side; so the projectile particles are neutron. But any scattering experiment, you have a projectile beam; normally, you the kinetic energy of each particle in the projectile is controlled, tuned. So, they are mono energetic particles falling on it on the target. And target has number of scattering centers; each one can interact with each one here. So, depending on which projectile particle is going close to a particular target scattering center, that interaction will take place in that event all right so this way.

On the other side of you, of this, you put a detector; normally this is quite small this beam size is may be one millimeter or even less. Now a days, you have, micro beams

where the accelerator gives you a beam of irons, which can be focused to say one micron or even less may be 10 to minus 3 millimeter 1 by a 1000 of a millimeter. So, you can have very focused iron beam right, so those things are there; but typically say millimeter or less. So, although, I am growing the diagram this is not to the scale But this portion is very small and this thickness is also normally small so you put a detector somewhere at some angle. So, if the particle are going in this direction this detector will pick it up and this, you take as a z axis and this angle is theta you can also change But normally this scattering is independent of phi it depends only on theta in this type of geometry.

So, for different values of theta that means, you put detector here, you can put detector here, you can put detector here, here and so on; and you get the distribution of scattered particles. How many particles are going at this theta, how many particles are going at that theta, how many particles are going at that theta, and so on. So, this is the experiment that data is analyzed and whatever information about this interaction is needed that is derived.

So, the number of particles at a particular theta; there are some trivial dependences, trivial dependences like suppose I collect this data for 5 minutes and in the next experiment I collect the data for 2 minutes. So, we know that in the second experiment we will get less number of particles, we have done only for 2 minutes in the previous experiment we did it for 5 minutes. So, it is an obvious dependence right trivial dependence on time, it has to be proportional to time; more time you allow for data collection, more number of particles will be collected.

Similarly, other is how big is, because these are particles are going in all directions not equally; some directions are preferred some directions are less preferred and so on. But in this direction itself, suppose your detector is placed at the double of the distance right in first experiment you put the detector here; in the second experiment you put the detector at double the distance. So, if these lines are diverging like this and you put the detector at double the distance; so less number of particle will enter that area of the detector. So, the solid angle subtended by the detector on the point, on the target where the beam is coming; it will be proportional to that. Because it is going in all directions, in all angles, so if you pick up a small cone the sudden particles going there; if this solid angle is increased; if your detector is placed close to the target detector window is large it will collect more particles.

So, it is proportional to the solid angle, these are some obvious dependencies. So let us see the dN; what is dN? Number of particles collected in the detector. This should be proportional to time, this should proportional to d omega then this should be proportional to the intensity of this projectile beam or sometimes it is called flux. That means, how many particles per unit area, per unit time are going in the beam. So, perpendicular to the beam direction if I construct an area and look for certain time delta t how many particles are crossing this area, right make it per unit area and per unit time that is called intensity of the beam number of particles crossing this perpendicular surface per unit area, per unit time that is intensity.

If that intensity is increased, the dN will also increase; more number of particles are falling here; so it is proportional to this I, intensity of the beam of the projectile beam right, which is number of particles crossing unit area in unit time and unit area does not mean 1 meter square; if you if you take 1 meter square your beam is only going here not in that entire thing.

So, in the beam cross section right, in the beam cross section or you can take the whole beam cross section itself and then look how many particles are crossing it per unit time; divide by the beam cross section that is intensity. And one more thing, since it is not one particle sitting here it is a target; so how many particles are there to scatter, how many scattering centers are there in the path of the beam. This is the path of the beam so the particles can be scattered from here, here, here, here, here, here, here and so on. So, how many target particles are there number of target particles; so that the number of target particles. So these are trivial or obvious dependences ((Refer Time: 15:11)) has to be proportional to that; whatever is the kind of interaction right But the proportionality constant, that is really related to the kind of interaction that the projectile particle and target particle are having.

So, you write this as dN is equal to either we use symbol d sigma d omega or you use symbol sigma as a function of theta, theta phi. If you write theta phi; this is the proportionally constant and then all these things t d omega I and number of target particles. Number of target particles is generally expressed as area of cross section of beam because only that much area is hit, right; area of the cross section of the beam projectile beam only that much area on this target surface is hit.

So, areal density means in this entire target how many particles are here and divided by this area, that is called areal density its actual density into thickness so areal density areal density into area. So, n a this is areal density into area so this that the number of target particles it depends on the density there in the target and the thickness so it is this. You can combine this I, this a, and this t, this becomes how many projectile particles are going through that target in that time. I is number of particles going per unit area, per unit time and here is area and here is time so multiply by that. This is a number of projectile particles which are hitting the target; so you can write inters of that also, sigma theta phi and then d omega and this n a areal density and then number of projectile particles hitting the target.

So, sigma theta phi is dN divided by d omega n a and np. This is how and this is called differential cross section; this is called differential scattering cross section and this contains the information about the kind of force, the kind of potential, kind of interaction which is taking place. All the geometrical factors we have already taken out; so this is purely it is representing the kind of interaction; which is taking place when projectile particles are close to each other, right which causes a scattering.

So, this experimental arrangement allows you to measure this cross section. You measure dN, you count how many particles are going there at that particular theta phi so dN is there; then you have placed the detector at some distance and some area of the detector window. So, you d omega how much is d omega and then areal density what target you are using? What is the density of that? What is thickness of that? Number of projectile particles during your experiment whatever time you have allowed how many projectile particles are hitting. So, all of that can be calculated, can be obtained, can be measured, counted and from there experimentally you can obtain the scattering cross section.

So, idea is do an experiment, find this cross section from the experiment; then the theory you have in mind from that theory you derive what should be the cross section and then match and see your model, your theory your calculations are correctly reproducing that section or not. And then you can adjust the parameter, you can change the model, you can modify, you can do something else; come out with some theory which is consistent with the measured results ((Refer Time 20:19)). So, the whole purpose was to tell you how people do this experiment; starting from Rutherford alpha particle scattering and

lots of in atomic physics, nuclear physics, scattering experiments are done abundantly even now alright. This basic formulation, the basic meaning of this cross section every time you ((Refer Time: 20:46)) hear this word cross section, cross section for this interaction, cross section for scattering, cross section for caption, cross section for absorption so many things, so this is it.

You can look at the unit of it or dimensions of it; this is number, this is angles, solid angle and this is again number so it is 1 by n a areal density number of particles per unit area. So, this is area inverse and since itself this is denominator this whole thing has dimensions of area. So, the cross section, the values of cross section will be coated in units of area and 10 to the power minus 24 per centimeter square; which is 10 to the power minus 28 meter square that is ((Refer Time: 21:44)) kind of the order of this cross section. So, this is given another name and is called what? So cross sections are generally coated in these units. So, let us see from the; this neutron proton thing what do we expect if assume that square well potential and so on.

(Refer Slide Time: 22:18)
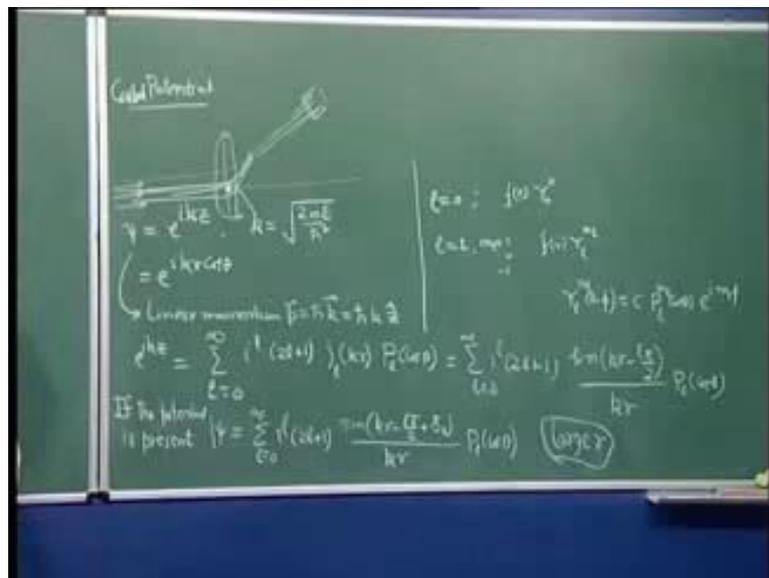
So, again some basics of scattering by central potential so here is the origin and then you have a central potential; that means the interaction between the projectile particle and the target particle is represented by a potential which depends on only r naught on theta phi, central potential. So, if the particle is coming; and then getting a scattered what? How we calculate from theory this scattering cross section for central potential? Now, if this

potential is not there; and only the beam is coming then there will be no scattering, the beam will just go and then wave function will be e to the power ikz; z is the direction of the beam and what is k right free particle going in z direction with an energy capital E is represented by this wave function right. You can write the ((Refer Time: 20:49)) and h psi is equal to e psi, no potential energy term only kinetic energy term and from there you can work out.

So, a particle which is going in z direction with energy E will have this wave function, free particle wave function. So, if there is no scattering center right, if there is no scattering, no potential v is equal to 0 everywhere; then this is the wave function. Since, we have to work with scattering and this is the geometry to put up origin here; and then as a function of theta phi, we are measuring the number of count and all that. So, we can write this as, in terms of r theta phi.

So, this accepts, this is e to power I k r cos theta; and in this what is the linear momentum; linear momentum p is h cross k. So, if you have fixed the energy, k is fixed and this linear momentum is also fixed k and this is h cross k in z cap direction. So, if the energy is fixed, this k is also fixed and if this k gets fixed then h cross k that means the linear momentum is also fixed. So, if you write the wave function in this form, it is a simultaneous Eigen function of the Hamiltonian because energy is fixed; and linear moment operator because the linear moment also fixed.

What about angular momentum? This is not an Eigen function of angular momentum operator. So, the same state of the particle; if you ask what is the linear momentum I have a definite answer this is the linear momentum. But if you ask what is its angular momentum about that origin it is not a definite angular momentum state it has got several momenta simultaneously present right; living out spin part the angular momentum corresponding to this motion. So, small l equal to 0 small l equal to 1 small l equal to 2, three they are all mixed up to make this state. You know if l equal to 0, you should have some r dependence and theta phi dependent should by Y 0 0. l is equal to 0, m equal to 0; if l is equal to 1 then you will have several states ml is equal to 1 and 0 and minus one so on, and this would be some function into Yl ml.

So, all those things are there; and our wave function is not of that type, right. This wave function is not of that type so it does not have a fixed angular momentum; it is not l equal

to 0, it is not l equal to 1, it is not l equal to 2. But it is a super position of those l equal to 0, l equal to 1, l equal to 3, l equal to 4 and so on. So, in what proportion do I super pose them to get that; so that expansion can be done and when you do that his e to the power ikz can be written in terms of this. First thing, you can see there is no phi dependence here it is only theta dependence. And in violence if this ml equal to 0 then you do not have phi, but if ml is there, you have e to the power iml phi right this Yl ml which is a function of theta phi, it is some constant then this pl lays under Pl ml cos theta then you have e to the power i ml phi.

So, if it does not contain any phi that means all this ml should be 0; if ml is not 0 you will have phi dependence. So, in making these combinations, only those ml equal to 0 terms have to be taken, l equal to 0 1, 2, 3, 4 you will be taking; this turns out to be submission over l going from 0 to infinity. Then I to the power l, 2 l plus 1 and then some Bessel function coming in jl, kr, and pl cos theta; ml equal to 0. The expansions are standard procedure if you know this, you first write as some constants time at this pl cos theta and so on; and then you have to do an integration to get that constant and then you can drive these expressions.

So, if there is no potential involved, no interaction involved then the wave function will be of this type, but there is an interaction; so the wave function will not be this, will be something different, right. Now, if detector is placed here and I am taking the particles here and this target atom or target nucleus is here and the projectile particle is coming and getting scattered from here; whatever this way or that way or this way in whatever kind of interaction it is, look at the distances this range of potential in nuclear case it will be in femtometers if this separation say 20 femtometers then almost no interaction.

So, whatever interaction is taking place that is taking place in femtometers or if it is atomic case atomic collision case this could be angstroms, right. Whereas, the detector you will be placing at least few centimeters this distance is very very large as compared to whatever r you will be writing for that interaction. So, we are interested in the wave function or what we can access right; we can access to wave functions at large values of r right. What is the wave function right here, I do not have probe to go there and look at that; our detector will find the particles only at large values of r. What happens when r goes to infinity what happens to this jlkr; jlkr turns out to be, it goes as sign of kr minus l pie by 2 right, very good, divided by kr that is how it goes. So, you can right this wave

function as summation over l and I power l, 2 l plus 1 and sign of kr minus l pie by 2 divided by kr and then legender polynomials. It turns out that, when you have the potential, you have the interaction; this wave function will get modified and the modified wave function can be written almost in the same format with one difference that all these sin arguments here, they are changed by certain amount.

And, that change that extra phase term coming here will be dependent on l for l equal to 0 that term could be different, l equal to 1 term can be different so on. So, the wave function that you write in presence of potential, the one which is already written is, when you have no potential; it is just e to power ikz that we have expanded. If the potential is present, if the interaction is present then this will be just changed to; the your psi will be changed to l equal to 0 to infinity and all those terms will be there and sin kr minus l pie by 2 plus delta l, some delta l will be there divided by kr pl cos theta. Remember, these things I am writing for large r; these delta l are called page shifts.

(Refer Slide Time: 34:46)
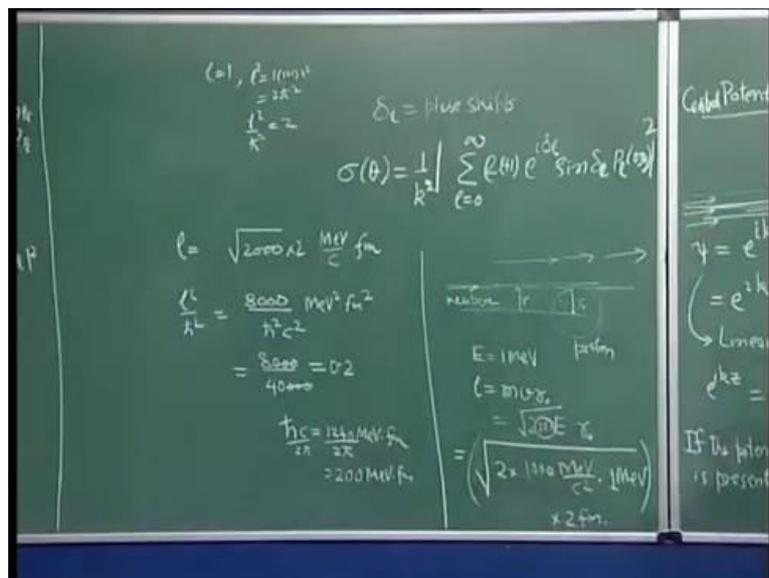
So, effect of this interaction is hidden in this delta l, alright; these are related to what kind of interaction is there. And therefore, these delta l's must be connected to the scattering cross section; scattering cross section gives you the strength of scattering the interaction between the target material and the projectile material in each event. And the effect of interaction is in delta l; therefore, this scattering cross section must be related to this delta a and that expression turns out to be sigma theta is equal to 1 by k square; k

you remember what is k; l going from 0 to infinity 2 l plus 1 e to the power I delta l, sin delta l, pl cos theta and n square, right, this full thing square.

So, now what is the next task? Next task is, start with the central potential that you assume, that you want to try out; take a central potential which you think is the potential corresponding to neutron, proton interaction; neutron proton force. With that potential, do some quantum mechanics and get delta l and from there you get the scattering cross section and compared with the result there so we will now do that.

So, you get contribution from all these l; l equal to 1, 2, 3, 4 and so on. But then if energies are small, you do not really get significant contribution from too many angular momenta. For very low energy only l equal to 0 will be significant, l equal to 1, 2, 3, 4 all the contribution will be neglible. If you have some more higher energy may be l equal to 1 will start contributing significantly so it is not that you really have to add large number of terms here, and we can make a small semi classical calculation to look at that energy angular momentum relation. Suppose, you have the particle here, target particle here, and the projectile particle is coming like this and the nuclear range is r naught not the interaction range is r naught, distance r naught.

So, that means in this region, when the particle reaches then there would be some interaction. But if, the particle goes like this then there is no interaction; it just goes straight. So, for really scattering to happen this particle must go into this sphere so the range is, let us say r naught, right. Now, let me take energy of this, this is say proton here and neutron is coming and going like this and energy is say 1 MeV. Let us try to calculate the angular momentum using our classical picture, particle going like this.

So, if this distance is r right if this distance is r the angular momentum will be m times, v times r; p into r. Now, this p is square root of 2 times, n times the kinetic energy and then r; let us take it at the edge I have want to have maximum value of linear momentum for which this scattering may occur. The angular momentum may be this distance multiplied by linear momentum; but that this scattering will not occur. So, I am taking this r naught so take it r naught.

What is this E kinetic energy, this is, p is p square by 2 m is kinetic energy I am using classical picture. So, 2 mE time ((Refer Time: 40:13)). Now, what is this? 2 times mass of the particle, mass of this proton is some 938 MeV or so let me write it as 1000 MeV to

make a quick calculation and mass is MeV by c square, right. Because mc square is thousand nine thirty eight MeV right proton rest mass energy mc square is 938 MeV so m is 938 MeV by c square. So, MeV by c square and kinetic energy let us take 1 MeV. So, it is just 1 MeV and into r naught r naught how much r naught I should take say 2 fmtometres so let us work it out. This will be square root of 2 thousand MeV by c MeV by c MeV by c and into 2 and femtometres that is the angular momentum. So, let us work out what is l by h cross? Because if l is equal to 1 l square is 1 plus 1 h cross square that is 2 h cross square. So, l square let us write l square y to do square root l square by h cross square l square by h cross square, should be 2 if l is 1 this is 2.

So, let us workout who much is this? This is 2 thousand into 4, 8 thousand right, divide by h cross square, c square MeV square, femtometer square. Now, how much is this h cross c? hc you remember h into c 1240 MeV femtometer, so h cross means divide by 2 pi, so 3 into 2 is 6. So, above 2 hundred so this is 8 thousand divided by 2 hundred; square of that so 4 double 0, double 0 and MeV femtometer square right and here also MeV femtometer square so that cancellers out it is number, should be number it should be number it is coming out to be number how much it is? 1 by 5 so instead of 2 what you are getting is point 2. So, that means l equal to 1 is aha not at all present; your l equal to 1 will happen if the angular momentum square is 2 h cross square and with this kinetic energy of 1 mega electron volt; ((Refer Time: 43:58)) the angular momentum so that it goes into the range. And that distance you cannot increase 2 femtometer is the range nuclear range so it has to be that r has to be less than 2 femtometer and kinetic energy is fixed 1 MeV.

So, linear momentum is fixed so you cannot increase the angular momentum with this parameters you increase the particle you can take it at larger distance. But then the scattering will not take place so for scattering to take place with 1 mega electron volt particles l has to be much, much smaller than that the root 2 h or so. And the only value which is available less than root 2 h in quantum mechanics is 0; l equal to 0 or l equal to 1 there is nothing in between. So, if you are not able to reach l equal to 1 you are in l equal to 0 right if it is 10 MeV if you take this as 10 MeV so it will be multiplied by 10 right here in state of this 1 MeV if you take 10 MeV so that means this will be 2.

So, around 10 MeV etcetera this of course, this calculation is very approximate and classical type and semi classical type and this and that. But this, tells that 10 MeV if you

are if you are 10 MeV kinetic energy particles if you are sending then vary about l equal to 1. 1 MeV is very safe, if we are lower than 1 MeV so even if this model is approximate and we are using classical physics arguments and calculations and quantum mechanically this is not true and so on, so on. But then giving all the margins if I say that I am confined I will my kinetic energy will be less than one mega electron volts then we are very safe to say that fully l equal to 0 will contribute to the scattering. Now, if you do that l is equal to 0 low energy scattering, for low energy scattering and low energy scattering means e less than 1 MeV so let us say; this sigma theta will be 1 by k square.

Now, l equal to 0 to infinity I only need l equal to 0 term and l equal to 0 suggest 2 l plus 1 is 1 and since there is no summation, no more summation only l equal to 0 term we are looking at this e to the power I delta l; when you do mod square will become 1. e to the power I delta l, mod square of that; so multiply by its complex conjugate e to the minus I delta l so that will become 1 so sigh square delta l, sign square delta 0, l equal to 0. And l is equal to 0 at first legender polynomial is one so this is also one so that is it. Differential cross section will be obtained just 1 by k square this is related to energy kinetic energy square root of 2 m e by h cross square and then sin square delta e, delta 0.

So, if you can you can get delta 0 from theory and this delta 0 from experiment, because differential; total cross section you can write, total all theta phi integrated. Since, in this you do not have any theta phi, in this expression you do not have any theta phi, theta is coming from here. Normally, it should depend on theta and that dependence is here but if you are in low energy so that only l is equal to 0 is operative then there is no theta dependence it is isotraphic, scattering is isotraphic. And in that case the total will be just integrate over all solid angles so 4 pi. This quantity can be obtained from theory and also from experiment and then compare and what do we check? The theory is correct or wrong that we are checking or the experiment is correct or not that we are checking.

Suppose, this, it does not match I work out from theory that cross sections should be this much and then I take theta from experiment and find that cross section is this much and these 2 are not matching theory; we have to look at experiment is experiment, make sure that we are done the experiment carefully. But once, you have done it carefully it is repeated and same results are coming all those things are taken care of work out your theory to explain that result.
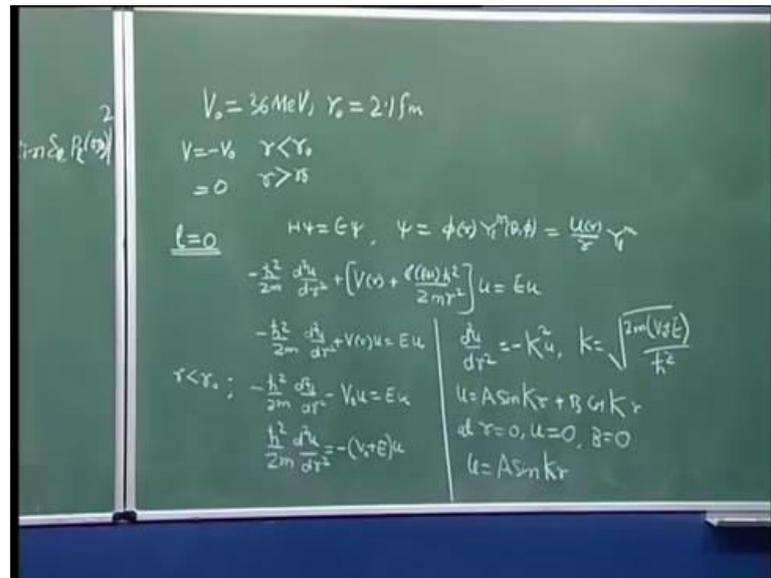
So let us see if we can, the potential between neutron and proton that gave us deuteron bound state. If I use that potential, that interaction what value of sigma do I expect for V naught square well potential. V naught is equal to 36 MeV and r naught equal to 2.1 femtometer, square well potential V is minus V naught for r less than r naught and is equal to 0 r greater than r naught. This potential was able to give me correct energy, bound state energy; can this potential give me correct scattering cross section at energies of few kilo electron volt or few hundred kilo electron volt. So, workout the theory how do you workout? First if you do not, first thing is at since we are in low energy region we will only be talking of l equal to 0 right and if, l is equal to 0 then the your ordinary question h psi equal to e psi h is you can right psi as the r part and times Ylm cos theta Y lm theta phi.

So, that, theta phi part is spherical harmonics you know that for central potential; and this part radial part is u by r. So, you define u in this fashion Ylm and this u and of course, since we are doing l equal to 0 this is this is going to be constant. And for u your differential equation is minus h cross by 2 m, h cross by 2 m; d 2 u d r square plus V r plus l is 0, l is 0; let me write first l l plus h cross square by 2 mr square times u is equal to eu. And then l is 0 so it is minus h cross by 2 m b 2 u br square and plus V ru is equal to eu. So, there are 2 region r less that r naught; where V is equal to minus V naught. So, you have minus h cross square by 2 m d 2 u dr square and minus V naught u is equal to eu. So, h cross square by 2 m d 2 u dr square is equal to minus V naught plus e times u,

correct. This is, with minus and this is also with minus so this; V naught remember is positive quantity 36 MeV and e is also positive quantity, we are talking of scattering not of bound state so we are sending the proton with some kinetic energy.

So, that is e the so this is also positive so this thing is now d 2 u dr square and equal to minus capital k so let me write it capital k square; u where capital k is square root of 2 m V naught plus v over h cross square, right. 2 m over h cross I bring it here minus I am keeping outside and this thing is capital k square. And then you know the relations, solution u is equal to a sign Kr plus b cos K you can write; but then you have boundary condition at r equal to 0 u is equal to 0 from where this is coming? This is coming from this very definition your ideal part is u divided by r and this has to be finite every were so at r equal to 0 this u should be 0 only than it can be finite. So, and that gives you b is equal to 0, u is a sign kr and for the other region r greater than r naught what happens V 0 the potential is 0 or r greater than r naught V is equal to 0 so V 0 is put V 0 is here.

(Refer Slide Time: 55:15)
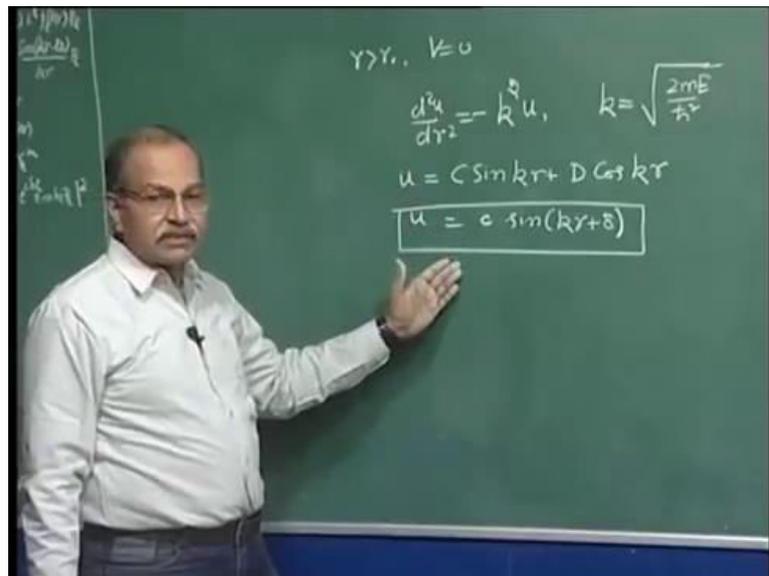
So, you have d 2 u dr square is equal to minus small k square u there small k is, square root of 2 me over h cross square. So, just I am putting this V naught equal to 0 so you will get this. And here, also your solutions will be like u is equal to some C sign kr plus some d kr. This region does not contain r equal to 0 so both term can be present you can write this as some other constant give any name e no e is energy. So, something some constant; let me write it is small c; although it does it look similar. But does not matter

sign kr plus 8, right. So, outside it will be this and inside it will be this; where K is given expression here and k is given by this expression here. So, remember these things we will start from here next lecture this r greater that r naught u is this, r less that r naught u is. This, we will do that matching conditions slope should be continuous and the value should be continuous and that will give me the face shift delta.