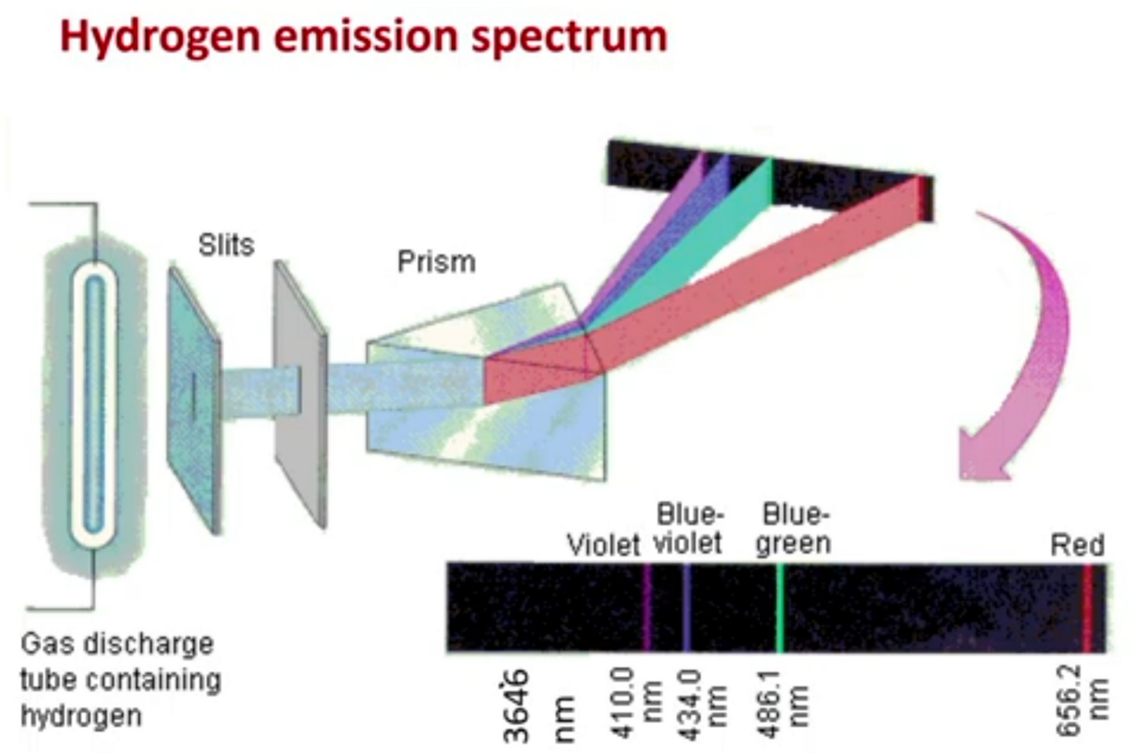
ELEMENTS OF MODERN PHYSICS

Prof. Saurabh Basu
Department of Physics
IIT Guwahati
Week-01

Lec 5: Hydrogen Spectra, Stern Gerlach, Compton Effect

To continue our discussion on the quantum theory, let us look at a few things such as hydrogen spectra. We will see this series, the Balmer series, Lyman series and so on. We will talk about the correspondence principle due to Bohr and then we will do a few, rather go over a few experiments which have established the quantum nature of matter that we see. So, we talk about Stern-Gerlach experiment, we talk about the wave particle duality and right now we just talk about the particle aspects of light and they would be shown in two experiments called as a Compton effect and the photoelectric effect. So, what happens is that, so you see a experimental setup where it is only a schematic setup. So there's a gas discharge tube that contains hydrogen. So why we study hydrogen emission spectrum to begin with is that hydrogen is the simplest atom which has got one electron and one proton. And so it's just a one electron atom and the simplest one and also that hydrogen is there everywhere. So we start with this hydrogen spectrum and try to understand that what are the features of that. And so here there is a gas discharge tube containing hydrogen. So this hydrogen comes out and it passes through two slits as it's shown here.

And then either a prism or a grating here prism is shown and they are made to fall on a spectrograph or this strip that you see the black strip. And the black strip is shown here where you see different wavelengths such as, you know, there are these. This has to be in 0.46. All right. So now you see that there are various wavelengths. you know, wavelengths that are shown on the screen. So, what happens is that this hydrogen atoms being, you know, discharged from this tube. So, they absorb energy and go on to these higher energy levels. And from the higher energy levels, because these have very short lifetimes, they cannot stay forever there. So they quickly come back to the lower levels or the ground state or I mean, ground state is just a sort of quantum energy state. That's the lowest energy state. And then, you know, they sort of. emit frequencies or emit photons which have these frequencies which you see there. So, these are the emission spectrum of hydrogen atom and you see a red spectra at 6562 angstrom or 656.2 nanometer and there is a blue green, there is a blue violet and so on.



So, these spectra are seen due to the emission lines of the hydrogen atom. Now, these electrons, they absorb different energies. So they go on to different excited states. And it's also true that they do not have a one step decay onto the ground state. So they take different routes for coming back to the lower energy state. So when I say a ground state, you should interpret it as lower energy states. And this is what happens. And that's why you see that on the plate or on this photographic plate that you see there, there are these lines that are produced. There's something very interesting that happens here. You see discrete lines from red, blue, green and so on. And then you see here it's almost continuous. It's shown as black, but this black. continuous shades of lines that appear here. And this is called as a series limit. And it is, you know, Balmer who found an empirical formula for these wavelength of these emitted lines that you see there. This 3646 is something that we just saw there. And in Angstrom, so this is written in nanometers. So, this is, it is about 364 nanometer or 365 nanometer. So, this 3646 and then there is a N square divided by N square minus 4 and this is in Angstrom. So, that is the wavelength of these lines that he could fit. So, these n equal to 3 is called H alpha line, n equal to 4 is called H beta, n equal to 5 is called H gamma and so on. So, these are the lines which are called as the H alpha, H beta, H gamma and so on. So, these are the lines, red lines and blue green lines and so on.

Balmer (1885)found an empirical formula for the wavelength of the lines

$$\lambda = 3646 \frac{n^2}{n^2 - 4} \mathrm{A}^0$$

$n = 3$ is called H_α, $n = 4$ is called H_β, $n = 5$ is called H_γ and so on.

He was able to predict 9 lines of the series.

So, that is what Balmer found out, and he was able to predict 9 lines of this series. Rydberg came, and this was I am sorry this should be 1890, okay. So, there is a correction here; please note that. So, in 1890, Rydberg expressed this Balmer formula. He found it more convenient to write it in terms of the inverse of the wavelength. And remember, the inverse of the wavelength appears here because the energy is actually proportional to the inverse of the wavelength. So, that is why if you take h-cross equal to 1, c equal to 1, the energy is nothing but 1 over lambda. And he expressed this 1 over lambda as RH into 1 by 2 square minus 1 by N square , where N equal to 1 will give the Balmer series, and then the Lyman series, and so on and so forth. So, n equal to 1 is sorry the Lyman series, and the Balmer series, Paschen series, Brackett, and Pfund series. And where do they lie? They lie in the ultraviolet region of the spectrum. The Balmer series is more in the visible or ultraviolet region. These are in the infrared, and just to remind you, we have this visible range from 4000 angstrom to 8000 angstrom. So, this corresponds to the violet end, and this corresponds to the red end, and we remember it by this you know

this kind of VIBGYOR, that is violet, indigo, blue, green, yellow, orange, red. And so, it is the same formula as Balmer, the same empirical formula, but expressed in terms of the inverse of the wavelength, with RH as the Rydberg constant, which is given by this value: 10967757.6 meter inverse. And as I said, if you put this 1 square minus n square, where n equal to 2, 3, 4, 5, then it corresponds to the Lyman series. Now you see that it is 1 by 2 square minus 1 by n square. Of course, there is a bracket everywhere that is missed here. And so, 2 square minus n square, this is called the Balmer series, which is what Balmer wrote down. And there's the Paschen series, which is 1 by 3 square minus 1 by n square. And there's the Brackett series, 1 by 4 square minus n square, where n equal to all these: 5, 6, 7, 8.

Rydberg (1890) expressed the Balmer formula as

1890

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

R_H is the Rydberg constant = 10967757.6 m^{-1}

$E = \frac{hc}{\lambda}$

4000Å → 8000Å

VIBGYOR

Series	Region of spectrum	Equation for wavenumber($\bar{\nu}$)
Lyman series	Ultraviolet	$\bar{\nu} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, n = 2,3,4,5...
Balmer series	Visible/ultraviolet	$\bar{\nu} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, n = 3,4,5,6...
Paschen series	Infrared	$\bar{\nu} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, n = 4,5,6,7...
Breckett series	Infrared	$\bar{\nu} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5,6,7,8...
Pfund series	Infrared	$\bar{\nu} = R_H\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, n = 6,7,8...

So you see that, you know, when you have n equal to 1 in the first term, you have all n's: 2, 3, 4, 5, all the way up to any number that you can think of. So, as I said, electrons absorb the energy because of the discharge electrical discharge or maybe putting them into high temperature, exposing them to high temperature. They absorb that energy and go on to different levels, and these are the different levels that we are talking about. So, emission from electrons. n equal to 2 to n equal to 1, or n equal to 3 to n equal to 1, n equal to 4 to n equal to 1, is called the Lyman series. Similarly, these n equal to 3 to n equal to 2, 4 to 2, and so on, are called the Balmer series. They lie in this region of the spectrum, which is shown here. It is visible to ultraviolet, which means that it is around this region, basically. And then you have the infrared region, which is here, which are all the other series wavelengths, okay. So, these are the hydrogen lines the emission lines of the hydrogen spectrum and when we say lines, we mean a wavelength, okay.

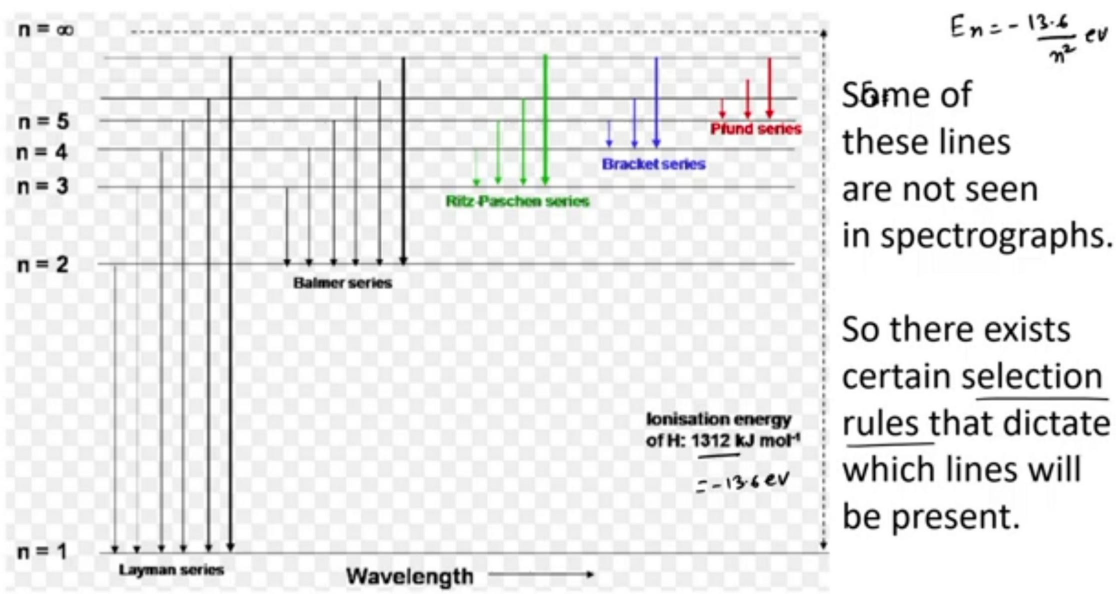
So, for heavier atoms such as alkali atoms, which are lithium, sodium, potassium, etc., the Rydberg formula is actually modified as follows: 1 by lambda equals R. R is again the Rydberg constant. Let us just come to that. So, it is m minus a minus 1 by n minus b whole square, m minus a whole square minus 1 by n minus b whole square. R is the Rydberg constant for a particular element. RH was for the hydrogen atom. And a and b are constants for a particular series. That is all the series that you saw for the hydrogen atom. m is an integer fixed for a given series, and n is a variable for that series. So, there are transitions from n, a variable number, to m, and m is less than n. So, this is how the Balmer series, or rather the Balmer formula, or the Rydberg formula, was modified for these elements alkali atom elements, alkali elements, okay. So, you see here there is a diagram of many transitions that are shown there. These Lyman series and Balmer series are just pictorially shown there. So, now from any of these lines, there is a transition to these lines. The n equal to 1 state is called the ground state, n equal to 2 is the first excited state, and you know that the formula is minus 13.6 divided by n square for the energy of the hydrogen atom. The electron has this energy in different levels, n equal to 1, n equal to 2.

For heavier elements, e.g. Alkali elements (Li, Na, K etc), Rydberg formula is modified as,

$$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{(m-a)^2} - \frac{1}{(n-b)^2}\right)$$

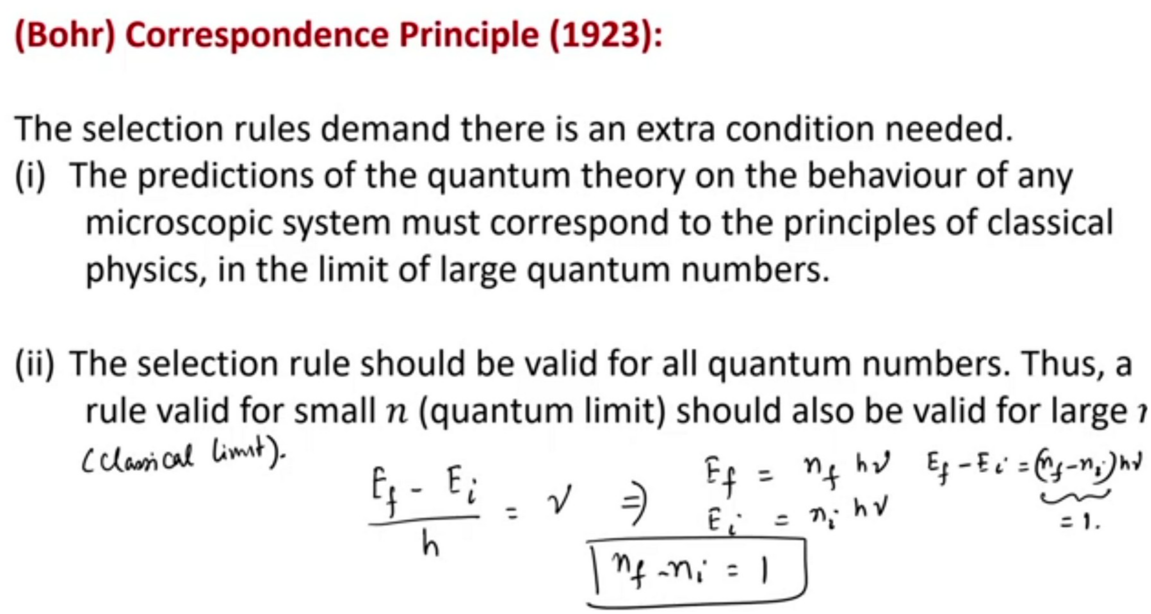
R is Rydberg constant for a particular element.
a and b are constants for a particular series.
m is an integer fixed for a particular series.
n is a variable for a series.

So, that is why these energy differences decrease as you go higher up in n, and this is just schematically shown. So, these are the Balmer series, also called the Ritz, Paschen, Brackett, and Pfund series, and so on. Okay. Now, the ionization of the hydrogen atom is taken as this thing, which is nothing but minus 13.6 electron volts. So, if you give that energy when the hydrogen atom is in the lowest state, the n equal to 1 state, it will lose that electron and become an ion by losing that electron, or you know, it sort of becomes positively charged without that electron. So, that is what it is. However, there is one problem with this diagram. Not all these transitions that are shown by these vertical arrows either black in color or green, blue, red in color are always seen. So, there is something more that is required in order to define these emission spectra or even absorption spectra. We do not discuss absorption spectra, which is pretty much more or less like the emission spectra with some minor differences that we do not want to elaborate here. So, some of these lines are not seen in spectrographs, and there exist certain selection rules. So, this is a very important thing. There are certain selection rules that dictate which lines will be present and which lines will be absent. So, I just write it here: E n equals minus 13.6 divided by n square in electron volts.



So, because of this selection rule, it demands that some extra condition is needed. And what are those extra conditions? Bohr stated that there is some correspondence principle. And it basically does not give you the selection rules per se, but it tells you that if there is a selection rule that exists in some region of the quantum numbers n1 and n2, these selection rules would also exist in another limit of the quantum numbers, n1 and n2, which are very large numbers, okay. So, the correspondence principle is really stated as the following in these two statements. So, it says that the predictions of the quantum theory on the behavior of any microscopic system must correspond to the principles of classical physics, and that is in the limit of large quantum numbers, okay. And so, basically, we will just elaborate on that in a moment. And so, the selection rule should be valid for all quantum numbers, and thus a rule valid for small n, which let us call the quantum limit, should also be valid for large n, which we will call the classical limit. Okay. So, that is an important thing.

So if you really think that you know there is an absorption of radiation, and this radiation gives you a sort of transition from a particle from one quantum state to another quantum state. So, this is actually the E_f minus E_i, which happens because of this quantum, because the energy absorbs a radiation of quantum, you know, hν. So, the frequency of the emitted photon if a particle goes from a lower energy state to a higher energy state or it decays from a higher energy state to a lower energy state, and this is exactly what we said about the hydrogen atom that absorbs this energy and goes to a higher state, and then because of the short lifetime in those higher energy states, they decay. So, this is equal to, you know, sort of E_f minus E_i equal to hν. So, E_f minus E_i by h equal to ν. So, this goes from, so it is E_f is equal to, for example, n_f hν, and E_i is equal to n_i hν, where i and f are the initial and the final state. So, E_f minus E_i, this is equal to n_f minus n_i.



hν, and so it is very clear that if there is one photon emitted in going from a higher level to a lower level or a photon is absorbed in making a transition from a lower level to the upper level, this has to be equal to 1, okay. So, n_f minus n_i has to be 1 because that is the energy of one photon, that is the frequency hν, okay. So, this is the selection rule which says that n_f minus n_i is equal to 1. That is the selection rule, and the selection rule will hold no matter what the values of n's are. So, if you are really here, at very large n, which is shown as n equal to infinity, or if you are at very small n, it does not matter, but these things will, you know, this selection rule will hold that n_f minus n_i equal to 1 in this particular case that we are talking about, okay. Let me give you another example. So, if you think about the, again we are talking about the hydrogen atom, okay. All right, so we are talking about the frequency of an electron in a Bohr orbit.

So, you see this sort of the expressions for velocity and radius that we have found out earlier. So, this is equal to, let us call it as ν_0. So, let us call this ν_0 frequency, this is equal to V over $2\pi r$. And this is equal to 1 by $4\pi\varepsilon_0$, ε_0 squared m e^4. This was done in the last day, so I will not repeat it here. It is n^2 over n^3. So, that is the frequency in a given Bohr orbit, and you can find that out because you know Vo and you know r, the radius of the orbit, and this goes as 2 over n^3 with this factor that is written here. Now, the same ν if you are talking about 2 levels, any 2 arbitrary levels which could be large, is again 1 by $4\pi\varepsilon_0$ whole squared, and there is a m e^4 by 2π, not 2π, there is a 4π, 4π h^3, and then we have a 1 by n_f squared minus 1 over n_i squared, and let us take n_f to be equal to some n_2 and n_1 to be n I mean, say some n, and so n_f to be say n minus 1 and n_i to be equal to n, and if you put it there, this you remember that this factor is same as that factor, the factor that you see here is same as that. So, if you look at this bracket that we have written here, so this is 1 by n squared and

And this is like, you know, so this is n. So we have n square and n minus 1 whole square. So there is n square minus n minus 1 whole square. So we can open up the bracket, which is n square. And there is a 2n and then there is a 1 there. So, this n square would cancel and we have this equal to 2n minus 1 divided by n square n minus 1 whole square. So, in the limit of large n. So, this goes as, so 1 can be neglected in terms in front of n when n is large or 2n when n is large and again 1 can be neglected in front of n. So, in the denominator, we have n to the power 4 and we have 2n in the numerator. So, this goes as 2n by n to the power 4, which is nothing but 2 divided by n cube and that is the factor that you get here as well. So, it really does not matter whether you are at large values of n or at small values of n, the frequency in the limiting case when n is very large would still be given by this formula that you see here, let us call it as equation 1.

H-atom

Frequency of an electron in a Bohr orbit.

$$\nu_0 = \frac{v}{2\pi r} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{me^4}{4\pi\hbar^3}\,\frac{2}{n^3}. \qquad (1)$$

$$n_f = n-1, \quad n_i = n.$$

$$\nu = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{me^4}{4\pi\hbar^3}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

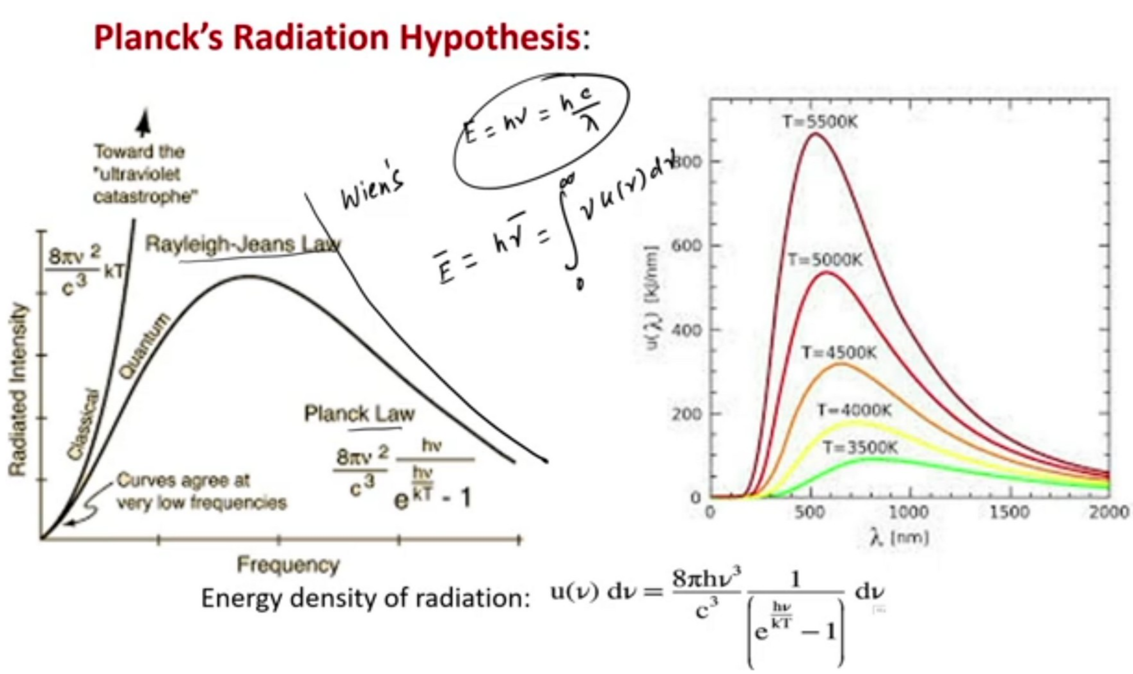
$$\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{n^2 - n^2 + 2n - 1}{n^2(n-1)^2} = \frac{2n-1}{n^2(n-1)^2}.$$

In the limit of large n, $\frac{2n}{n^4} \simeq \frac{2}{n^3}$.

So, let us go into an important discovery, which happened in the early 1900s and so on. And that is called Planck's radiation hypothesis, okay. And it is often called Planck's law as it is written here. I would still like to call it a hypothesis because this is the experimental data for the radiated intensity or the radiation intensity due to a black body. So, a black body is, you can read up the definition, the black body is basically whatever is incident is absorbed, okay. So, there is or rather, you know, it has this emissivity equal to 1. So, that is why it is called black; it is completely, you know, sort of absorbs all the radiation that is incident on it. So, like the sun is a black body. So, here we are showing this; if you see the right picture, that will be even better, which shows the radiation intensity as a function of wavelength and you see that there is a sort of non-monotonic behavior. For all the temperatures, of course, this is at 5,000 Kelvin, which is a large temperature.

Then it is 5,500 Kelvin, 5,000 Kelvin, 4,500 and so on. So, what happens is that there is a peak behavior that is seen, but the peaks shift slightly towards larger wavelength. That is one thing. And so, what it means is that if you start heating up an object, the emitted radiation changes its color, okay, so changes its wavelength and the emitted radiation has, you know, first it becomes white hot and then red hot and then blue hot and so on. So, these emitted intensities are what is, you know, plotted as a function of the wavelength. And one feature, as I said, is that there is a non-monotonic behavior. And this non-monotonic behavior could not be explained by what is called the Rayleigh-Jeans law, okay, which only shows that there is a, the classical theory says that which goes as 8 pi nu square by c cube k T. So, this tells you that it should go, you know, infinitely large at low frequencies. And there is this side by Wien's law, which talks about large frequency.

Remember that in the left figure, we are talking about frequency. And in the x-axis and in the right figure, we are talking about wavelength on the right figure. And this is intentional. It is just to remind you that the frequency is inverse of the wavelength and so on. So, E is equal to h nu or it is equal to hc over lambda. So, that is an important thing that you should remember. So, if you look at the left figure as a function of wavelength and the right figure as a function of frequency, they would sort of interchange their high and low notions. So, what was very important at that time is that both these Rayleigh's and Wien's law they could not explain the observed spectrum or the non-monotonicity that is observed in this spectrum. And that led Planck to propose that the energy density of radiation which between frequency nu and nu plus d nu you can equivalent to write it in terms of the wavelength by using this relation that we have written here. So, u nu d nu is equal to 8 pi h nu cube divided by c cube and then there is a exponential h nu by kT minus 1 and this helps you to obtain this non-monotonicity in this observed intensity spectrum and how one calculates is a following that one can calculate the average frequency, you know, or the average energy, say, for example, by using this frequency.

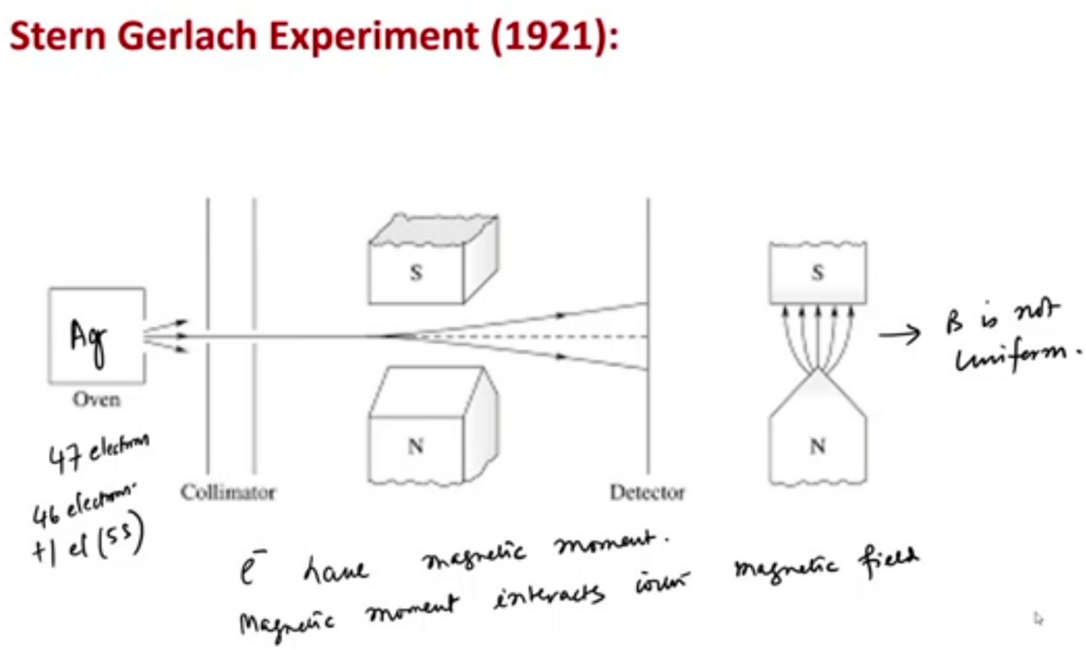


So, this is like this and h nu and then you can calculate a nu, u nu, d nu from, you know, from 0 to infinity and then you multiply it by c, that will give you the intensity, the c is the speed of light or the speed of this radiation. Okay. And there could be some, you know, factors which could be there. But in any case, this is what has been proposed by Planck. And that's why I call it as a hypothesis, because this had no sort of proof that has been presented by Planck at that time. And it was later S N Bose, he came and proposed this form for bosons. So, photons are known as or they are called as bosons with certain, you know, symmetry properties and statistics. And he consciously, you know, sort of derived this spectrum or rather this distribution function, which reproduces the blackbody radiation. But one very important thing that emerges out is that what Planck said, if you keep these radiation, electromagnetic radiation or whose quanta are known as photons, then you can never keep the number of photons to be fixed.

the walls of the container is always going to emit and absorb radiation rather you know it will exchange radiation with the surrounding and the packet in which it exchanges radiation it is a quantum and that quantum the energy quantum has a value which is h nu. So this was quite important in those days and the birth of you know all these quantum mechanical ideas of quantized energy levels that follows afterwards. Okay, so let us go into another quantization which at that point, you know, this Stern and Gerlach experiment. Stern first did it alone in 1921 and in the same year, later on he repeated the experiment with Gerlach in 1921. So, there is an oven that you see, which is of course, at high temperature or which produces high temperature and there are Ag atoms, silver atoms. And if you ask why it is Ag, why not anything else? So, first of all, Ag is a stable atom, Ag is a little, you know, kind of big atom. And these atoms have the electronic configuration. It has 47 electrons.

And if you start filling up, you'll see that 46 electrons form a filled shell and plus that one electron which is you know in the 5s configuration that is only free. So, it is like a one electron atom. So, you get the properties of an electron manifested by choosing Ag. Of course, there are other ones which have one electron in the outermost orbit, but they are small atoms and they would be vaporized inside the oven. And nothing will come out. So that way silver was a better option or rather at that point was the only option that was available. Okay. So it was sent through two collimator slits and then it was sent through two pole pieces of the magnet. You see that S is the south pole and N is the north pole. But there is something interesting about these pole pieces. There is a sort of this roof of this is like this roof of a house and that makes this magnetic field to have gradient in the

Z direction and that was very needed and is shown in the right panel. You see that there is a North Pole and the South Pole. There is a bending of the field lines and the magnetic field is not uniform here.



Okay, so this is an important thing and understood that this why do you need this B to be non-uniform, we just come to that in just a while and let me sort of then do this simple calculation which tells you that this field is indeed needed to be non-uniform and so these electrons have magnetic moment. So, we can write you know electrons to be E minus have magnetic moment and this magnetic moment would interact with the magnetic field. So let us see what happens because of that. So there is a force that arises because of this interaction and that force is given by for simplicity you can think this to be the Z direction. Which means that B has only a variation in the z direction. So, B has z direction dependence. So, f of z that is f in the z direction is del del z of mu dot B that is the interaction term and that is how the magnetic moment of the electron the only electron that we are talking about the rest are all field shell of silver atom. So, there is a mu dot B interaction and so the force is obtained from an interaction potential by the gradient of this we are just talking about the magnitude there is indeed a negative signs a negative gradient.

But, so this is equal to mu z del B z del z. This is where the non-uniform field comes into the picture, because if field is uniform, then we do not have sort of force that exists, okay, because mu is constant and B has a variation in the z direction. So, now if you really think of mu z to have any value, so classically mu z has values between you know plus and minus mu okay. So, which means that if that is true then what you should get here is that there is a patch I will draw it with the color. So, you should get a patch here of silver atoms that is on this detector or the screen, you should get a patch of that. But what was seen experimentally is that there was no patch and there are two sharp images, let me again use another color and there are two patches, there are two just sharp spots that are observed and so this is observed experimentally and not the patch. So, this is not observed. So, that tells you that it takes two, you know, extreme values, but do not take any value that is, you know, in between that, in between these two values. And this tells you that, you know, these mu actually has only two possible values.

And in those days, it was referred to as the space quantization. But now we know that this called as you know basically the quantization of the spins electronic spin and this mu is proportional to the spin with some you know constants and so on and this is. So, mu z is corresponding to this you know it corresponds to the z component of the spin which has two values which are known as h cross by 2 and minus h cross by 2 and this is interesting because h cross is has the unit of angular momentum. and this is, the S is known as the spin angular momentum, okay. And so, h cross has a value which is 1.0546 into 10 to the power minus 34 joule second and this is called as a Planck's constant, as we are quite familiar with this. So, once again this quantization of the z component of the spin and the fact that this mu which is not a classical object or quantity rather it is not a classical quantity as it was thought about that you know there is this electrons have a magnetic moment and this magnetic moment is just a classical variable which can take values any value. So, these quantization of spin of spin or spin angular momentum is a main inference of Stern-Gerlach experiment.

Let me in short write SG experiment. Now, apart from that, there is a very important inference that emerges out of this experiment, which was less stressed about and one should talk about that as well. Before that, you have to understand that if the magnetic field varies in the x direction, Then, we will have a force in the x direction and then we will have this again two spots, but that would correspond to Sx equal to plus h cross by 2 and Sx equal to minus h cross by 2, where Sx is the component of this angular momentum spin angular momentum vector. Or if the magnetic field is non-uniform in the y direction, then we will have Sy and so on, okay, having values which are plus h cross by 2 and minus h cross by 2, okay. So, let me sort of give you another perspective of this Stern-Gerlach experiment in addition to the spin quantization. We will talk about a series of experiments, so SG experiments.

$$F_z = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B}) = \mu_z \frac{\partial B_z}{\partial z}$$

Classically μ_z has values between $\pm \mu$.

$\hbar$: angular momentum.

S : spin angular momentum.

$\mu \propto S \qquad \mu_z \rightarrow S_z$: $+\frac{\hbar}{2}$, $-\frac{\hbar}{2}$.

$\hbar = 1.0546 \times 10^{-34}$ J-s   Planck's Constant.

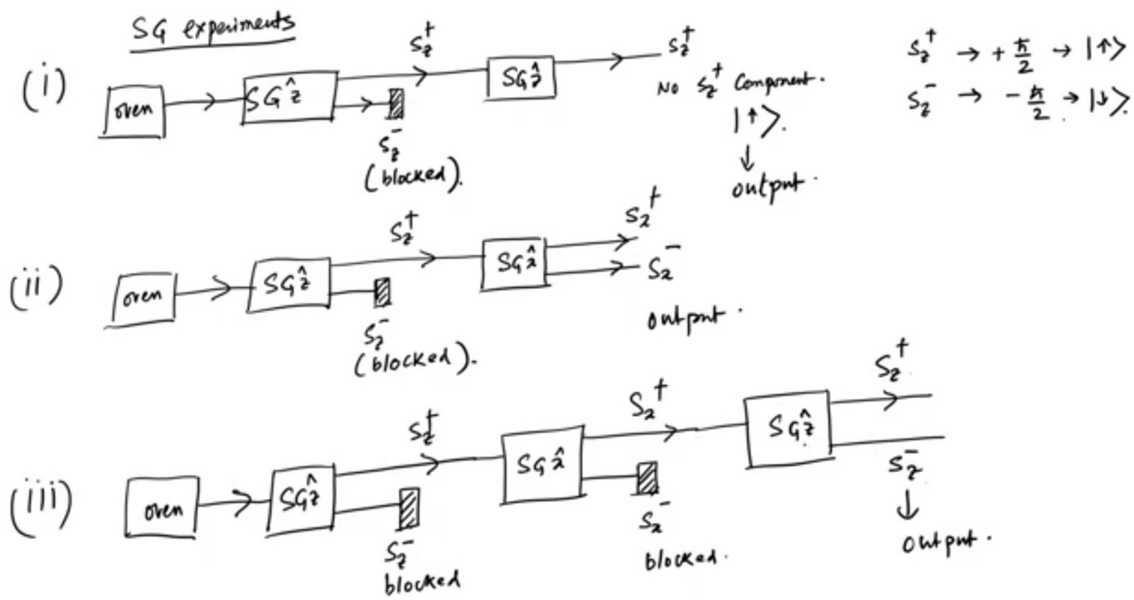
Quantization of Spin angular momentum $\rightarrow$ SG experiment.

And let us just put them as thought experiments. So, there is an oven that is there, which is what we have shown. So, there is oven and there is a collimator. Let us not draw that, but there is a collimator and there is a, we will call it as SG z cap okay, which means that the magnetic field is non uniform in the Z direction. So, that is SG is for Stern-Gerlach and z cap is when the magnetic moment is or the magnetic field is non uniform in the z direction okay. And then, of course, you will have these Sz plus and Sz minus coming out and then it can send it through this again, this Sz. So, this is that Sz plus. So, Sz plus, this is just a new notation used for these plus h cross by 2 and Sz minus that corresponds to minus h cross by 2, the notation. And what we do is that suppose we clip this thing. So, we put a stopper for this Sz minus. So, this is blocked. And so we will again send it through another SGZ apparatus and then we have this SZ plus and there will be no SZ plus component. That makes sense.

So, output is only that of say you call this as notationally you call it as up spin and this you call it as a down spin. So, there is only an up spin that emerges. So, this is the output in two such SG apparatus. Consider a second case where we have there is an oven here and so there is this. Now, we have a SGZ and again we let this SG plus go through and send it through say SGX. which means the magnetic field is non-uniform in the x direction or along the x direction. So, again we block Sz minus component, so it is blocked. So, what will come out is Sx plus component and you get a Sx minus component. So, we will have corresponding to Sx, we will have a component which is having an eigenvalue plus h cross by 2 and minus h cross by 2.

We do not want to represent it by up and down because up and down are the, you know, the eigenstates of Sz. So, you get Sx plus and Sx minus. So, these components come out. So, that is the output for this case. Now, this is, it makes one think that whether this output actually has 50% SX plus and 50% SZ plus. And 50% of it is Sz plus and the other 50% is Sx minus and Sz plus. So, is that the composition? But if that is the composition, then that runs into a trouble as we will see in the third step. So we have we add one more layer of complexity and put this oven and put a SGZ. That is the first and again let only the SZ plus component go through the SZ minus one is clipped. This is made to go through a SGX. And then, of course, Sx plus is let go while Sx minus is blocked. So, here we have blocked Sz minus and here we have blocked Sx minus. So, these are blocked and then this is made to pass through finally another Sz, okay?

That means the magnetic field is in the non-uniform in the z direction. So, you have this as Sz plus and Sz minus. So, that is the output that you see here, okay? Now, this is strange because in the first case itself, you have blocked this SZ minus. So, if in the second stage, you would have thought that you have these N0 by 2 number of atoms. So, N0 is a number of atoms that are emitted from the oven. So, if there are N0 by 2 and N0 by 2 in the second stage which comprise of Sz plus Sz plus and Sz plus Sz minus and then you have blocked the Sz minus and then you have to interpret that you know the final these output in presence of these three sequential SG apparatus is Sz plus Sz plus and Sz minus Sz plus.



Then that is not true because what is observed is that you get Sz plus and Sz minus. So, that tells you that even though Sz minus was blocked in the first level itself after the first SG apparatus, it still manifests in the final outcome. So, it changes the state that you see here at the final outcome, and that tells you that these Sz and Sx are really incompatible observables. So, this is a very important statement: Sz and Sx are incompatible observables; they cannot be measured simultaneously. So, this is a very important statement that two components of spin cannot be measured simultaneously. And this would be reflected later: if there are two observables and these two observables correspond to two different operators. These two operators, if they are incompatible or cannot be measured simultaneously, then we say that the commutation of Sz and Sx is not equal to 0. Indeed, the Sz-Sx commutation is not equal to 0, and it is in fact the third component, which is Sy here. So, the general statement is that if there are two observables and they correspond to two operators, and these two operators do not correspond commute, then they cannot be simultaneously measured. Or, if they cannot be simultaneously measured, then they do not commute, okay? And these two are non-commuting operators. We will see that X, the position variable, and the momentum variable or their corresponding operators are non-commuting observables or non-commuting entities.

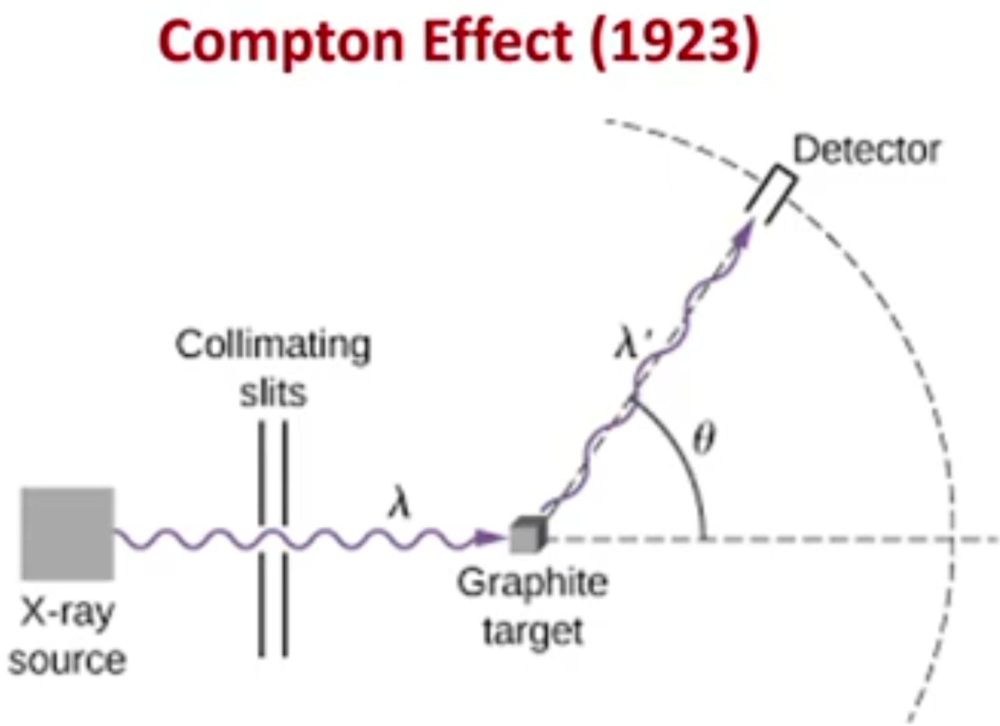
S_z and S_x are incompatible observables.

They can not be measured simultaneously.

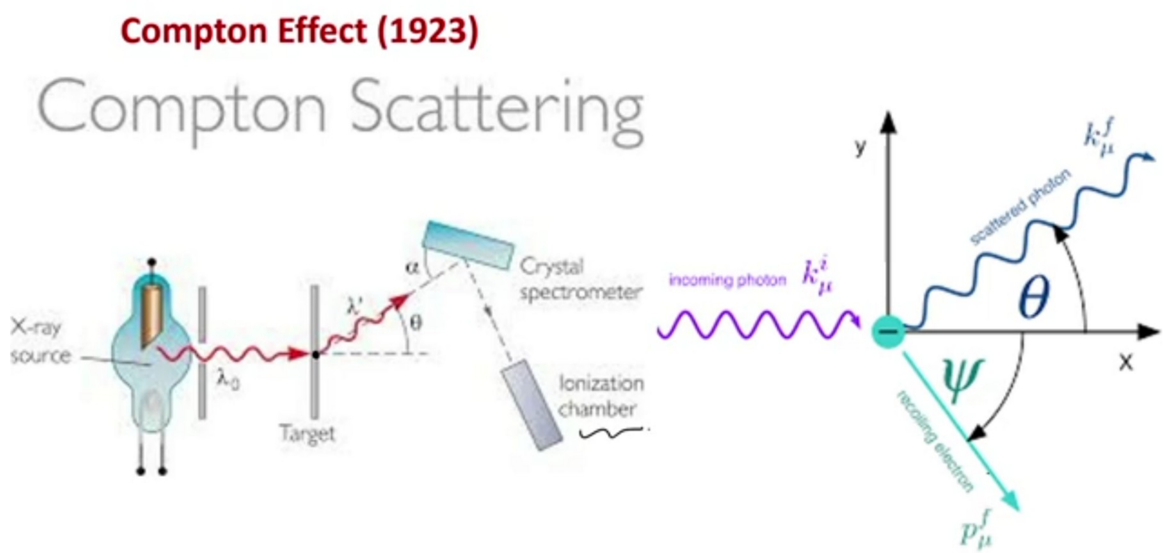
$$[S_z, S_x] \neq 0.$$

$$[A, B] \neq 0.$$

Okay, let us go through this Compton effect. Why do we study the Compton effect? We want to understand the particle nature of this radiation, okay? In some other experiments, we will see that the wave nature of electromagnetic radiation shows up. But here, we will talk about the particle nature and how we show that the particle nature will demonstrate that they obey energy-momentum conservation relations, which are applicable to particles. So, what do we have? We have an X-ray source here where photons are coming out photons of wavelength lambda. They are coming out, passing through collimating slits, and hitting a target. In this case, it is a graphite target. These photons finally get collected in the detector, and because photons are represented by this wavy line. So, there is an incoming photon with a wavelength lambda, and the outgoing photon has a wavelength lambda prime. The angle by which it is scattered is called the scattering angle, and the detector is kept at that angle. So, you can understand that the detector can be placed anywhere on these dotted or dashed lines in an angular fashion, okay.



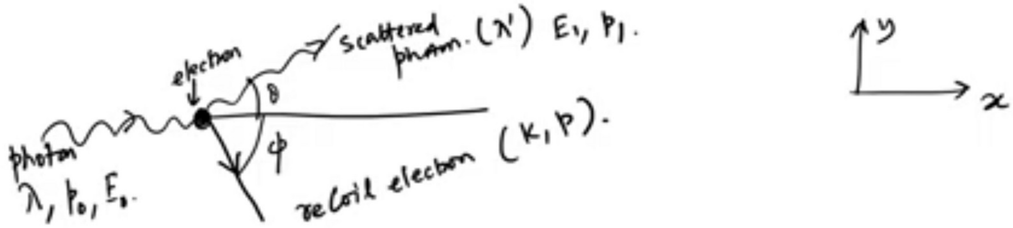
So, this is the experiment that Compton did in 1923, okay. If you really want to see the actual experiment, there is an X-ray source, and the photons come out, hit a target, and then hit a crystal spectrometer or there is this crystal there and finally get detected in the ionization chamber. So, this is the detector that we have drawn on the periphery of a circular arc, but there is this crystal that we have not shown. In the actual experiment, this crystal was used. It, of course, won't matter to us because we will write down the energy-momentum relation between the photon, which is coming from the X-ray source, and some electrons that are there. So, it is just like a classical scattering. There is an electron there. The photon collides with the stationary electron and then gets scattered, and we measure the scattering angle. So, let us draw this thing. You can simply draw this or rather, schematically, it can be shown. There is an incoming photon with a frequency or some momentum k i. Of course, photons carry momentum and have a pressure associated with that. It is called the radiation pressure or this radiation intensity blackbody radiation intensity. You know, measures. And then there is a scattered photon, shown by the wavy lines, that is the scattering angle theta.



It is also true that there is the electron also gets scattered which is called the recoiling electron in a different direction. Let us call that angle as psi. So let us see in Compton effect what actually happens, particularly in experiments. So if you allow the monochromatic beam of X-rays to fall on a crystal, specifically a graphite crystal, if you know graphite is a 3D form of graphene, so these X-rays will be scattered from the target, from the atomic nuclei as well as from the orbiting electrons of graphite. So the scattering from the nucleus is elastic so that there is no change in energy, while that from the electrons is inelastic. So, this causes a shift in the scattered wavelength and we shall precisely calculate the change in wavelength because of this scattering. We can calculate a delta lambda as a function of theta which means that this is the incoming wavelength and this is the scattered wavelength. So, lambda minus lambda prime or the other way around maybe lambda prime minus lambda is equal to delta lambda theta. And this delta lambda can be found in as a function of theta. And how we do that is let me draw a simplified picture for our purpose. So, this is the photon and there is a stationary electron that is there. And then there is this photon that so, this is the incoming photon and so, this is the

the recoil electron and this angle is theta and this angle, let us call it instead of psi, we call it phi. So, this is electron and stationary electron. So, there is a photon coming and the photon has wavelength lambda, momentum p 0 and energy E 0 and so this is the electron. So, this is scattered photon with lambda prime and this is recoil electron. This is say k and p. Initially, it is at rest. So, this is say for example, energy E 1 and p 1. So, what is the momentum conservation? The momentum conservation says that it is along the direction and perpendicular to the direction of propagation. So, this momentum conservation yields p0, which is the initial photon momentum. There is no electron momentum because it was at rest. So, this is p1 cosine theta, theta is this angle, so it is in the direction of propagation and p cos phi, p is the recoil electron momentum and k is the kinetic energy. So, this is let us call it as 1a and perpendicular to the direction of propagation which is p1 sine theta, this is equal to p sine phi.

So, this is in the direction of propagation because perpendicular to the direction of propagation there was no initial momentum either of the photon or of the electron but for the photon and the electron they are in opposite direction one is in the positive y axis if you like because this is say x axis and this is y axis. So, positive y axis and negative y axis they would simply be equal. because there was nothing in the initial part, okay. So, if you square 1a and 1b, then we get P0 minus P1 cosine theta square is equal to P square cos square phi. So, this is 2a And this squaring 1a and squaring 1b, we get a p1 square sine square theta equal to p square sine square phi 2b. I hope all the variables are clear and all the angles, etc., all these things that are written, they correspond to some of them, correspond to those of photons and some of them correspond to those of electrons.



Momentum Conservation.

$$p_0 = p_1 \cos\theta + p\cos\phi \qquad 1(a).$$

$$p_1 \sin\theta = p\sin\phi \qquad 1(b).$$

Squaring 1(a) & 1(b).

$$(p_0 - p_1\cos\theta)^2 = p^2\cos^2\phi. \qquad 2(a)$$

$$p_1^2 \sin^2\theta = p^2\sin^2\phi \qquad 2(b)$$

So, if we have these written as 2a and 2b, what we do is add 2a and 2b. That gives us P0 square plus P1 square minus 2P0P1 cosine theta is equal to P square. Let us call that equation 3. So, that is adding 2a and 2b. Just to remind you, we are trying to arrive at this by applying all these momentum and energy conservation laws. We have not done it yet, but we will do it. What we are trying to establish is that this scattering process or this collision process is indeed like two balls colliding in classical physics, where we apply classical laws of conservation of momentum and energy. So, conservation of energy, but now you have to think about using relativistic expressions. So, this tells you that E0 plus m0 c square is equal to E1 plus k plus m0 c square again. This is the incident photon's energy because it is coming from a source. The incident photon this is the rest mass, sorry, this is the electron this is the rest mass energy of the electron because it was at rest. So, that is the rest mass energy.

This is the scattered photon's energy. This is the recoil electron's kinetic energy, and this is again the rest mass energy of the electron. This term will not be there for photons because photons have zero mass. So, this will tell you that E0 minus E1 is equal to k, and let us call that equation 4, okay. So, the relativistic expression for energy is, generally, the relativistic expression for energy is E is equal to p square c square. This is for a general relativistic particle, like for the electron, this will be there, m0 square c4, so this is equal to E square. So, for a photon, of course, m0 is equal to 0. m0 is equal to 0, so E is equal to p c. So, p is equal to E over c, which is equal to h nu over c. Let us call this equation 5. From 4 and 5, we have c p0 minus p1, which is equal to k. Let us call that equation 6.

Adding 2(a) & 2(b).

$$p_0^2 + p_1^2 - 2p_0p_1\cos\theta = p^2 \quad (3)$$

Conservation of energy.

$$E_0 + m_0c^2 = E_1 + K + m_0c^2$$

E_0 → inc. photon; m_0c^2 → electron (rest mass energy); E_1 → scattered photon; K → KE of recoil electron; m_0c^2 → rest mass energy of electron.

$$\text{or,} \quad E_0 - E_1 = K \quad (4)$$

Relativistic energy : $E^2 = p^2c^2 + m_0^2c^4$

for photon, $m_0 = 0$, $E = pc$.

$$p = \frac{E}{c} = \frac{h\nu}{c} \quad (5)$$

And for the electron, the energy is k plus m0 c square whole square, which is E square, is equal to p square c square plus m0 square c4. And if you open the bracket and simplify, this will simply give you k square plus 2k m0 c square plus p square c square sorry, this is equal to p square c square. Because the other term cancels, okay, and so this tells you that k square over c square plus 2 k m0 is equal to p square. Let us call that equation 7, okay. So, putting now p square on the RHS of 7 from 3, so from equation 3. Let us see where equation 3 is that is the P square that you see there on the right-hand side so you put that into the right-hand side of this equation 7, and what one gets is also k from 4. k from 4, okay.

From (4) & (5),

$$c(p_0 - p_1) = K \qquad (6)$$

For electron, the energy is

$$(K + m_0 c^2)^2 = p^2 c^2 + m_0^2 c^4$$

$$K^2 + 2K m_0 c^2 = p^2 c^2$$

$$\text{or, } \frac{K^2}{c^2} + 2K m_0 = p^2 \qquad (7).$$

Putting p^2 from Eq (3) and K from (4) on the RHS of (7)

$$(p_0 - p_1)^2 + 2 m_0 c\,(p_0 - p_1) = p_0^2 + p_1^2 + 2 p_0 p_1 \cos\theta$$

So, these have to be put on the right-hand side of 7, and that will tell you that one gets this equation, which is equal to p0 minus p1 whole square plus 2 m0 c p0 minus p1. This is equal to p0 squared plus p1 squared plus 2 p0 p1 cosine theta. Simple algebra, but you have to do it carefully to get all the terms correct. So, if you simplify, then what happens is the following. m0 c p0 minus p1 equal to p0, so this minus, so p0 p1 into 1 minus cosine theta, and so 1 over p1 minus 1 over p0 is equal to 1 over m0 c and 1 minus cosine theta. And if you multiply it by h on both sides, and if you recognize that h over p1 so, h over p1 is lambda 1, h over p0 is lambda 0, which are respectively the lambda 1 or lambda prime.

Did we say, okay, so let us call it as lambda prime, and this is lambda, okay. So, lambda prime, which is the scattered photon wavelength, and this is lambda, the incident wavelength of the photon. I think that is what we have started with, or we might have started with, okay. So, there is lambda and lambda prime, correct. So, this means that we have lambda prime minus lambda equal to delta lambda, this is equal to h over m0 c and 1 minus so there is a 1, this square will not be there h over m0 c 1 minus cosine theta, and this is written as lambda c 1 minus cosine theta, and this. So, lambda c is equal to h over m0 c, which is called the Compton wavelength.

Simplifying

$$m_0 c (p_0 - p_1) = p_0 \cdot p_1 (1 - \cos\theta).$$

$$\frac{1}{p_1} - \frac{1}{p_0} = \frac{1}{m_0 c} (1 - \cos\theta).$$

Multiply by h

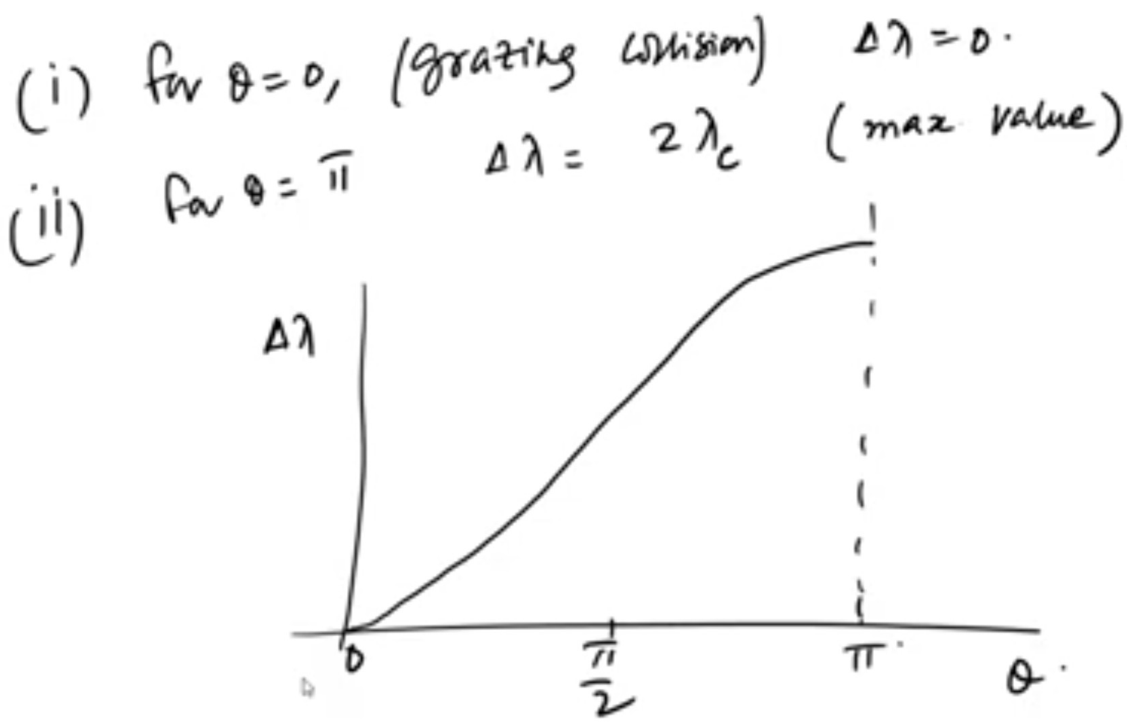
$$\frac{h}{p_1} = \lambda' \qquad \frac{h}{p_0} = \lambda$$

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c} (1 - \cos\theta) = \lambda_c (1 - \cos\theta).$$

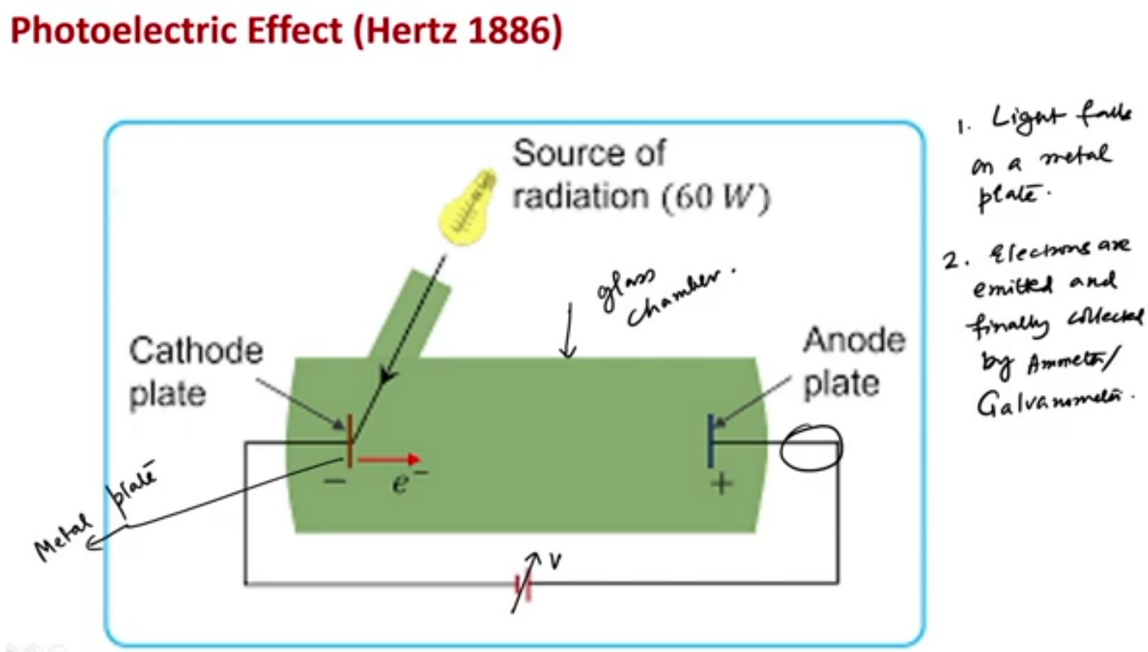
$$\lambda_c = \frac{h}{m_0 c} = 2.43 \times 10^{-12}\ m = 0.0243\ \text{Å}.$$ Compton wavelength.

$\Delta\lambda$ = Compton shift depends on θ (but not on ϕ).

So, this 2.43 into 10 to the power minus 12 meter equal to 0.0243 Angstrom, and this is called the Compton wavelength. Okay, so this delta lambda is called the Compton shift because of Compton you know, they performed this experiment it is called the Compton shift, and interestingly, this shift depends on theta but not on Phi. So, we are only concerned with the scattered photon and not with the recoil electron, and there are, you know, two limits that can be seen: one is that for theta equal to 0, we have this called the grazing collision. That is, after the collision, it does not change the direction; then there is no change in the Compton wavelength. And for theta equal to pi, that means it kind of returns back in the direction of incidence, and then delta lambda is equal to twice lambda c, and that is the maximum value. Okay, so what it means is that if you plot delta lambda versus theta, then it sort of looks like, you know, like this. Okay, so this is the maximum value, which is at pi, and somewhere in the middle is like pi by 2, and this is 0.

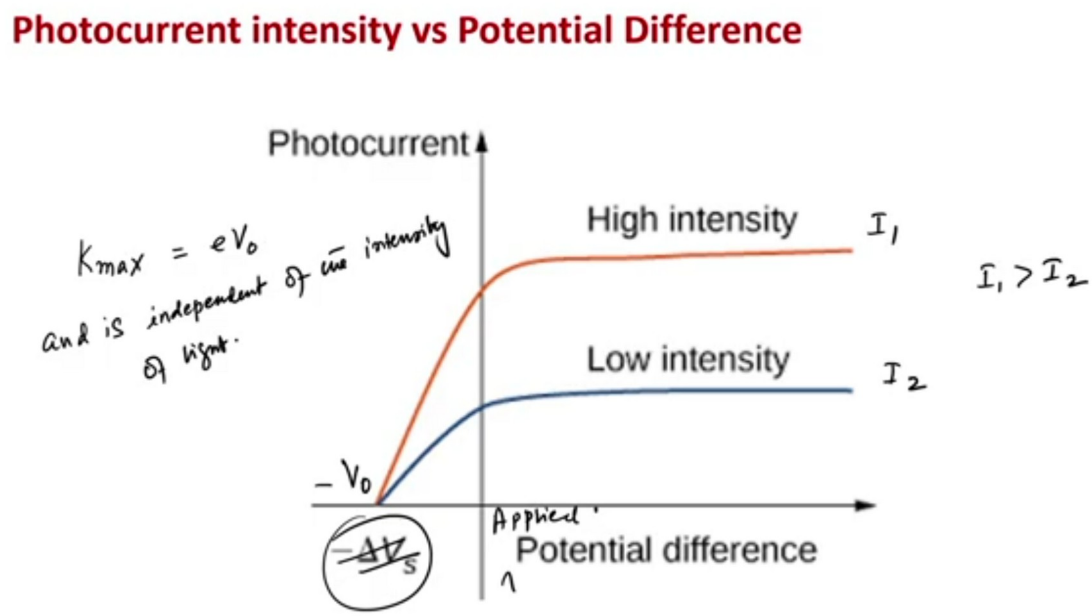


So, this is what the scattered or the difference between the wavelengths before and after scattering that looks like this and which is proportional to 1 minus cos theta. So, this is Compton effect. And we can quickly go through the photoelectric effect, which was performed by Hertz in 1886, in which there is a source of radiation, which can be a bulb or which can be any source that you can think of. And there is a metal plate, which is called as a cathode plate here. So this is a metal plate. It incidents on a metal plate. And there is an anode. So there is an electron that is ejected from the metal plate and it moves towards anode. And there is a variable voltage source that you have here. And this is like a glass chamber. And this was, you know, done by Hertz in 1886. So what happens is that so light falls on a metal plate. Number two, electrons are emitted. And finally collected by I mean they are sort of this thing is is connected to say a galvanometer or an ammeter which is connected to the cathode cathode to the anode sorry. Okay, so this is the situation that we have.



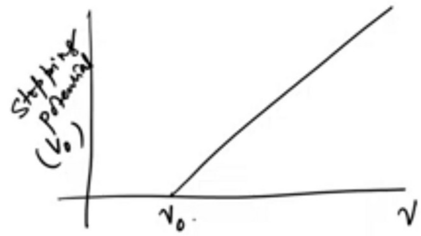
And what else do we have? We have one finds that there are two observations mainly. So one is that if you make this metal plate that you see here to be more and more negative with respect to the anode, which is done by this voltage that you see down there, Then the photo current that is the current due to the emission of the electrons and which go and deposit in the anode and anode is connected to a measuring device of the current that current completely stops for a given you know this stopping potential this called as a stopping potential. So, if you make it negative and make it negative enough and then the photo current stops then And this photo current is performed for say true intensity, let us call it as I1 and I2, where I1 is greater than I2, it has the same stopping voltage. So, this is one of the things that is of importance here. And not only that, if you increase the potential difference, it sort of increases, you know, linearly at the beginning, but then it saturates at large potential difference. So, this photo current has no dependence on the potential difference that is applied between the metal plate and the anode. So, this is basically this is the applied potential difference, you can call it as applied potential difference. And what is another important thing is that the maximum kinetic energy of the photoelectron, let us call it as a K max.

That is equal to E V 0 or which let us call this as minus V 0 instead of delta V s minus V 0. So, it is equal to E V 0 and is independent of the intensity of life. This is what has been told that is whether it is a high intensity or low intensity, this is same. Okay, so what is the question then? The question is that whether these observations are consistent with the classical theory or the classical ideas that we have so far, okay. And in order to strengthen that, what Millikan did, this is done somewhere around 1923, but Millikan, I think 1914, not 1923, but he got the Nobel Prize in 1923. So what he did is that he wanted to know whether the stopping potential is associated with the cutoff frequency and how it depends on the cutoff frequency.



So what he found out is that if you have this stopping potential, which is that V0, so the stopping potential and the frequency. So, this stopping potential is associated with a cutoff frequency nu 0 below which there is no photo current that is observed. And he performed this experiment Millikan for sodium where nu 0 was observed to be 5.6 10 to the power 14. And this is basically the V0 versus nu which shows you know a straight line behavior with a stopping potential to be at nu 0 okay. And this has won him the Nobel Prize and we are asking this question that what are the abnormalities with regard to classical theory or what cannot be explained with regard to classical theory. So, the abnormalities that we have that K max that is the maximum kinetic energy of the electrons should have been proportional to the intensity of light, it is not as we have shown that there is for two different intensities, they merge at the same value. Second is that there should have been no cutoff frequency. That is, there should be classical theory would say that if you impinge it with a very small frequency also, it will eject an electron, but that does not happen.

Millikan (1914) → Noble prize 1923.



For NA, $\nu_0 = 5.6 \times 10^{14}$ Hz.

Abnormalities wrt classical theory of light.

(1) K_{max} should have been proportional to the Intensity of light.

(2). Should have been no cut off frequency.

(3) Should be a time lag between incidence of photon & ejection

And if you apply classical wave theory, then it would demand that there should be a time lag between incidence and ejection. but there is no time lag that was noted. And this is where Einstein came and Einstein proposed that this kinetic energy of the electrons is equal to h nu minus some W, where W is called as a work function. And what it means is that there is the work required to eject an electron from the metal surface, okay. And so the maximum kinetic energy should obey this h nu minus some W0 and W0 is the property of a metal.

I mean, basically this W0 is the work function, it is a property of a metal. So, it will vary on which metal you are talking about. So, it will vary, the value will vary from one metal to another. And if you substitute W0 is equal to the stopping potential e V0, then sorry, the kinetic energy, the maximum kinetic energy to be equal to the stopping potential. So, then e V0 is equal to h nu minus W0 and V0 equal to h over e into nu. minus W0 and this is the equation that Millikan found. So, once again what we have done is that we just said that this is the experiment that you have very simple looking experiment. There is a metal plate which is called as a cathode plate which is full of electrons.

So there is a source of radiation which is incident on the metal plate; electrons are ejected, electrons go towards the anode, get collected, and there is an ammeter or a galvanometer that is connected which measures the current. If you make this cathode plate or the metal plate more and more negative with respect to the anode, then after a certain or at a certain value of this applied voltage, the current stops, and it is independent of the intensity of the light used, and this K max is equal to e V0, which is what you know comes out and which is what we have seen as well. Okay, Millikan extended this and wanted to find out whether the stopping potential has any dependency on the frequency of the radiation or the light, and he found that there is indeed a stopping potential nu 0 below which there is no emission of or I mean for frequencies below nu 0, no photoelectrons will be ejected out of the metal plate. And this led to a few abnormalities with regard to what we already know.

And one of them is that Kmax should have been proportional to the intensity of light, which it is not. The two intensities or any intensity that you use would have the same stopping potential. There should have been no cutoff frequency. That is what classical theory says. And there should be a time lag between the incidence of photons and the ejection of photoelectrons, okay. None of these are seen there. So, Einstein said that let us have a relationship between the kinetic energy of the ejected electron and the frequency or the energy of the photons by this relation, which is h nu minus w, and this w 0 is called the work function, which is a property of the metal. And by putting in K max equal to e V 0, we got a new 0 or rather the V 0 to have this e here as well.

Einstein proposed

W_0: work function. property of the metal

$$K = h\nu - W$$

$$K_{max} = h\nu - W_0.$$

$$K_{max} = eV_0.$$

$$eV_0 = h\nu - W_0$$

$$V_0 = \left(\frac{h}{e}\right)\nu - W_0.$$

Millikan found this !!

So, V0 is proportional to nu, and there is also a constant there which is found in the work by Millikan. So once again, we have applied the conservation of energy and treated this problem as if there is this photon and electron; the collision is like the scattering between or collision between two spheres, and we have gotten this result, which is seen in experiments or rather observed in experiments, and so there is a triumph of the particle nature of radiation, but we will have to also see that in some other experiments it shows up as waves. We will see that later. That is all for today. Thank you.