ELEMENTS OF MODERN PHYSICS

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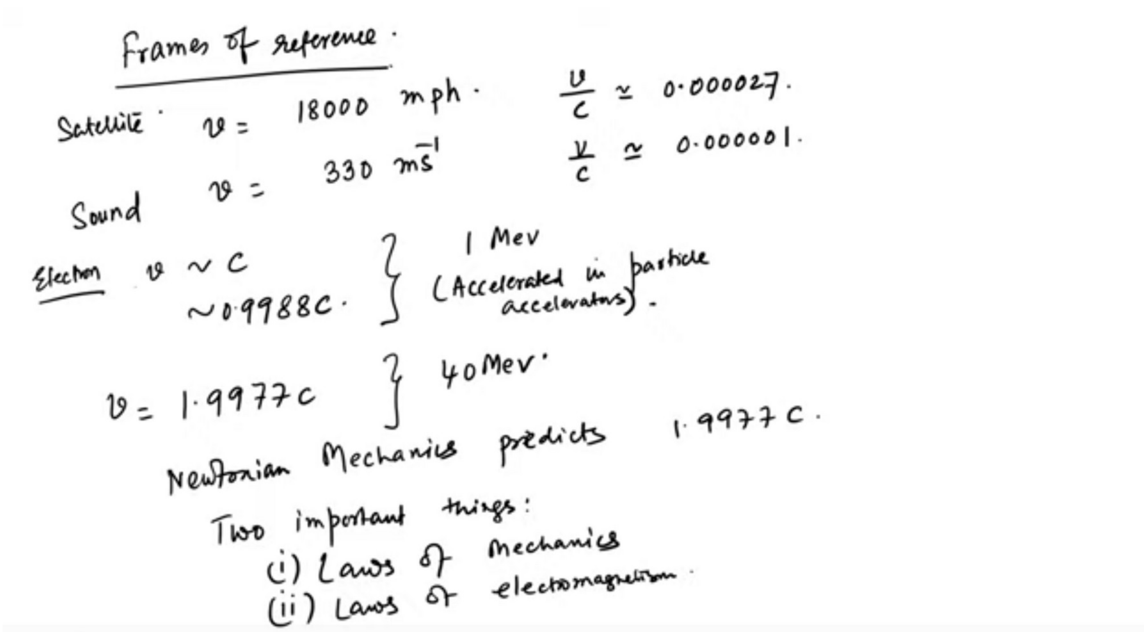
Week-01
Lec 3: Special Theory of Relativity

Good morning, everyone. So, we have done Newtonian mechanics and Lagrangian mechanics in brief. Now, we move on to relativity, and we discuss a few topics such as sending signals and the emergence of the speed of light as the ultimate speed that one can achieve. Then we'll talk about Galilean transformation, different frames of reference, in particular the inertial frames, and why Galilean relativity breaks down. We'll talk about the Michelson-Morley experiment, which rules out the presence of a preferred frame of reference called the ether frame. We'll also talk about the postulates of special relativity. And finally, we'll talk about length contraction and time dilation. So, we start with sending signals in space. It can be done with, say, for example, radio waves or electromagnetic waves. And that's the fastest mode of communication that one can achieve in space by sending signals through light. So, anything greater than the speed of light is not possible, which also emerges out of the special theory of relativity.

So, let us just talk about the frames of reference. And what we mean is the following: suppose there is an artificial satellite that is circling the Earth at, say, a speed of 18,000 kilometers. meters per hour, okay. So, the v by c that comes out, as you know, is 0.40027, whereas the speed of light. So, this is for a satellite, and for sound, v is equal to somewhere around 330 to 332 meters per second. This is in a vacuum. So, V by C comes out as 0.00001 and so on. So, these are some of the speeds that we are familiar with, very large speeds. And in the real world, of course, it is possible to accelerate electrons, which are, you know, by, say, about, I mean, one MeV of energy if you want to accelerate or if you accelerate the electrons, then they reach a speed that is close to c, maybe 0.9988 c. This is for one mega electron volt.

And now what happens is that if you sort of accelerate it by, say, about 40 MeV, then Newtonian physics says that this is going to be 1.9977 c. So, this is the speed of the electron. So, this is for an electron, and this is accelerated by or this is in particle accelerators. And it looks absolutely fine because you have used the equation of motion, and it gives 1.9977 c. And this is what Newtonian mechanics predicts: 1.9977 c, okay. And there seems to be nothing wrong with this.

But of course, we know that the ultimate speed by which signals travel or by which anything can travel, the maximum speed is that of the speed of light, okay. Newtonian mechanics does not distinguish between the speed of light. In fact, if you look at all of the discussion of Newtonian mechanics, it never has a real special space for, you know, the speed of light. So, it sort of predicts it just the way it predicts the speed of any particle, okay. So, we are really, you know, worried about two important things while we talk about relativity, and these things are the laws of mechanics and, number two, the laws of electromagnetism. So, with this, let us talk about both the frames of reference and that of Galilean transformation. So, we discuss what is called an event. Okay, say an event is anything that happens in spacetime.



So, suppose an event happens in x, y, z, and t. For example, suppose in some reference frame, okay, whw ich is that could be a stationary reference frame. Something happens with x equal to one meter. So, let me write that. So, x equal to one meter, y equal to 4 meters, z equal to, say, minus 2 meters, and t equal to 5 seconds. And we write this event as 1, 4, minus 2, and 5 in some reference frame. Now, in another reference frame, which is either stationary or moving, this event can still be represented by another x prime, y prime, z prime, and t prime, where they are a set of four numbers once again, but they have different values. For example, if a car is parked in a parking space, and someone who is standing very close to the car observes this event of the car being parked. And he notes down this x, y, z, and t. Another person who is probably standing 30 meters away also records these data of x, y, z, and of course, the time will be the same because they are recording at the same time.

So, this event of parking the car at two reference frames is represented by x, y, z, and t and x prime, y prime, z prime, and t. So, is there an unambiguous description of the event? What I mean is, can all these reference frames converge? So, is there one unambiguous description of the event? Which tells you that there is a need for a preferred reference frame. So, when we say a preferred reference frame, we usually mean what is called the inertial frames. And by inertial frames, what we mean is a frame of reference that is moving with respect to another frame of reference with constant velocity, either it is at rest or moving at a constant velocity with respect to a stationary frame. Okay, so these are called inertial frames. For example, consider, you know, a boy sitting on a stone in a field, say, and for example, a satellite is floating in outer space and it is not spinning; it is simply floating in outer space.

So that way, you know, very strictly speaking, the boy standing or rather sitting on the stone would not be called an inertial frame because the Earth is rotating. So if there is a rotational motion, then there is an acceleration, and the acceleration really prohibits the boy, you know, who is stationary, who is sitting there, from being considered as a reference frame. But then we know that this acceleration is so slow that for all practical purposes, anything that is stationary on Earth can be considered as an inertial frame. Now, a car speeding or a train moving continuously where its velocity is increasing cannot be called an inertial frame, and they are called non-inertial frames. So, an inertial frame is one that moves with a constant speed with respect to a stationary frame or is at rest with respect to the stationary frame.

And a non-inertial frame is one that accelerates with respect to a stationary frame, okay. So, that is the definition of stationary or rather inertial frames. And in fact, STR deals with the special theory of relativity, as it is called. So, let me write that as the special theory of relativity, which was established, formulated by Einstein, and that deals with inertial frames, and in fact, the general theory of relativity deals with any kind of frame, including inertial and non-inertial, okay. Let us talk about the Galilean relativity or Galilean transformation. So there are two frames, okay. And these are, you know, moving with respect to one another but with a constant speed. So these are the two frames, and one could call this, you know, the S frame and the S prime frame, where the S prime frame is moving with respect to the S frame with speed u. And we make matters a little simpler by taking into account that this is the x-axis, this is the y-axis, and say this is the z-axis.

Event $\rightarrow$ (x, y, z, t).

$x = 1m$, $y = 4m$, $z = -2m$, $t = 5s$.

$(1, 4, -2, 5)$ in some reference frame.

(x', y', z', t')

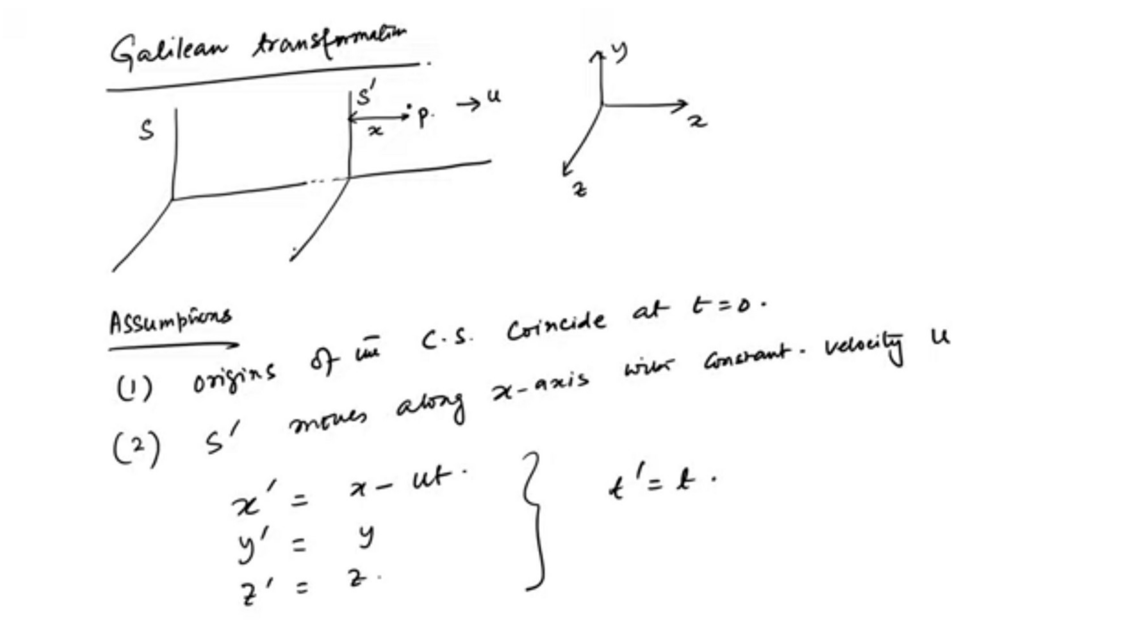
Unambiguous description of an event?

Need for a preferred reference frame: Inertial frames.

Special Theory of Relativity STR deals with inertial frames.

So in principle, this speed or rather the velocity of this S prime frame can be in any arbitrary direction, in which case it becomes a vector, but for simplicity and no loss of generality, we take this u to be moving along the x-direction, and in addition, you can enforce other constraints such as at t equal to 0, the two coordinate systems, you know, their origins are at the same place, etc. So, suppose there is a point P, okay, which is at a distance x prime from this, and they are sort of aligned along this direction. So, these are the assumptions that we should write down. So, one assumption is that the origins of the coordinate system, which we write as C S, coincide at t equal to 0. Second, S prime moves along the x-axis with a constant velocity u. Okay, so these are the two assumptions that we make here, in which case, x prime is clearly equal to x minus u t.

y prime does not change, and z prime does not change either. And, you know, in classical physics, there is no transformation of time, or in Galilean relativity, there is no transformation of time. So, t prime is equal to t. So, the interval of two events, consider there are two events, P and Q, and they occur at t P and t Q. So, say a projectile is taken off, and then it probably bursts into different fragments. So, there's t P for the first one and t Q for the second one. The taking off is considered as t P, and this other one is taken as when it bursts into different fragments, which is taken as t Q. And if you look at it from another reference frame, it's t P prime and t Q prime. Okay, so this is in, say, for example, S, and this is in S prime, then t P minus t Q is equal to t P prime minus t Q prime.



Okay, so, and also the distance between the two points also remains the same. So, that is x, say, for example, x P minus x Q is equal to x P prime minus x Q prime. Well, you can consider this P and Q to be, I can write it with, you know, different indices. Probably that's less confusing. So, let's call it as x A and x B, x prime A and x prime B, where A and B are the two ends of a rod. And this rod is stationary in S and is moving with a constant speed in S prime with speed u. So, this is all that is known to us. So, x B prime that is in the S prime frame is x B minus v t u t B, that is because, so, if S prime is moving with respect to S with the speed u.

S is moving with respect to S prime with a speed minus U. So, x B prime is x B minus u t B, whereas x A prime is equal to x A minus u t A. So, that gives x A prime minus x B prime is equal to x A minus x B, and that is u t B minus t A, and this being equal, we conclude that this is equal to x A minus x B. So, this is what we have said that these are the transformation equations, and we again get this t B prime equal to t A prime if we do the inverse transformation between this x A and x A prime and x B and x B prime. So, the time interval and the space intervals are absolute in the Galilean transformation. That is, they are the same for all inertial observers or they are the same for all inertial observers. And so, in the measurements of length and time of two different events, the relative velocity of the two frames does not come into the picture.

Consider 2 events P & B.

$\underbrace{t_P \ \& \ t_B}_{S}$ / $\underbrace{t_P' \ \& \ t_B'}_{S'}$

$$t_P - t_B = t_P' - t_B'$$

Dist. between 2 points.

$$x_A - x_B = x_A' - x_B'.$$

A & B are the 2 ends of a rod — stationary in S, moving with u in S'

$$x_B' = x_B - ut_B.$$

$$x_A' = x_A - ut_A.$$

$$x_A' - x_B' = (x_A - x_B) - u\underbrace{(t_B - t_A)}_{=0.}$$

$$= x_A - x_B$$

As you saw that neither in the length nor in time, the speed u, which is the relative speed between the two frames, does not come into the picture. So, if we add, mass also remains invariant, that is, in addition to the length and time, the mass also remains invariant. What I mean by invariant is that the speed u does not enter into the transformation, that is, mass in frame A and mass in frame B, they are the same. Then, of course, we get the length, mass, and time all remain invariant and independent of the relative speed of the two frames, okay.

So, there is a result that we get from Galilean relativity, okay. So, how do the velocities add? The velocities would add as you know, so the velocities of the particles would be related by some u prime equal to u minus v and so on. And so this velocity in u prime. So let me write it with a v here. So this v prime v minus u. So this is the velocity in S prime, and this is the velocity of the particle in S, and this is the relative speed or relative velocity between the two frames.

If 'mass' also remains invariant.

L, M, T all " " and independent of relative speed.

$\vec{v}' = \vec{v} - \vec{u} \rightarrow$ relative velocity.

$\vec{v}'$ ↓ velocity in S'

$\vec{v}$ ↖ velocity in S

$\vec{a}' = \vec{a}$

$\vec{F}' = m\vec{a}' = m\vec{a} = \vec{F}.$

$\boxed{\text{Newton's laws are invariant.}}$

We would call it a velocity; we are writing it as a vector. So, these are the same, and for the acceleration, now we have A prime equal to A. Because you take the derivative of this equation that you see above with respect to t, because u is a constant, that is a constant speed, so a prime becomes equal to a, which tells you that if I multiply the mass, which has already been taken to be an invariant, so your F prime, which is equal to m a prime, that is equal to m a, that is equal to F. So, that tells us that Newton's laws are invariant. So in Galilean relativity, Newton's laws are invariant. But this poses another problem because, so why Galilean relativity? Let me write it as Galilean relativity breaks down is one of the questions that we asked on the front page itself, and one can really see that suppose there are two electrons ejected in opposite directions from either some nuclear reactions or, say, from some radioactive atoms in the lab, so these atoms are at rest in the lab, and so the speed of one electron, say for example with respect to another is 0.67 c, okay. So, what is the speed of, or the relative speed of, you know, one, and they are in opposite directions; they are emitted in opposite directions So the speed of, you know, if you look at these two frames as S prime and think that this speed of the S frame, where the electron is at rest, is 0.67 c again in the negative X direction. So your addition of velocities tells you that your u x and u x prime, that is the speeds of the particle or the electrons in the stationary frame or the S frame with respect to the S prime frame is u x prime. And now both of these is 0.67 c.

plus 0.67 C, that makes it equal to 1.34 c. And since we know that there cannot be anything that is larger than the speed of light or no speeds can be larger than the speed of light. So this is really an absurd result in special relativity. And that is not allowed. And we then need to, you know, sort of change the notions of these transformation equations. And they carry certain consequences of this. You know, this exercise says that a collision between two particles or two masses occurs. cannot really determine the relative velocity between the two frames. For example, say two tennis players are playing on a moving ship.

Why Gal. rel. breaks down?

Two electrons are ejected in opposite directions from radioactive atoms (at rest) in lab. The speed of 1 el. wrt another is 0.67c.

$$u_x = u_x' + v$$
$$= 0.67c + 0.67c = 1.34c$$

$\boxed{\text{— Absurd result in STR!}}$

which is the ship, you know, cruising in calm water with a constant speed. Now, the two tennis players, by looking at the velocities of the balls that they, you know, hit towards each other, cannot determine the speed of the ship. Okay, so that's a simple thing, and we sort of understand. So, if you look at any collision events, looking at the velocities of these before collision and after collision, one cannot determine the relative velocity between the two reference frames, okay. So, now it was for quite a while that it was thought that there is indeed one reference frame that's preferred over another. And this reference frame is called the ether reference. OK. And light actually moves through this preferred frame. And this would give rise to, you know, and this preferred frame or the inertial frame has a certain velocity with respect to another frame, which is completely at rest.

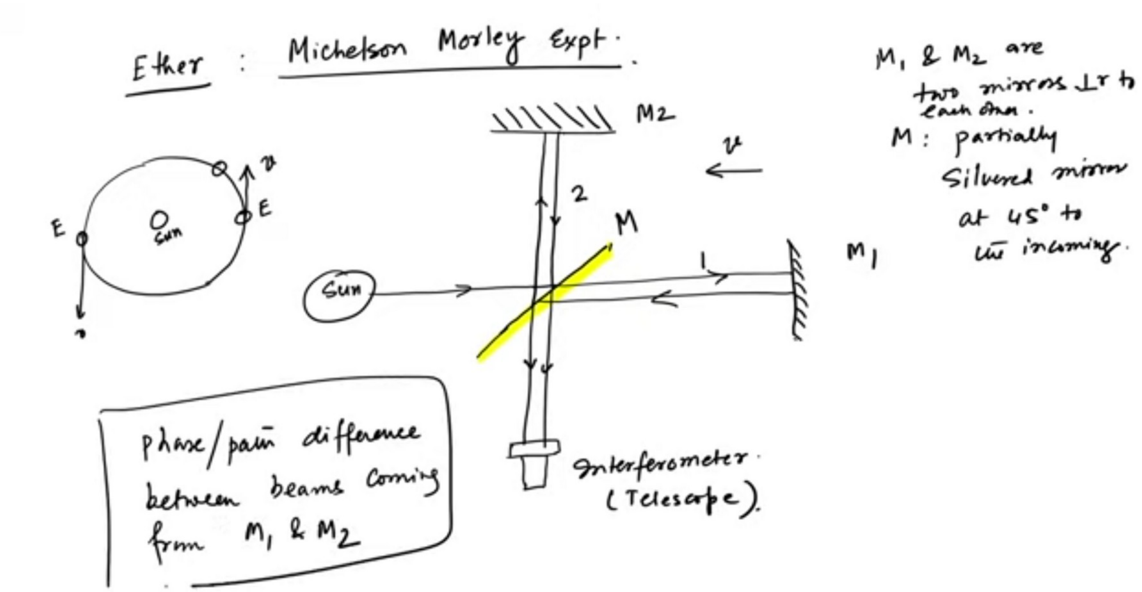
And so, if light is moving through the ether, then we should be able to detect the speed of ether. Now, since in those days when these experiments were being done, there was no direct way of confirming the presence of ether or the absence of ether. This preferred frame of reference was not available in those days. So, they had to do some measurements, indirect measurements, in order to detect the presence of ether. This led to the famous experiment by Michelson and Morley, and we'll discuss this experiment because it is quite important. Okay, so just to put things in perspective. So, this is the Sun that we have, and this Earth is moving around the Sun. So, when it is here, its velocity is tangential to the orbit, and when it is here, the velocity is tangential to the orbit. This is the position of the Earth. So, this is the position of the Earth; let us call it E. This is the position of the Earth at the two extremities, and so on, okay.

So, now let us try to see how this comes. We will have to draw a neat diagram for that. So, there is a mirror; let me finish this, then it will be clear. So, there is a partially silvered mirror. All right. There is another mirror here. Just to show that it is partially silvered, we have shown it by a patch of yellow. There is a Sun here, which gives light. And this Sun, the light comes like this. The light from the Sun comes; it gets reflected from this mirror, let's call it M2. This mirror is at M1, and then it comes back. Because this is partially silvered, this part of it goes, and then it comes back as well. Okay, and they get collected by a detector, which is an interferometer.

So, this is an interferometer. M1 and M2 are two mirrors, and M is actually a partially silvered mirror. So, a mirror and a partially silvered mirror; the difference is that a partially silvered mirror allows the light to go through it. So, it's like a glass plate, but it also has the ability to reflect, as we see here. These are proper mirrors; that is, they don't allow any light to pass through them. It's completely silvered at the back surface. This is an interferometer, which can be a telescope, and so on. So, this is beam one. This is beam two, and there is an apparent relative velocity, which is the velocity of ether. We will see that in the calculations. This is oriented at a 45-degree angle to the incoming beam, and it is important to make this distinction.

So, these two mirrors are perpendicular to each other. M1 and M2 are perpendicular to each other. That is the setup they have used. And so, what they have predicted is that if beam one gets transmitted and gets reflected by mirror M1, it comes to this mirror M and travels downward towards the interferometer. Whereas this light comes and goes to The mirror M2 gets reflected, and then that gets transmitted and comes to the telescope. Now, if there is a relative speed, that is, if there is movement in between these things, if this whole thing moves, then there will be interference because that is from a coherent source, that is, from the same source. That is, the Sun, and there'll be a change in the fringe pattern or the cross-wire of the telescope would move through a few fringes. We should be able to detect that movement of the few fringes.

And, that is to be looked at, and one needs to, you know, the fringe pattern needs to be sort of studied, okay. So, we need to calculate the phase difference between the two, or phase and path difference. I mean, basically, the path difference gives rise to the phase difference between beams coming from M1 and M2. So that's the job that we have at hand. And so there are two things that are important here. They arise from the different path lengths traveled. So there are different path lengths traveled by light, so different speeds of travel for the beams due to ether wind.



So, what I mean is that this ether moves, and then there is a so this ether wind velocity which is called as v. So basically here, the 1 is more important. So by 1, what happens is that, so the time for the beam 1 to travel from M to M1 and back, okay. So, it goes from M to M1 and back, and let us call this length as, you know, so the stationary length to be say l 1, and that is equal to t 1 equal to l 1 c minus v plus c minus v and l 1 c plus v. So there are two L1s which are traveled with two different speeds. One is c minus v and c plus v. c is the speed of light. of light and we have defined what v is, that is the different speeds of travel etc.

So, this is 2 c l 1 c square minus v square. So, this is equal to 2 l 1 by c 1 divided by 1 minus V square by C square. So, that is the time taken. So, it is clear that the light has an upstream velocity of c minus v and a downstream velocity of c plus v. So, while going at c minus v and while coming at c plus v. For beam 2, we need to have a little more clarity on the situation. So, this length is say l 2, and l 1 can be equal to l 2 or l 1 can be different than l 2. So, let us see what happens for beam 2. So, this is for beam 1. For beam 2, just to remind you that beam 2 travels from this M to M2 and back. But here you have to understand that by the time that it comes back getting reflected from the mirror 2, M itself has shifted by a distance, and that has to be taken into account.

(1) The different path lengths traveled by light

(2) Different speeds of travel for the beams due to ether wind velocity (v).

By (1)

Time for beam 1 to travel from $M \rightarrow M_1$ & back

c : speed of light

$$t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v}$$

$$= \frac{2cl_1}{c^2-v^2} = \frac{2l_1}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right).$$

So what I mean is the following that this is M2 and this was M earlier. And, you know, at a later time when this thing happens. So there is a reflection that has happened here, and that will give rise to thing here. So, this is that once again just to make things. So, we use a highlighter, and so this yellow patch says that it is actually M, okay. All right. So, this is where we are here. So, this is again M2, and this is position M, and this is a later position M, and this is nothing but l 2, okay and this is mirror M, and this is this distance that it moves is v t 2 because of this wind speed or the speed that you have is v t 2 okay.

So, that is there and this is l2 and so on, okay. So, there is a speed v here. So, the transit time again from M2, M2 and back, so M2, M2 and back, this transit time is given by c t2, that is what the speed of light takes in a time t2. It is equal to twice of, is basically the, you know, if you consider this as a right angle triangle. So, you are calculating this hypotenuse, and this hypotenuse is l2 square plus v t2 by 2 whole square. So, that will give you your t2 and l2 square is understandable, that is the perpendicular distance, and v t2 is the horizontal distance, and because it is this v t2 by 2 whole square, that is the half the distance.

So, this distance is v t2 by 2. So, that gives you t2 equal to 2 l2 divided by root over of c square minus v square and this is equal to 2 l2 by c and 1 divided by 1 minus v square by c square. So, we got t1 and t2. So, t2 is done in the ether frame, while t1 is done in the frame of reference or rather in the frame of the apparatus. So, this I think is an important thing. So, t1 is made in the frame of the apparatus, this setup basically, and t2 is made, this measurement is made in the ether frame. Alright, so we now have both t1 and t2, and we can find out the difference between the two times, which is equal to t2 minus t1, which is equal to 2 over c.

(1) The different path lengths traveled by light
(2) Different speeds of travel for the beams due to ether wind velocity (v).

By (1)

Time for beam 1 to travel from $M \rightarrow M_1$ & back

c : speed of light

$$t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v}$$

$$= \frac{2cl_1}{c^2-v^2} = \frac{2l_1}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right).$$

l2 divided by 1 minus v square by c square and minus l1 divided by 1 minus v square by c square. Of course, there is a difference between the two, which tells that the path lengths are, they take different times to travel. And then the path lengths are, of course, different. And then that will give rise to phase difference. And this phase difference will give rise to an interference pattern to, you know, the crosswire to shift from one fringe to another, okay. Now, what you do is that you interchange l1 and l2.

So, if you do that, that is you make these lengths to be completely different or rather what you do is that you rotate the apparatus by 90 degrees. So, if you do that, let us call that as delta t prime, which is equal to t2 prime minus t1 prime, and then this becomes equal to l2 divided by 1 minus v square by c square minus l1 divided by root over of 1 minus v square by c square. So that is the change that you have. So now if you calculate a delta t prime minus a delta t, in experiments it is easy to change the lengths because that is where you keep the mirrors. And so this is equal to 2 over c and you have l1 plus l2 divided by 1 minus v square by c square and a minus of l1 plus l2 divided by root over 1 minus v square by c square. That is the change in time. If you do a binomial expansion for v by c to be smaller than 1, do the expansion, and then one gets the delta t prime minus delta t that is equal to 2 over c 2 l1 plus l2

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left[\frac{l_2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{l_1}{1-\frac{v^2}{c^2}}\right]$$

Interchange $l_1 \to l_2$

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left[\frac{l_2}{1-\frac{v^2}{c^2}} - \frac{l_1}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$\Delta t' - \Delta t = \frac{2}{c}\left[\frac{l_1+l_2}{1-\frac{v^2}{c^2}} - \frac{l_1+l_2}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

for $\frac{v}{c} << 1$

$$\Delta t' - \Delta t = \frac{2}{c}(l_1+l_2)\left[1+\frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right] = \frac{l_1+l_2}{c}\frac{v^2}{c^2}$$

And 1 plus v by c square, so we do it for both the terms, 1 plus v by c square for the first term and 1 minus half v square by c square, okay. And this gives you l 1 plus l 2 by c, v square by c square, okay. So, this rotation of the apparatus by 90 degrees should cause a shift in the fringe position, okay, and because that changes the phase relationship between the two beams coming from M1 and M2 in these two. So, the optical path length difference Between the beams changes by one wavelength, one wavelength of light, so if the optical this thing changes, then there will be a shift of one fringe shift of one fringe at the crosswire. I think all of you are familiar with this crosswire because this is where you see that there is a plus type of thing there which tells you that probably is there either in the middle of a minima or a maxima and by whatever it is, it gets shifted by certain few fringes maybe, okay. So, if delta N, that is the fringe shift or number of fringes shift, this is the fringe shift, then delta N is given by delta t minus delta t prime, delta t prime minus delta t divided by t where t is equal to the time period of one vibration. So, that is lambda by c, where lambda is the wavelength of light and c is the speed. So, this is nothing but equal to like 1 over N or 1 over the frequency and so on. So, this is equal to l 1 plus l 2 by c t.

And v square by c square, and this is equal to l 1 plus l 2 by putting t equal to lambda by c. So, this is lambda and v square by c square. So, in the experiment, in the actual experiment, they have kept these lengths to be the same, and they are 22 meters. They have used the wavelength of light to be 5.5 into 10 to the power minus 7 meters. And v by c is taken as 10 to the power minus 4, that is like the speed of ether, then delta N should come out to be, if you put in all these values, it should come out to be 0.4. So, this fringe shift should have been observed. And unfortunately, nothing was observed there. And there was no observation of such things.

If optical path length difference between the beams. changes by one wavelength. then shift of one fringe at Cross wire.

ΔN : fringe shift

$$\Delta N = \frac{\Delta t' - \Delta t}{T} = \frac{l_1 + l_2}{cT}\frac{v^2}{c^2} = \frac{l_1 + l_2}{\lambda}\frac{v^2}{c^2}.$$

$$T = \frac{\lambda}{c}$$

In actual expt. $l_1 = l_2 = 22m$, $\lambda = 5.5 \times 10^{-7} m$, $\frac{v}{c} \sim 10^{-4}$.

$$\Delta N = 0.4.$$

fringe shift should have been observed!!

And it was, in fact, for the accuracy of the experiment, they mounted it on a massive stone to eliminate any vibration, other kind of vibration, which otherwise would have affected the experiment. And then they have mounted it on mercury so that it can be smoothly rotated. But nothing gave rise to any fringe shift, and this experiment has been done by a number of people, a number of groups, and they have been carried on for about 50 years, around 50 years, and at different times, in different seasons, so that whether this ether wind has any effect or whether you are experiencing winter or summer or, in fact, day and nights, whether the rotation of the earth is coming into the picture, but there is absolutely no change in the fringe shift that is observed through this experiment, and later in 1958 Sederholm and Townes carried out an ether wind experiment using microwaves, not using light, but microwaves with a different frequency, and concluded that if this ether theory is to be correct,

the Earth's orbital speed through ether would have to be a thousand times less, which means that all these Earth's rotation and revolution that give rise to day and night and different seasons would be completely, you know, destroyed by this theory. So, which means that finally, it is proved beyond doubt that there is nothing called ether, and that light actually does not require any material medium to propagate from one point to another, and it also tells us that there is no frame of reference that is preferred. So, all frames of reference are equivalent, and this is one of the upshots or one of the headlines of the special theory of relativity. So, let us write down the postulates of the special theory of relativity. And these postulates are the laws of physics are identical, that is, they are invariant in all inertial systems.

There is no special inertial system. So, all are equivalent, which is what I just said, and the speed of light has the same value. In all inertial systems and irrespective of the direction in which the light is propagating. So, we can really talk about the relative motion of two frames, and there is no way to infer, or rather no way to say for sure, that the motion is absolute, or there is no notion of absolute motion or absolute rest. And that probably justifies the name relativity. And of course, we have seen that Galilean relativity of this transformation equations they break down and we need to have new transformation equations which would make these laws such as you know the laws of mechanics and the laws of electrodynamics to be invariant and so we need transformation equations I am not saying of what.

Postulates of Special Theory of relativity.

(1) Laws of physics are identical in all inertial systems. There is no Special inertial system.

(2) Speed of light has same value in all inertial Systems.

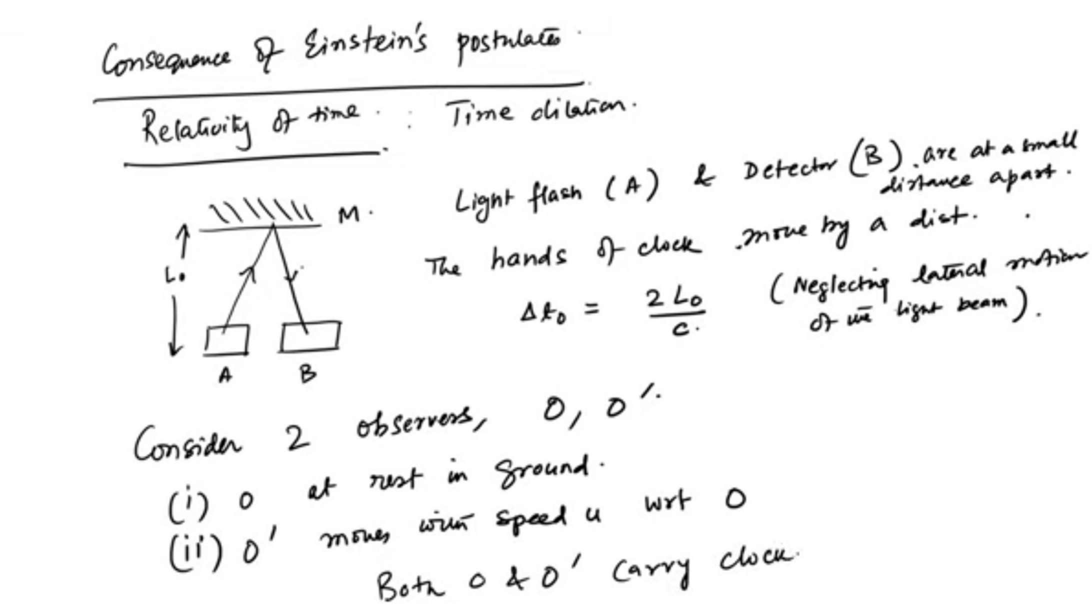
Need transformation equating various quantities are needed.

But they belong to everything: velocity, distance, and if time is not the same again, then there needs to be a transformation equation for time as well, and these are important to get. So, transformation equations equating various quantities are needed because we had to discard the Galilean transformation equations, and these are called the Lorentz transformation equations. So, we will just do the Lorentz transformation equations and stop with this discussion of relativity. So, these are basically the consequences of

Einstein's postulates. And let us call it the relativity of time, okay. And in particular, what you would be more familiar with is called time dilation, okay.

So, let us have a mirror like this, okay. Let us call it M. And you have A and B. So, light goes like this and comes like this, and so on, and this length is L0. We will tell you what we are drawing and so on. So, light flash A and detector B. So, these are the two things that we have. So, they are at a small distance apart. So, there is a small distance. Alright, so the distance is shown between this A and B and the mirror is L 0. So, a light pulse from A is sent towards M, the mirror gets reflected and it is registered by this detector B, okay.

So, the hands of the clock by that time, okay, move by a distance. So, this moves by a distance which is given by a delta t0, which is equal to 2 L0 divided by c, and we neglect any lateral motion of the light beam. Okay, so this is equal to delta t 0 equal to 2 l 0. So, this is the time interval that the clock is at rest with respect to the observer. Okay, so now consider two observers, and let us call them O and O prime. So, O is at rest on the ground, and O prime moves with speed u with respect to O.

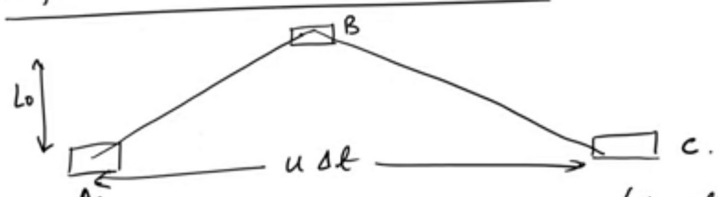


And again, it is just a velocity, but we have simplified it so that it can move only along one axis. So, that is the observer. And both O and O prime carry their own clocks. So, let us see what happens in O's frame of reference of O and O prime. So, what happens is that there is this A and there is the mirror which is B, and this is the one that it moves through. So, this is the picture that O sees for O prime. So, this is L0.

So, there is a light flash that is sent to the mirror, and the mirror reflects it, and it is obtained at, or rather detected by, the detector at C. So, this happens in O's case or O prime's case as seen by O. So, there is a sequence of events. So, the sequence of events is that a flash of light is emitted from A, gets reflected from B, and then it gets detected by C. So, in this time interval, let us call it delta t, and this is like this is u delta T. So, in this time interval delta t, O observes O prime's clock to move. by u delta t. So, this u delta t is the clock that has moved by this distance in space, which is what O registers about O prime, okay. So, what does O conclude? So, that observer concludes the light beam travels a distance 2 L.

2 L, where L is given by L 0 square plus u delta t by 2 square. Again, the same thing split it into two right-angle triangles and calculate the length of the hypotenuse of the two of them; I mean, that is why it is 2L, okay, and where L is given by that, okay. So, the time interval measured by O is delta t, this is equal to 2 L over c, which is equal to 2 into L 0 square plus u delta t by 2 whole square divided by c. So, that is the time interval between these two events of flashing a light and then detecting it. So, this is the time interval that O registers. And if you go to this and call this as equation 1 and maybe this as equation 2 and this as equation 3.

In frame of reference of O and seeing O'



In this time interval Δt, O observes O''s clock to move by $u\Delta t$.

O concludes light beam travels dist. $2L$.

where $L = \sqrt{L_0^2 + \left(\frac{u\Delta t}{2}\right)^2}$ (2)

Time interval measured by O is

$$\Delta t = \frac{2L}{c} = \frac{2\sqrt{L_0^2 + \left(\frac{u\Delta t}{2}\right)^2}}{c}. \quad 3$$

So, if you put 1 in 3, basically you eliminate L 0. So, if you do that, then delta t becomes equal to delta t0 divided by the square root of 1 minus u square by c square, and this is let us call it as equation 4, and this is called time dilation, okay. This is an important concept in relativity called time dilation. So, O measures a longer time interval for the clock of O prime. So, which means that the moving clock actually runs slow. So, if there is an occurrence of an event for a time delta t not, delta t not is the time that is noted for this event to take place in the frame of O prime,

okay, which is moving along with the speed u as compared to the observer O. So, the beginning and the end take place at the same point in space according to O prime but not with respect to the observer O. So, that means that the observer O moving with respect to O prime, or rather if O prime is considered to be at rest, then the observer O is moving with respect to him. So, he measures a longer time interval than the proper time delta t 0. So, delta t 0 is called the proper time interval, and delta t is the longer time. So, moving clocks actually run slow. Let me do the other thing, which is length contraction. So, we will call it relativity of length length contraction.

Putting (1) in (3). → Eliminate L_0.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}} \qquad (4).$$

→ Time dilation.

Δt_0 : proper time.

Moving clocks run slow.

Now, we make one important adjustment here; now, the light flash emitted by O prime is in the direction of motion. See, here we have done it perpendicular to the direction of motion, okay? And now it is in the direction of motion, which means that O, the observer at O, which is at rest, will see that the clock itself has a length, okay? So, you can think of it like this: this is the, you know, the so this is a, and there is a mirror which is like this. And this whole thing, the whole thing has a length L. So the clock has a length L in O's reference frame. Again, we are showing the thing in the frame of reference of O, okay.

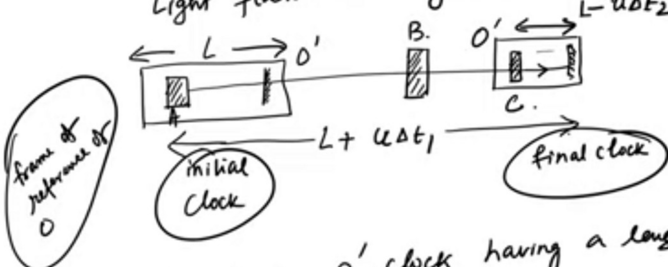
So now what happens is that there is a light pulse that is emitted. And we see that in B's reference, or rather this O prime. So that is like this, and then there is a. So the clock is actually represented by that there is a flash, and the flash is now releasing a signal, or a light flash is being released, and that gets reflected by a mirror, which is exactly what we did. So, this is, you know, at a later time, so this is, we can, so this initial clock will make this thing clear. So, this L plus u delta t 1 is this entire length, okay, that is there. And this is, so these all, you know, O primes, okay.

But these are O prime, but they are actually seen in the frame of reference of O, okay. So, this distance is L minus, L minus u delta t 2, and this L plus delta, u, uh, delta t 1. And so this is basically the initial clock, and this is the final clock. And now the motion takes place or this emission of this light flash takes place along the direction of motion. So this light goes and here and then gets detected and so on. So this is the detector which is what we have called as A, B and C.

So, A, B and C, that is here. So, this is like A and so this is like B and then this is again like or you can call the B to be somewhere at the intermediate regime and this call this as C. All right. So, maybe if you just So, according to O, the length of the clock is L. So, O sees O prime's clock having a length L. Okay, so this length, as we will see, is different than the proper length. So, a light flash is emitted from A, it reaches the mirror at B, so this is B, and at a time, you know, delta t 1 later, and in this time interval, light has traveled a distance which is 0. c delta t1 which is equal to the length L of the clock plus this additional distance that is L plus c delta t1 and as seen by the observer at O, okay.

Relativity of Length : Length contraction.

Light flash emitted by O' is in the direction of motion.



O sees O's clock having a length L.

$$c\Delta t_1 = L + u\Delta t_1 \quad (1)$$

$$c\Delta t_2 = L - u\Delta t_2 \quad (2)$$

So, the mirror moves forward in this interval and we have a c delta t 1 is equal to L plus u this u delta t 1 as well. So, the flash of light that travels from the mirror to the detector in time delta t 2. So, this is that delta t 2 from the mirror to the detector. So, this is the mirror to the detector back. So, this C is the position of the detector. And so your c delta t 2 is now equal to L minus u delta t 2. So if you solve for equation 1 and 2, which is fairly simple to do, so we solve for delta t 1, delta t which is equal to delta t 1 plus delta t 2. Which is equal to L divided by c minus u plus L divided by c plus u. This is equal to 2L by c and 1 divided by 1 minus u square by c square.

So, all these things were happening. So, all this diagram may be a little confusing, but if you think that this B was actually in a perpendicular direction earlier and if it is moving in the same direction, O sees that the clock itself has a length, which is what is denoted by this box, okay, and then it is again detected after it gets reflected, is detected by the detector, and because O sees the length to be still there, maybe a different length is what we are going to find out, so that is why the two boxes have different, you know, extensions. And then this light is again detected by the detector, or the flash of light is detected by the detector when it comes from that or reflected from that mirror and reaches C. And all these lengths, etc., are shown in the figure itself. And now we are calculating the time: that c delta t 1, so within the time interval delta t 1, light travels this distance c into delta t 1, which is nothing but L plus u delta t 1. L is the initial length of the clock that is seen by O, that is O prime's clock.

That is seen by O. So, these are delta t 1 and delta t 2 is the time within which, you know, the light gets reflected from the mirror and reaches the detector. So, this is equal to that, and if you do a bit of simplification here or rather if you write this delta t as delta t 0 divided by 1 minus u square by c square, this is equal to 2 l 0 by c. This is what we are using the time dilation formula. So, this is equal to 2 l 0 by c, 1 divided by 1 minus u square by c square. So, L becomes equal to L0 divided by 1 minus u square by c square, and this is called the length contraction. Okay, so the observer O prime, which is, you know, at rest with respect to the object, measures a proper length, which is L 0.

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-u} + \frac{L}{c+u} = \frac{2L}{c}\,\frac{1}{1-\frac{u^2}{c^2}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{2L_0}{c} = \frac{2L_0}{c}\,\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$L = \underbrace{L_0}_{\text{proper length (rest length)}}\sqrt{1-\frac{u^2}{c^2}}.$$: Length Contraction.

$\boxed{\text{Moving sticks are shorter.}}$

So this is called the proper length. Just like proper time or length, it's also called rest length. Okay, so all observers who are in motion with respect to O prime, because O prime is in motion with respect to O, which means that with respect to O prime, O is in motion. So all the observers who are in motion with respect to O prime, the primed observer, would measure a shorter length. Okay, so this length is contracted. So I write once again, moving sticks are shorter. This is called length contraction and time dilation, and using this, one can formulate the equations for the transformation of velocities and acceleration, etc., from where one can formulate the equations of motion and by looking at how the electric field and the magnetic fields transform under these length and time. One can also formulate Maxwell's equations, which are equations for electromagnetism that we study in electromagnetism. So, and they are found to be absolutely correct. So, even if Newton's laws fall short in the velocity regime where v is comparable to that of c, the electromagnetic laws of Maxwell's equations are invariant. So, we stop here for today and then we will carry on with more information about early quantum theory and the development of quantum mechanics in the next lecture.