ELEMENTS OF MODERN PHYSICS

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Lec 29: Radioactive Decay, Half Life

Welcome to the course called Elements of Modern Physics. So we have been doing nuclear physics and today we are going to talk about radioactivity. We have seen some Sort of glimpses of it yesterday, but we'll talk about radioactivity elaborately today. We have seen that the bombardment of atoms with the alpha particles that had set the platform for the determination of atomic structure.

And this was done by Rutherford by bombarding these alpha particles, you know, towards this gold foil, thin gold foil, a few hundred nanometres. and that made this structure of atoms apparent to the community. So there is also an analysis of the chemical relationships among the radioactive elements that resulted in the discovery of isotopes. I have shown you these radioactive elements in the periodic table on the last class. And radioactivity is undeniably connected to the development of nuclear physics.

In fact, a lot of nuclear physics and nuclear engineering, they revolve around this radioactivity. And we'll talk about this discovery a little bit. But you have to read some relevant literature and sort of understand that how accidental can it become And so this was one of the upshots of this discovery of radioactivity, that it was just by chance that it was discovered. And the theory of radioactivity that we are going to formulate will depend on the ability to precisely measure these radioactive changes that take place in the nuclei.

and substantiate them by simple equations which govern these radioactive changes or these, you know, how the activity sort of change with time or the number of particles of a given radioactive element, nucleides basically that change with time. And these are going to be some of the discussion topics for this class on radioactivity. So I show you this APS 125 celebration which took place in Feb 25, 2008, and in which it was said that March 1, 1896, Henry Becquerel, he is also called Antoine Henry Becquerel, he discovers radioactivity, that's a picture of Becquerel, And he got the Nobel Prize in 1903, along with this Curie couple, Madame Curie and Pierre Curie. And it says that this, I'm quoting it from the citation of this APS News, says it's one of the most well-known accidental discoveries in the history of physics.

On an overcast day in March 1896, the French physicist Henry Becquerel opened a drawer and discovered spontaneous radioactivity. It is written in a dramatic fashion, but then you should look up the history and it's very interesting. So I elaborate a little bit again from the same APS News article. It says that the story of Becquerel's discovery is a well-known example of an accidental discovery, which is what we have told.

Somewhat less well-known is the fact that 40 years earlier, which means that 1856 and so on, because this was 1896. So 1856, someone else had made the same accidental discovery. And his name is Abel Niepce de Saint-Victor. He was a photographer by profession. He was experimenting with various chemicals, including the uranium compounds.

Uranium is one of the most radioactive or common radioactive elements that one knows about. So like Becquerel would later do, means at that time, 40 years before Becquerel, he exposed them to sunlight. So these uranium compounds to sunlight and placed them along with pieces of photographic paper. He was a photographer, so it was natural to work with photographic paper. And he placed these uranium compounds along with the photographic paper in a dark drawer.

When he opened the drawer, he found that some of the chemicals, including uranium, had exposed the photographic paper. And Niepce thought that he had discovered some new sort of invisible radiation. He reported his findings to the French Academy of Sciences. However, probably because his profession was not that of a scientist, no one investigated the effect further until decades later—four decades later. Becquerel repeated essentially the same experiment on the 1st of March.

So, this is the discovery of radioactivity. And the properties of radioactive decay are when nucleus undergoes an alpha decay or a beta decay, the Z or the atomic number that changes and it becomes a different element. So, we have a different element when the nucleus of a certain element undergoes radioactive decay. And the energy of these decay processes comes from within the nucleus. That is, there is no excitation from any external source.

It just spontaneously happens. And it's a statistical process and obeys, hence obey laws of probability. And, um, why these, uh, you know, this two point number two, which says that, uh, where does this energy come from? Because there is no, um, excitation that has happened from any external agency. And, that was explained by, Einstein's mass-energy equivalence, that is, mass is, you know, can be converted into energy, and he wrote down that E equals mc squared relation. So, which means that the mass itself has a form, or rather it is in a form of energy, which gives rise to this decay. Here, schematically, some different radiations coming out from a radium sample kept inside a lead box is shown. And what you see, those X's or the crosses, these are actually the magnetic field lines that are, you know, going into the screen that you see. So it's into the paper.

Properties of Radioactive decay:

(1) When a nucleus undergoes α or β decay, Z changes and it becomes a different element.

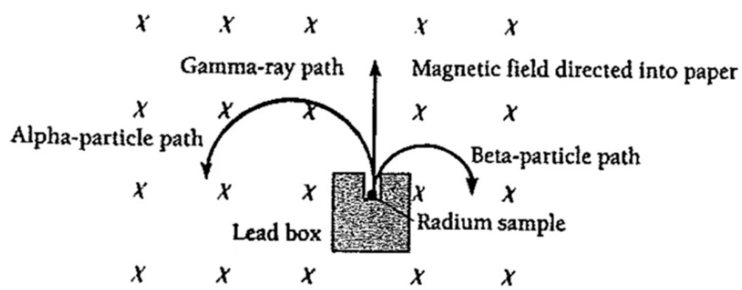
(2) The energy of such decay processes comes from within the nuclei (no external agency!!)

(3) Radioactive decay is a statistical process and obeys laws of probability.

Einstein's mass energy equivalence explains (2).

Different radiations From Radium.

Alpha: He nuclei
Beta : Electrons
Gamma : Photons



Plane of the paper, and then you see all these radiations coming out, and the alpha particle paths are shown on the left, which kind of bend in the magnetic field, and which means that, of course, they are charged, and the beta particle paths are shown on the right, which has got the opposite curvature and smaller than the alpha particle curvature. And also, because it bends in a magnetic field, so there must be the centripetal force that, you know, is acting there. So what we are trying to mean is that we have mv squared over r, and that's equal to qvb and so on. So this equation is You know, they are balancing each other.

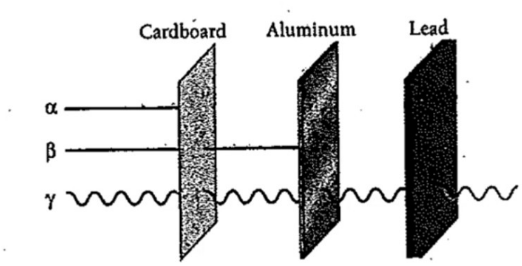
So there's a low range force that balances the centripetal force. And then there is these radials that tells you that how much is the curvature of this or the trajectory of the particle. OK, so these are the beta particle. And then you see that there's a gamma ray has also come out. So it's alpha, beta and gamma.

But the gammas are charge less and they do not change. you know bend in the magnetic field. So these gammas are the photons and alpha are of course we know as the helium nuclei which is written as this 2Hg4 that we have seen earlier and these betas are nothing but the electrons and gammas are the photons. So this is the schematic diagram of these emergence of radiation from a radium sample. So different decay processes are we have from the nucleus.

So that's alpha decay and then there's a beta decay. Then there are positron emissions and then there are electron captures and then there are gamma decay and so on. And each one is shown by this equation that there is this. X with atomic number Z and mass number A. When it undergoes an alpha decay, you see that there is the Z becomes Z minus 2 and A becomes A minus 4. For a beta decay, it becomes Z becomes equal to Z plus 1.

Different Decay Processes

Decay	Transformation	Example
Alpha decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{238}_{29}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$
Beta decay	${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^{-}$	${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-}$
Positron emission	${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^{+}$	${}^{64}_{29}Cu \rightarrow {}^{64}_{28}Ni + e^{+}$
Electron capture	${}^{A}_{Z}X + e^{-} \rightarrow {}_{Z-1}^{A}Y$	${}^{64}_{29}Cu + e^{-} \rightarrow {}^{64}_{28}Ni$
Gamma decay	${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$	${}^{87}_{38}Sr^{*} \rightarrow {}^{87}_{38}Sr + \gamma$



2.2 Alpha particles from radioactive materials are stopped by a piece of cardboard. Beta particles penetrate the cardboard but are stopped by a sheet of aluminum. Even a thick slab of lead may not stop all the gamma rays.

And, of course, there's an electron that comes out. You also have helium there. So, in positron emission, it's just the opposite of beta decay—or, you know, this Z becomes Z minus 1. and you get a positron there, the electron capture, you get, so this electron is being captured by this, this element X, giving rise to Z minus 1, and the mass number remains same, and then it becomes a different element Y. You see that X element goes to Y, and so on in every place, that's what we meant by these, you know, it becomes a new element, and of course, the Both A and Z could change, or A could remain the same and Z could change, and so on and so forth.

OK. In gamma, of course, it sort of—you know, the mass number doesn't change, but there's a gamma that comes out in each one. There's an example given. So there's uranium going to thorium plus helium. Helium, then carbon going to nitrogen plus an electron, that's for the beta decay, and then copper goes to nickel plus a positron, and then copper plus an electron becomes a nickel, then a strontium, and it becomes a strontium and gamma and so on and so forth.

Now, you see that it's like schematically showing that what are these the penetration power of each of these alpha, beta and gamma particles. So they are, you know, from the radioactive materials, the alpha particles are easily stopped by just a cardboard that we use on our everyday office work. They can be stopped by cardboard. Beta particles cannot be stopped by cardboard, but they can be stopped by an aluminum sheet—a sheet of aluminum. And gamma particles—they have enormous, you know, penetrative power and in fact the thick slab of lead also cannot stop these gamma rays.

So these are various, you know, sort of penetrative power or penetration power of all these alpha, beta and gamma particles or this decay. So let's talk about these radioactive decay equations and sort of discuss what half-life is. And so we talk about radioactivity. So we talk about the measurement of activities of a radioactive sample and how they really fall off with time. So, that is the main thing that we want to know.

Radioactive decay equations: Half-life

Measurement of the activities of a radioactive sample shows that they fall off exponentially with time. The decay process obeys:

$$R(t) = R(0)e^{-\lambda t} \qquad (1)$$

$R(t)$ is activity at time t, $R(0)$ is activity at $t = 0$, λ is disintegration (decay) constant.

The half-life of a radioactive sample $T_{1/2}$ is defined as the time at which the activity drops to half of its initial value, $\frac{R(0)}{2}$,

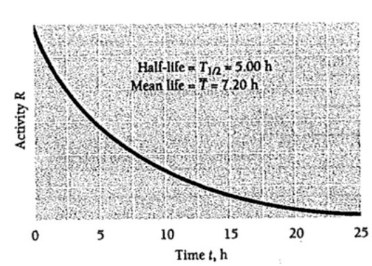
$R(t) = \frac{1}{2}R(0)$

$$\frac{R(0)}{2} = R(0)e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2$$

$$\lambda T_{1/2} = \ln 2 = 0.693$$

$$T_{1/2} = \frac{0.693}{\lambda}$$



So, the decay process obeys this decay equation where R T is the activity at T equal to at some time T and R 0 is the activity at time T equal to 0 and lambda is either called a disintegration constant or it is also called as a decay constant that you see that it is in the exponent and it's getting multiplied by t and clearly because the exponent has to be dimensionless or rather the argument of the exponential function term has to be dimensionless so lambda has these unit of 1 over t or if t is in seconds or hours this should be in hour inverse or second inverse and so on. So, we also define a quantity called the half-life of a radioactive sample, and it is written as this t half. The way it is written there, it is defined as the time at which the activity of the element or activity of the nucleoid that drops to half its initial value. So, R0 becomes or rather Rt becomes R0 by 2.

And so this becomes, so RT becomes, you know, R0, half of R0. So at what time it falls into that is called as the half-life. So we can put it in this equation, if you call this as equation 1, then we can put it in this equation and can write down these RT equal to R0 by 2 and then, you know, cancel R0 from both sides because that is not 0. So it gives you exponential lambda to the power t half. A little bit of notation that you have to form of your own.

It's this entire thing is in the exponent. It might give you a feeling that it is not in the exponent, but it is. So exponential lambda t to the power half is equal to 2. So, lambda into the t half, which is called as a half-life is nothing but log 2. Log 2 has a numerical value of 0.693.

So, t half is equal to 0.693 by lambda. So, if you are asked this question that, you know, after how much time? or rather this activity of this sample will fall to half or what is the half life and you are given the disintegration constant or the decay constant, you can simply use this relation to get T half in terms of lambda or if you are given T half, you can find out lambda using the same equation. This is 0.693 is nothing but the log 2. So, this activity curve is shown for some radioactive element and it sort of the activity really falls off as exponential minus t and this lambda is can be found out from this.

And then you can use that lambda to calculate t half. It turns out that t half is equal to five hours here. Now, there's another quantity we'll just come to that called as the average life. So that's t bar and that t bar is not equal to t half will define what is t bar in this slide. Okay.

So, for example, if you take T half to be equal to 5 hours, then lambda comes out to be equal to 3.85 into 10 to the power minus 5 second inverse. And we have simply used this thing that T half is equal to log 2 by lambda and log 2 is 0.693. But this entire description, you know, in terms of activity makes it a little ill-defined, not really ill-defined, but it is less, you know, intuitive. But if we represent it in terms of the number of undecayed nucleides, then it becomes more clear because now we are talking about number, some number that, you know, decays over time and you can count that number. So the number dn that decays in a time dt is the product of these number of nuclei and is the probability lambda dt and that each one will, you know, decay each of these nuclei.

These nucleides will decay in a time dt. So dn is actually proportional to n and dn is of course proportional to dt, the time over which you are taking it. And if you take both these things and then take this proportionality constant as lambda, that is also one way of looking at this. And lambda is, as we told that it's a decay constant. And there's a negative sign because this, you know, is decreasing.

For the $T_{1/2}$ to be 5 h, $\lambda = 3.85 \times 10^{-5}\ s^{-1}$. $T_{1/2} = \frac{\ln 2}{\lambda}$.

A more familiar way of representation is in terms of the number of undecayed nuclides. The number dN that decays in a time dt is the product of the number of nuclei N and the probability λdt that each will decay in dt.

$$dN = -\lambda N dt \quad \Rightarrow \quad \frac{dN}{N} = -\lambda dt$$

$dN \propto N$
$dN \propto dt$

Integrating, the radioactive law is expressed in terms of

$$N(t) = N(0)e^{-\lambda t}$$

There is also a mean lifetime, $\bar{T} = \frac{1}{\lambda}$ with $\bar{T} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}$

DN is is basically, you know, has will decrease and this negative sign simply talks about that. So if you try to integrate this equation by writing dn by n equal to minus lambda dt, so you get n of t which is equal to n0 exponential minus lambda t. Now this n is the number of nucleides left at a time t, n0 is the number of nucleides at the beginning, that is at t equal to 0, whenever you have said t equal to 0 is, and exponential minus lambda t.

As I was saying that there is also a mean lifetime, and the mean lifetime tells you that when this number really falls to a value which is n0 by e, that is the exponential. So, nt becomes n0 by e at, you know, t equal to t bar. So, that is the definition of t bar or the mean lifetime and if that happens then t bar becomes equal to 1 over lambda and lambda we know that is equal to 0.693 by t half.

So, there is a simple relationship between t-bar and t-half. They are slightly different than each other and make sense because this E is really something like 2.7, whereas when we define half-life, this factor in the denominator instead of E, it was just 2. So, it is 2 and 2.7, that made the difference between this T half and T bar, which are respectively the half-life and the average life. So we sort of, you know, go back to these activity picture and want to understand that what does these or how these activities connected to the number of nuclei. And R is nothing but defined as the activity is defined as D and DT with a minus sign.

And you can write this down as r equal to lambda n0, which is n0 is the number of nucleides at t equal to 0, okay, and exponential minus lambda t. So, if r0 equal to lambda n0, okay, then you get RT equal to lambda NT. So, that is the relationship between the number of nucleides and the activity at a given time, and the same relationship actually holds for all time. Because you see the left relationship is R0 equal to lambda N0. So, the same relation holds at any subsequent time.

Let us see an example. The question is to find the activity of 1 milligram of radon, which is written as this 222 Rn, and the atomic mass we want is given as 222 units or atomic units. And the decay constant we can find out by taking this 0.693 divided by T half. And this T-half is picked up from, you know, some data sheet. And that gives you that this is equal to 2.11 into 10 to the power minus 6 per second.

So that is lambda. And now the number of atoms in 1 gram of radon or 1 gram of this Rn. Radon is equal to this 1 milligram, which is 10 to the power minus 6 kg, divided by its mass. Then you take the mass of this proton, which gives you there are so many atoms at t equal to 0. So if you want the activity, we will have to multiply this lambda that you got and this, getting this activity in terms of you know, these decays per second. It also has these units as TBq and Ci and so on.

We can restrict ourselves to these decays per second. And so R equals lambda n, this is 5.72 into 10 to the power 12 decays per second. So that is the activity. So that is what is being asked. Find the activity of 1 milligram of radon.

The activity of a sample is defined as

$$R = -\frac{dN}{dt} \Rightarrow R = \lambda N(0)e^{-\lambda t}$$

at $t=0$

If $R(0) = \lambda N(0) \Rightarrow R(t) = \lambda N(t)$

Example Find the activity of 1.00 mg of radon, ^{222}Rn, whose atomic mass is 222 u.

Solution

The decay constant of radon is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(3.8\text{ d})(86{,}400\text{ s/d})} = 2.11 \times 10^{-6}\text{ s}^{-1}$$

The number N of atoms in 1.00 mg of ^{222}Rn is

$$N = \frac{1.00 \times 10^{-6}\text{ kg}}{(222\text{ u})(1.66 \times 10^{-27}\text{ kg/u})} = 2.71 \times 10^{18}\text{ atoms}$$

Hence

$$R = \lambda N = (2.11 \times 10^{-6}\text{ s}^{-1})(2.71 \times 10^{18}\text{ nuclei})$$
$$= 5.72 \times 10^{12}\text{ decays/s} = 5.72\text{ TBq} = 155\text{ Ci}$$

What are the other applications of radioactivity? So one does radioactive dating or these are also called as carbon dating. So the ages of geological or biological specimens, which are very old specimens, can be established by radioactive dating. The ratio of the amount of the nucleoid and the stable daughter, which is called as a resultant or it's called the daughter. So this the parent nucleoid, so it's often called as a parent and daughter.

So parent one is the one that's at t equal to zero. And then, of course, it undergoes a transformation from that element to a different element. And that's called a daughter element. And this is it says that it depends in a specimen depends on the age of the latter. I'll read it once again.

The age of a geological or biological specimen can be established by radioactive dating. The ratio of the amounts of nucleoid and its stable resultant in a specimen depends on the age of the latter, which means the age of this daughter of this thing. So by counting the age or knowing the age, so we can know what is the age of the specimen. Greater the proportion of these daughter nuclei, it is likely that the older is this specimen. So one can know that approximately how many years old a rock is and so on and so forth.

So radioactivity is one of the primary applications of radioactivity. For example, rocks or for example, hills and all that, one knows that by doing this procedure, how old it is. Let us give an example of such a determination. So, a piece of wood from the ruins of an ancient dwelling was found to have a carbon-14 activity of 13 disintegration per minute per gram of the carbon content. The 14C activity of living wood is 16 disintegration.

That is living wood means that this is, you know, at the T equal to zero and so on. So this is known. So there's 16 disintegrations per minute per gram. So how long did the tree die from which the wood sample came? OK, so that's the question.

So if the activity of a certain mass of carbon from a plant or an animal, so carbon is the constituents of all plants and living beings, so that was recently alive is R0, and the activity of the same mass of carbon from the sample to be dated is R. So these activity R and the R0, which is at the recent, or rather that's when it was alive, where the initial at t equal to 0 is r0. So they have this relation r and r0 is r0 exponential minus lambda t. Now to solve for t, that's the age of these ruins or this wood rather. So this exponential lambda T is R0 by R and lambda T is nothing but a log of R0 by R and T is nothing but 1 by lambda log of R0 by R. So if you know the disintegration constant, if you know the R0 and R1 can find it and some equations, say, for example, let us not worry about that. The decay constant is found to be like this, which is, you know, and so here one can give this R0 by R is 16 by 13 because it was initially 16 disintegration per minute and now is 13 disintegration per minute.

Example

A piece of wood from the ruins of an ancient dwelling was found to have a ^{14}C activity of 13 disintegrations per minute per gram of its carbon content. The ^{14}C activity of living wood is 16 disintegrations per minute per gram. How long ago did the tree die from which the wood sample came?

Solution

If the activity of a certain mass of carbon from a plant or animal that was recently alive is R_0 and the activity of the same mass of carbon from the sample to be dated is R, then from Eq. (12.2)

$$R = R_0 e^{-\lambda t}$$

To solve for the age t we proceed as follows:

$$e^{\lambda t} = \frac{R_0}{R} \qquad \lambda t = \ln\frac{R_0}{R} \qquad t = \frac{1}{\lambda}\ln\frac{R_0}{R}$$

From Eq. (12.2) the decay constant λ of radiocarbon is $\lambda = 0.693/T_{1/2} = 0.693/5760$ y. Here $R_0/R = 16/13$ and so

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{5760\text{ y}}{0.693}\ln\frac{16}{13} = 1.7 \times 10^3\text{ y}$$

And so on. So T becomes equal to 1.7 into 10 to the power 3 years, which is like more than 1,000 years, 1,700 years. That's the power of this radioactivity that one can really find out from this carbon dating or this radiometric dating. So what about the radioactive series? There are four radioactive series.

Each series in different daughter nuclei starting from the same element. OK. And why there are only four. And I mean, what is the justification for these four series? So the four series come from the alpha decay, which actually reduces the mass number of the atomic mass A by 4 units.

So after that 4, this gets repeated. So if you take the mass of a certain nucleus to be equal to 4N, so A equal to 4N, where N is just an integer, they would decay into descending orders of the mass numbers of 4N to 4N minus 1, 4N minus 2, and so on. and the other three series will be 4n plus 1 4n plus 2 and 4n plus 3 well not 4n plus 4 this is wrong okay so this is written by mistake so 4n plus 3 because 4n plus 4 will become equal to 4n same as 4n you just go to n from one value to its next value so that will be 4N plus 1. So these are the three series whose mass numbers are given by 4n plus 1, 4n plus 2, and 4n plus 3, and where n is just an integer.

Radioactive series

There are 4 radioactive series, with each series yielding different daughter nuclides starting from the same element.

α decay reduces the mass number by 4 units. Thus, for nuclides whose mass numbers are given by $A = 4n$ (n: an integer), they can decay into descending orders of mass numbers.

The other 3 series are given by the nuclides with mass numbers $A = 4n + 1$, $A = 4n + 2$
$A = 4n + 3$

Different members of this series too can decay into one another.

And this gives rise to four radioactive series. So different members of this series can also decay into one another. We'll give an example of that. So the four radioactive series are 4N (thorium), which has half-life 10 to the power 10 years, very long half-life and the stable end product is the lead 82208.

4n plus 1, it's called neptunium, which has a half-life of 10 to the power 6 years. And the stable one is the one that becomes bismuth. And then 4n plus 2, it starts with uranium and ends with 82206 Pb. And 4n plus 3 is actinium, which starts with, again, 92, 235 uranium, as opposed to 92, 238 uranium for the 4n plus 2 series. And the half-life is again like 10 to the power of 8 years or close to 10 to the power of 9 years.

Four Radioactive Series

Mass Numbers	Series	Parent	Half-Life, Years	Stable End Product
$4n$	Thorium	$^{232}_{90}Th$	1.39×10^{10}	$^{208}_{82}Pb$
$4n + 1$	Neptunium	$^{237}_{93}Np$	2.25×10^{6}	$^{209}_{83}Bi$
$4n + 2$	Uranium	$^{238}_{92}U$	4.47×10^{9}	$^{206}_{82}Pb$
$4n + 3$	Actinium	$^{235}_{92}U$	7.07×10^{8}	$^{207}_{82}Pb$

And the stable end product is again lead, but with a different mass number than that of the 4n plus 2 series. Okay, so we are sort of closing in this discussion or rather closer to the end of the discussion on radioactivity. And let us just, you know, tell us what alpha decay is. So the alpha decay is the attractive forces between nucleons are known to be short range. So these are called the strong forces.

So we are talking about strong forces between the neutrons and the protons. Of course, there are strong forces among quarks, but we are not talking about that. And the total binding energy, say, is proportional to A, which is the mass number, which is equal to Z plus N. Because it is short range, so it will just depend upon how many constituent nucleons are there. and there are A constituent nucleons, which is A equal to Z plus N. Now, the repulsive energy between protons is proportional to Z squared. This is, we know, just the Coulomb energy, which depends on the charge of these protons.

So, for a number of nucleons larger than 210, Then the Z square will be very large and it will be so this enormous repulsive energy between the protons would be unable to or will fall short of compensating the attractive potential energy between the nucleons. OK. And alpha decay actually occurs in such nuclei and whereby decaying this alpha particle, there is an effort to reduce the size and hence make these resultant, you know, element to be stable. OK.

So alpha decay is it's a spontaneous thing that happens, happens for nucleons, the mass number to be, you know, to be large, larger than maybe 200 or so. And the alpha particles are emitted, which is like 2He4 are emitted. So the atomic number goes down by 2 and the mass number goes down by 4. Thereby, it transforms into a different element and a smaller element. And this smaller element means that there's a reduction in size of the element.

Alpha decay

Attractive forces between nucleons are short-range (strong forces).

The total binding energy, $E_b \simeq A$.

While the repulsive energy between protons $E_C \simeq Z^2$.

When the number of nucleons > 210, the enormous repulsive energy is unable to compensate for the attractive potential energy.

Alpha decay occurs in such nuclei, where there is reduction in size to compensate for the instability.

And that contributes to the stability. And that's how it counters, you know, being coming into a collapse by emitting these alpha particles. So why alpha particles are emitted, not protons? Because protons can also be emitted. These are charged particles.

And so now, to escape the nucleus, one should have sufficient kinetic energy. And we saw it yesterday that if the electron has to reside in the nucleus, its kinetic energy is of the order of 60 MeV. So the alpha particle and electrons are, of course, different. But nevertheless, alpha particles among the nucleus are the lightest ones. So they would have very large kinetic energy, which would allow them to escape, whereas protons have much larger mass.

So they have lesser kinetic energy for them to escape. Or rather, it is not, as we have said, of the order of a few MeV, and that is not the energy with which the protons can escape the nucleus, whereas the alpha particles can escape. So there is this empirical relation, energy of the alpha particle is like A minus 4 divided by A, A is the atomic mass into this

Q, where Q is known as the disintegration energy defined as Mi minus Mf, minus mx into c squared, that's like the effective mass of the total net mass, and it's multiplied by c squared, that gives you the disintegration energy, mi is the initial mass of the nucleus, so this, let's call it as mi, and then mf is the final mass of the nucleus, or mass of the final nucleus, and mx is the particle mass, and so this will tell you that What is the kinetic energy of these alpha particles in a certain nuclei?

Why α-particles are emitted, and not protons?

For escaping the nucleus, one has to have sufficient kinetic energy.
α-particles (among the nucleons) have smallest mass for them to achieve the required kinetic energy.

$$KE_\alpha \approx \frac{A-4}{A} Q$$

Where **Disintegration energy** $Q = (m_i - m_f - m_x)c^2$

where m_i = mass of initial nucleus
m_f = mass of final nucleus
m_x = particle mass

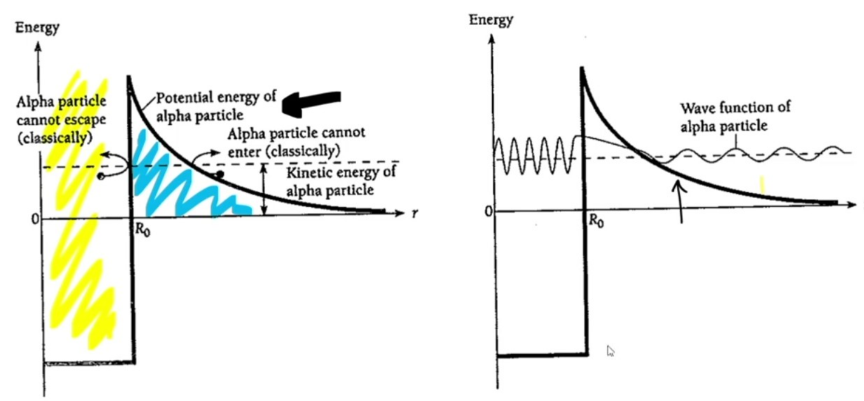
And if you calculate these, these will come out to be much larger than about 3, 4 MeV, which is the energy that the nucleus can hold. Now this picture could be familiar from the first course on quantum mechanics or whatever has been discussed even in this course as well. So there is this potential that these alpha particles feel, and it is based on this tunneling theory. We are not doing any explicit calculations here, but if you see this left region, that is this region here, So this is the region where, you know, alpha particles cannot escape from if they are there.

OK. And the alpha particles also cannot enter this region, this region. So this region that they cannot enter because their energy is lower than the potential that the alpha particles face. And so if they're coming from this side, then if they're coming from this side, they cannot enter into the nucleus. And if they are inside the nucleus, they cannot come out of the nucleus.

And the energy is shown by the dotted line. So this is classically, you know, an alpha particle would not have been able to come out. But then we know that it's inside the nucleus. It's like a wave. It's like a, you know, a propagating wave.

Now, when it enters into the classically forbidden region, it kind of drops and then it goes on again as a propagating wave, you know, outside the barrier when it acts like a free particle, nearly a free particle, because it's the energy is larger than V. So V is given by this dark line, which is the nuclear potential that exists, okay. So this is the tunnel theory, and we have done problems on tunnel theory. We, of course, haven't kind of taken this equation, but this is like an approximation of the Lennard Jones potential, and the form here is a little difficult to handle, whereas we have, you know, We talked about, say, for example, we have talked about this kind of a barrier and we have talked about barrier of this kind and so on.

Tunnel Theory of Alpha-decay



OK, so these are called step. These are called barriers. But it's something similar to that. So now what's beta decay? So one is that the electron energy is observed in beta decay, you know, of a particular nuclei to found to vary continuously from zero to a maximum value of maximum kinetic energy value.

So K max, which is a property of the nuclei. Okay. And so let's not worry about this thing. So I haven't included this thing here. But the maximum energy of these beta decay is, you know, E max is equal to the rest mass energy and this maximum kinetic energy.

So that's carried out by the decay electron is equal to the energy equivalent of the mass difference between the parent and the daughter nuclei. Only seldom, however, there is an emitted electron is found with an energy which is equal to Ke max. So this actually is taken from Beiser. You can read this book. There's a very nice book on modern physics.

And so when the directions of the emitted electrons and of the recoiling nuclei are observed, they are almost never exactly opposite as required for the linear momentum to be conserved. And then the spins of the neutron and proton and electron are all spin half. So if beta decay involves just a neutron becoming a proton and an electron, the angular momentum is not conserved. So beta decay poses a big problem in terms of the conservation of charge, linear momentum, and angular momentum.

Beta decay

1 The electron energies observed in the beta decay of a particular nuclide are found to vary *continuously* from 0 to a maximum value KE_{max} characteristic of the nuclide. ~~Figure 12.10 shows the energy spectrum of the electrons emitted in the beta decay of $^{210}_{83}Bi$; here~~ $KE_{max} = 1.17$ MeV. The maximum energy

Beiser

$$E_{max} = mc^2 + KE_{max}$$

carried off by the decay electron is equal to the energy equivalent of the mass difference between the parent and daughter nuclei. Only seldom, however, is an emitted electron found with an energy of KE_{max}.

2 When the directions of the emitted electrons and of the recoiling nuclei are observed, they are almost never exactly opposite as required for linear momentum to be conserved.

3 The spins of the neutron, proton, and electron are all $\frac{1}{2}$. If beta decay involves just a neutron becoming a proton and an electron, spin (and hence angular momentum) is not conserved.

Beta decay violates charge, linear and angular momenta conservation.

And there was this attempt by Pauli to remedy this, in which he said that if an uncharged particle of small or zero rest mass and spin half is emitted in beta decay together with an electron, That's what happens. Then the above discrepancies would not occur. So he said that, you know, there's another extra particle that's emitted, which is later called as a neutrino. And once you take into account the neutrino, there is, of course, anti neutrino also that should be found.

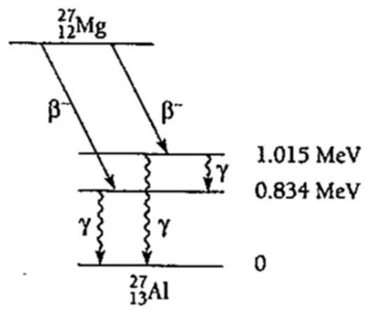
Fixing the conservation maladies:

In 1930 Pauli proposed a "desperate remedy": if an uncharged particle of small or zero rest mass and spin $\frac{1}{2}$ is emitted in beta decay together with the electron, the above discrepancies would not occur. This particle, later called the **neutrino** ("little neutral one") by Fermi, would carry off an energy equal to the difference between KE_{max} and the actual KE of the electron (the recoiling nucleus carries away negligible KE). The neutrino's linear momentum also exactly balances those of the electron and the recoiling daughter nucleus.

Subsequently it was found that *two* kinds of neutrinos are involved in beta decay, the neutrino itself (symbol ν) and the **antineutrino** (symbol $\bar{\nu}$).

Beta decay $$n \rightarrow p + e^- + \bar{\nu}$$

but never found. And the beta decay is actually given by this equation where the neutron decays into a proton and an electron and an antineutrino. So, these antineutrino is, you know, shown by these new bar and these will sort of these things that we have talked about that it violates charge linear momentum and angular momentum, they are now valid or rather they would be satisfied. Finally, gamma decay, so a nucleus is in its excited state, can return to its ground state by emitting gamma rays, and these gamma rays are, we have seen them, they have high penetration power, so they have energies which are the difference between the, so there are two levels, and if a nuclei actually, you know, makes a transition from the first excited state, let us write it by 1, and by zero, so they would emit a gamma, which is having an energy, which is the E1 minus E0. So the gamma rays have these energies of the order of MeV. And as I said earlier, that they have high penetration power. Finally, I mean, some diagram showing from this magnesium, 1227 magnesium that decays into aluminium 1327 by these beta decay. And finally, it comes from these from these level that you see here to the ground state, which is the lower state, which is 1327 AL.



Successive beta and gamma emissions in the decay of $^{27}_{12}$Mg to $^{27}_{13}$Al via $^{27}_{13}$Al'

and by a gamma decay and so on or it can also do a beta decay to a lower energy and the difference between these energies is approximately of the order of 0.2 MeV. So there are these successive beta and gamma emissions shown from these 1227 magnesium to 1327 Al via this 1327 Al. So we'll stop here with this radioactivity. It is mostly a description of these phenomena. You may want to read more from various books.

One of them, as I said, is Beiser. Then there's another book called Kaplan. These are nice books on these nuclear physics and one can look at them. We will do another topic which is the nuclear models and we will close this discussion on this nuclear physics. So we stop here.