ELEMENTS OF MODERN PHYSICS

Prof. Saurabh Basu
Department of Physics
IIT Guwahati

Week-01

Lec 1: Reviews of Classical Physics- Newton's Laws of Motion

As the name of the course suggests, it's the elements of modern physics. So then the topics that we are going to cover throughout the course, let me give you an overview of that. We'll talk about reviews of classical physics, Lagrangian formalism. We will do special theory of relativity where we talk about length contraction and time dilation. We will talk about the Lorentz transformation equations, mass energy equivalence.

Then we will talk about this old quantum theory, the structure of an atom, Bohr quantization, angular momentum, rather force scattering correspondence principle, the spectral lines seen in hydrogen atom. These are Balmer series. Sorry, there is a... Spelling mistake, it is a Balmer series, Lyman series, Bracket and Pfund series. We will talk about the Stern-Gerlach experiment, basically the space quantization.

We will talk about Planck's radiation law, wave-particle duality, de Broglie relation, photoelectric and Compton effects, the probabilistic interpretation in quantum mechanics, uncertainty principle, phase and group velocities, Then we will talk about the postulates of quantum mechanics, the superposition principle and the Dirac's notation and a bit of linear algebra solution of the Schrodinger equation in simple 1D problems such as particle in a box, barrier transmission problems, harmonic oscillator and bound states. We will also talk about Schrodinger equation in three dimensions and that would include hydrogen atom, the degeneracies in hydrogen atom, elementary ideas of perturbation theory, the splitting of the energy levels, that is how the degeneracy is lifted in presence of a perturbation, Stark effect and Zeeman effect. We'll also talk about time-dependent perturbation theory, Einstein's AB coefficients, stimulated emission and absorption, electron spin, spin-orbit coupling, the angular momentum, the total angular momentum, LS coupling, JJ coupling, the Clebsch-Gordon coefficients. Then we will also have basics of electromagnetism, electric and magnetic fields, poynting vector, Maxwell's equation and propagation of electromagnetic waves in vacuum and medium.

Then we will take on statistical description of matter, that is elementary idea about ensembles, the micro canonical, canonical and grand canonical ensembles, Liouville's theorem and Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distributions, these are known as the statistics, different classical and quantum statistics. Application of Bose-Einstein and Fermi-Dirac statistics, Bose-Einstein condensation properties of free Fermi gas, and as an example, we shall see Pauli paramagnetism. We will also talk about solid state physics in which we start with lattice vibration. We talk about specific heat of solids.

So, mainly, you know, we divide into two subtopics in solid state physics. One is properties of the lattice and the other is the properties or rather the behavior of the charges of the electrons there. We will talk about semiconductors P and N type semiconductors, elementary ideas of magnetism and superconductivity. We shall also do elementary nuclear physics. the binding energy, shell model, liquid drop model, nuclear forces, radioactivity, the half-life that plays an important role in knowing the age of very old objects, the alpha, beta and the gamma, radiation, fission and fusion.

And at the end, we shall have a very short introduction to elementary particles, Leptons and quarks and elements of the standard model. So, as you see the syllabus is very vast and it is intended to be because it should give you a horizontal view of a large number of topics. So, we start with reviews of classical physics. Classical mechanics is a mathematical formalism that studies the motion of bodies or the displacement of bodies under the action of different forces. Galileo started the modern era of mechanics by using the mathematical tools for describing the motion of bodies.

And his book called The Mechanics, published in 1623, that introduced the concepts of force and described it being proportional to the constant accelerated motion of objects near the surface of the earth. So that's proportional to acceleration of bodies, which is denoted as the change in velocity with respect to time. 60 years later, Newton formulated his laws of motion, which he published in 1687 under the title Philosophy Naturalis Principia Mathematica. In English, it's called Mathematical Principles of Natural Philosophy. And in the third book, which is called De Munde Systemat, or On the System of the World, Newton solved one of the greatest scientific problems of his time by applying the universal law of gravitation to determine the motion of planets.

And this was really a big challenge in those days to know about the planetary motion and this revolution of the Earth and the related topics that came along with in terms of formation of different seasons, etc. and so on. So, Newton established a mathematical approach to analyze the physical phenomena. in which he stated that it was really unnecessary to introduce hypothesis without any experimental basis.

He strongly believed that each of the physical theory or physical laws that we should formulate should have an experimental basis or an observation supported by observation. So the physical models are framed from experimental observations. And this led to the emergence of principles of Newtonian mechanics, which we are familiar with today. Very vast of it is taught in the class 12th as well and then in undergraduates and in postgraduates as well. Euler performed a systematic study of three-dimensional motion of rigid bodies leading to a dynamical set of dynamical equations known as the Euler's equation of motion.

Alongside, the concept of energy emerged, resulting in the discovery of principle of conservation of energy, which finds several applications everywhere, including in mechanics and thermodynamics. Conservation principles, not only of energy, but to other quantities, physical quantities as well, they became central to the study of mechanics, such as conservation of momentum, conservation of angular momentum, along with, of course, conservation of energy. And in fact, one studies these scattering problems or where the momentum is of course conserved, but whether energy is conserved or not, that will depend upon whether we are talking about elastic scattering or inelastic scattering. So during this formative period, the mathematical tools of Newtonian mechanics were applied to non-rigid system as well. So when we say rigid bodies or rigid systems, that means the distance between two particles or two objects rather constituting the body, they remain same.

Now this has been extended to non-rigid systems and development of continuum mechanics emerged. The theories of fluid mechanics, propagation of waves, electromagnetism emerged leading to the development of wave theory of light. Initially, there were perplexing aspects of the wave theory of light. For example, light propagates through a medium. This called as the ether. And then a series of experiments were performed to sort of demonstrate the presence of ether. And it was found to be negative. And the most important of those experiments was performed by Michelson and Morley in 1887, which completely ruled out the hypothesis of ether. In fact, now we know that the majority of the space that light travels from the sun to the earth is devoid of any atmosphere, is a total vacuum.

So the basic concepts of absolute time and absolute space, which Newton had defined in Principia, they were found to be incorrect to explain a host of experimental observations. And when they were found to be incorrect, when the velocity of particles become too large, and this led Einstein to propose the fundamental rethinking of concept of space and time, where he showed that they being independent, they are actually coupled to each other. So, X and T, the position and time are not independent variables, they are coupled to each other, they are linked to each other and he wrote them down in his special theory of relativity. which along with this coupling of space and time had said that the speed of light is the ultimate speed that one can achieve. So any information that has to be sent or has to be propagated, this will take a speed of light and nothing can move faster than speed of light.

So, STR or the special theory of relativity proposed in 1905 provides the necessary framework for describing the motion of rapidly moving objects whose speeds are comparable to. So, this is comparable to c or the speed of light so that v is it is always less than c as I said. but it cannot be more than c. So, this greater than is not, it is actually meant to be greater than, you know, say 0.1 C or something. So, it is like 0.1 c, if you at all think of greater than. So, then the special theory of relativity comes into the picture and it is very important for them to be taken into account.

So another limitation of Newtonian mechanics, they appear to occur at the microscopic length scale. So for atoms and ions and elementary particles, these Newton's laws do not really appropriately describe the equation of motion. So, a new field, statistical mechanics was developed relating the microscopic properties of individual atoms and molecules to explain the macroscopic or the bulk thermodynamic properties. This was one aspect, but there is another aspect which was even more important, that is, in the middle of the 19th century, new observations were At very small length scales, that is microscopic length scales, revealed anomalies in the predicted behavior of gases and it became increasingly clear

that the classical mechanics or the Newtonian mechanics that we are familiar with did not adequately explain a wide range of phenomena at the atomic and the subatomic length scales. And an essential realization emerged towards the beginning of 20th century when quantum mechanics was developed and classical mechanics was found to be inadequate to qualitatively describe most of the microscopic phenomena that we come across. So, by the early part of 20th century, quantum mechanics provided a mathematical description of the microscopic phenomena. This was in agreement with our empirical knowledge of all the non relativistic physics, which are found to be valid for velocities, which are much, much lesser than the speed of light, which is denoted by c. On a parallel ground, in the 20th century, this experimental observation led to more detailed knowledge of the large-scale properties of the universe and Newton's law of gravitation, they were found no longer to be accurately modeling the observed universe and needed to be replaced by general theory of relativity.

By the end of 20th century and the beginning of 21st century, many new observations, for example, the accelerated expansion of the universe, etc., required introduction of new concepts such as dark energy that may lead once again to fundamental rethinking of the basic concepts in order to explain several observed phenomena. So let us take a very quick recap of Newtonian physics and in terms of solving some problems. And we are familiar that this mechanics is broadly based on two formalisms. One is deals with kinematics. where we describe the motion numerically or we sort of under the application of some force, which is never talked about. But in terms of these acceleration, distance, time, velocity, initial and final velocities, we describe these kinematics of bodies. And we also talk about dynamics, how force affects motion. This is precisely the Newton's laws that we are familiar with. So, the kinematic equations are easy and they are easy to apply and they relate various quantities such as velocity, distance, time, etc. and acceleration.

So, v equal to u plus a t, s equal to u t plus half a t square, v square equal to u square plus 2s and so on. And the dynamics is obtained via the Newton's law of motion. So just for the sake of completeness, let me state the equations of motion. First law states that an object at rest remains at rest and an object in motion remains in motion at constant speed. in a straight line unless acted on by an unbalanced force. Now, this is very important to note that there is something called an unbalanced force, which would cause it to accelerate, which is given by the second law. So, the acceleration of an object depends on the mass of the object and the amount of force that is applied. Third law says that whenever an object exerts a force on another object, the second object, that is the other object, also exerts an equal and opposite force on the first object.

It is to be kept in mind that it is always a pair of bodies that are involved. You cannot lift yourself by pulling up your bootstraps. Or you cannot accelerate a boat by putting a fan pointing towards its mast. So it has to take place this action and reaction that they have to take place between a pair of bodies. So there are many things about Newton's laws and we will not be able to cover all of them. But some of the things that are of importance and you should definitely have a look at it are these free body diagram, the concept of friction, how it takes place and which direction it is and what it depends on, etc., Then we talk about momentum, mainly the linear momentum we are talking about here.

We talk about collision problems, scattering problems, and then do a center of mass analysis. One does a variable mass problem as well. That is, the mass is not constant in Newton's second law. Ordinarily, the way we write it is that F is equal to m a, and m is taken as a constant, but it may not be. Suppose in a leaky bucket or a train moving with its, you know, the bogies are containing, say, sand, and it's somehow leaking out. So, there are various examples of that. And we also deal with work and energy. motion in other coordinate systems that is in polar, spherical polar, cylindrical coordinate system. This would aid us in learning circular motion, angular momentum, rigid body dynamics, inertial and non-inertial frames, etc.

Applications of Newton’s Laws

1. Free body diagram
2. Friction
3. Momentum, Collision problems, Center of mass analysis
4. Variable mass problem
5. Work and energy
6. Motion in polar coordinate systems
7. Circular motion, Angular momentum
8. Rigid body dynamics
9. Inertial and non-inertial frames
10. Fictitious forces etc.

And we also talk about fictitious forces and one actually understands that in non-inertial frames, how these fictitious forces allows us to write down Newton's laws just in the way as we do it in inertial frames. Okay, so, we will not be able to cover all of these things, but you must be having ideas how to, you know, sort of, I mean, deal with some of these problems. And there is a general framework, let us talk about that, where, how we attack these problems. So, the general framework would say that, we have problem to solve which is m x double dot equal to f of x and this is nothing but Newton's second law. Okay, so we really talk about f of x which is obtained from a potential function V of x and we are talking about the conservative force here. A conservative force means that a force that can be derived from the negative gradient of a potential. This is really the gradient but written in one dimension. That is why it is written as dV dx. But if V is a function of x, y and z, then we have to write it as a gradient of V. So this is the concept of conservative force.

And so what you do is that if you take this as equation 1 and if you multiply equation 1, by x dot and integrate then what we have is we have a M by 2 x double dot square plus V of x equal to E which leads to the conservation of energy. And if you need to solve for this, we get this equation and this equation can be solved for where you write this as 2 over m E minus v of x. So that gives you, if you can solve this for a given value of v, then you will be able to get x as a function of t and that is what you want. One wants x as a function of t. So you know that at each time what is the position of an object under the influence of a potential V of x and you can tabulate these values that is at t equal to t1, the position is x1, t equal to t2, the position is x2, t3, x3 and so on. So then you can make a table and can really write down these quantities of these variables. So here, of course, as you see that t is an independent variable and or x is a dependent variable. So you find x at a given time. And this is how if you can integrate this equation, then, of course, you know x as a function of t.

General framework.

$m\ddot{x} = f(x)$ → Newton's second law. (1).

$f(x) = -\dfrac{dV(x)}{dx}$ → Conservative force.

Multiply Eq. (1) by $\dot{x}$ and integrate.

$$\frac{m}{2}\dot{x}^2 + V(x) = E.$$

$$\dot{x} = \pm\sqrt{\frac{2}{m}\left(E - V(x)\right)}.$$

One wants $x(t)$.

So let me tell you a particular problem and we'll solve the equation of motion. And so let's take an example of say charged particle in a magnetic field. It's a very well-known problem. And so we have, say, for example, an electron, which is placed in a magnetic field B. And this is a classical problem. So we'll just talk about, you know, the Lorentz force that comes into picture. And so it's M and dv dt. And this is equal to q into v cross B. So, of course, this is a moving charged particle, charged particle with a velocity V and it enters into a region where there is a magnetic field B. So, the force that this particle or the charged particle experiences is given by q into v cross B and this is called as a Lorentz force. So one may want to know what is the trajectory of the particle in presence of such a force. And it is easy to understand or most of you already know about it that the particle actually executes a circular motion.

When you have a v which is perpendicular to B, so v is perpendicular to B, it is a qvB and this would be, you know, sort of compensated by the m v square over r which is the centripetal force and so this will, the magnetic field will sustain such a circular motion. And but we just want to, you know, I mean, the circular motion has a characteristic frequency, which is called as the cyclotron frequency, which is given by omega equal to qB by m. So this is qB by m, and so on. And this radius of this motion is given by mv over qB. Okay. This is special case when v is perpendicular to B. So now we'll try to arrive at a full solution as a simplification. Let us assume that B is fully in the z direction. That is, there is no component along x and y and it's in the z direction. And v, of course, has all the three components vx, vy and vz.

okay and so we have this v cross B that comes out as v y B v x B and 0 so that tells you that we can write down the equation of motion which is F is equal to m dv dt so it will be m dv x dt that is equal to q v y sorry v y into B And so this is, this gives you dvx by dt. So that's the velocity in the, or the change in velocity in the x direction, which means the acceleration in the x direction, which is qB over M and a vy. And we also have for the y direction, we have a dvy dt, which is equal to minus qvx into B. that gives you a d v Y d t that is equal to minus qB by m and a v x. You see, this is a coupled equation in which the vx involved or the variation of vx which is acceleration in the x direction that involves velocity in the y direction whereas acceleration in the y direction that involves velocity in the x direction. So let me just number these equations and we have this as 2 and maybe 3 and 4. So these 3 and 4 are the equation of motion.

Example

Charged particle with a velocity $\bar{v}$ in a magnetic field.

$$\vec{F} = m\frac{d\vec{v}}{dt} = q(\vec{v}\times\vec{B}) \qquad (1) \text{ Lorentz force.}$$

$$\left.\begin{array}{l} \omega = \dfrac{qB}{m}. \\ r = \dfrac{mv}{qB}. \end{array}\right\} \text{when } \vec{v} \text{ is } \perp\text{r to B.}$$

$$\vec{B} = (0, 0, B) \qquad \vec{v} = (v_x, v_y, v_z) \qquad (2).$$

$$\vec{v}\times\vec{B} = (v_y B, -v_x B, 0).$$

$$m\frac{dv_x}{dt} = q v_y B. \Rightarrow \frac{dv_x}{dt} = \frac{qB}{m} v_y. \qquad (3)$$

$$m\frac{dv_y}{dt} = -q v_x B \Rightarrow \frac{dv_y}{dt} = -\frac{qB}{m} v_x \qquad (4).$$

$$\omega_c = \frac{qB}{m} \quad : \text{cyclotron frequency}$$

We can make a simplification of writing this omega or now we call it omega c just to say that this is a cyclotron frequency, the c, sub c frequency. That indicates that it is called as a cyclotron frequency. So, equation 3 and 4 becomes a little simpler to write. So, we have these dvx dt is equal to omega c vy and dvy dt equal to a minus omega c v of x and we number them as 5 and 6. Okay, so these coupled equations they need to be solved and what you do is that differentiate 5 with respect to t, one gets d2 vx dt2 which is equal to omega c which is a constant because it is a constant magnetic field and omega c depends upon only that.

only B. So, this gives you now from 6 one can use this relation dv y dt equal to minus omega c vx and use 6 so that tells you that it is a omega c square and a vx. Now at the second order level it becomes decoupled so vx the double derivative of vx double derivative with respect to time, what I mean, is equal to minus omega c square vx, where this has a solution which is, let us call it equation 7, and 7 has a solution that is v of x, t is equal to v0 cosine omega ct, and v0 is some initial velocity at t equal to 0. So, maybe this is 8 and so on and we can solve for vy in the same fashion that is you differentiate 6 and put 5 there and one gets a vyt is equal to a minus v0. sin of omega c t. You can do this and convince yourself that one has a vx and vy and now what we do is that we want x and y both as functions of t. So, x if you integrate 8 and 9,

one gets x of t equal to some initial x0 plus v0 by omega c sine of omega ct and y of t is equal to y0 plus v0 by omega c cosine of omega ct. These are say 10 and 11 and so we have now gotten the solutions of this particle in the presence of the magnetic field and here of course there are unknown quantities which are x0 and v0 which we have to determine. So x0 initial position at t equal to 0. And this has to be, you know, figured out what it is or what they are rather. So, now if you look at equations 10 and 11 and, you know, square them up, they will have a sine square and a cosine square with same coefficients, which will give rise to 1. So, it will give rise to an equation of a circle where X square plus Y square gives you an equation of a circle. Okay.

$$\frac{dv_x}{dt} = \omega_c v_y \qquad (5)$$

$$\frac{dv_y}{dt} = -\omega_c v_x \qquad (6)$$

Differentiate (5) wrt t, and use (6)

$$\frac{d^2 v_x}{dt^2} = \omega_c \frac{dv_y}{dt} = -\omega_c^2 v_x \qquad (7).$$

v_0 : initial velocity at $t=0$. (8).

$$v_x(t) = v_0 \cos(\omega_c t) \qquad (9)$$

$$v_y(t) = -v_0 \sin(\omega_c t) \qquad (10)$$

Integrate (8) and (9)

$$x(t) = x_0 + \frac{v_0}{\omega_c} \sin(\omega_c t). \qquad (11)$$

$$y(t) = y_0 + \frac{v_0}{\omega_c} \cos(\omega_c t).$$

x_0 : initial position at $t=0$

So, that is what is important here to know and so this circle has a radius. So, we write that the EOM that is equation of motion is a circle of radius r c this is equal to v0 over omega c. putting the value of omega c which is mv0 over qB and we also have there is the centre of the circle is located at x0, y0. Okay. So, we know now the where the center of the circle is located and then we also know that what's the radius of the circle. Okay. Now, there is also a third equation of motion where we have this as m dvz dt equal to 0 because if you look at this form of the v cross B, that is equal to 0. So, m dvz dt equal to 0.

So, that is the equation that we should not neglect because that will give you the real trajectory or rather the complete trajectory of the particle. So, this is, let us call it as equation, these two included as equation 12 and this as equation 13. And this is useful because it can have the particle can have an initial velocity in the z direction. And let us

EOM is a circle of radius $r_c = \frac{v_0}{\omega_c} = \frac{m v_0}{qB}$ } (12)

The center of the circle at (x_0, y_0).

(13).

$$m \frac{dv_z}{dt} = 0$$

Let the particle has initial velocity along z is v_{0z}. (parallel to the direction of B).

(14)

$$z = v_{0z} t$$

(10), (11) & (14)

x - y plane — circular motion

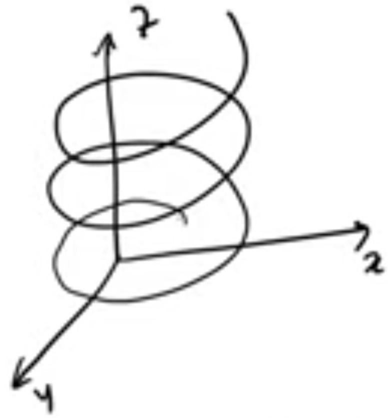
z direction → free motion

call this that the initial velocity in the z direction as. along z is V 0 z which is parallel to the direction of B. because B is purely in the z direction, okay.

So, this gives you Z is equal to V0 z into t, where it, there is a uniform motion in the z direction. So, what we get is that, this from 10, 11 and 14, we get that in xy plane, there is circular motion and So, this is from 10, 11 and 14 and z direction, it is a free motion. So, this helps us to construct, you know, the total trajectory to be like this. So, let us just draw this x, y and z axis. So, we call this as y, call this as x and call this as z. So, we have this trajectory to be like this and so on, Okay. So, and this is like a helix. just like DNA that you have seen, a picture of DNA. So this actually moves ahead in the Z direction and executes circular motion in the xy plane.

So if you take a cross section, it's a circle, but otherwise it's moving, that is a moving circle in the x direction. So the time period of this motion is given by T equal to 2 pi by omega c and one can also calculate there is the pitch of the helix. So, the pitch of the helix is given by let us call it as pitch to be n. So, n is equal to this v Z into T. So, this v Z divided by 2 pi over omega c equal to v z into 2 pi, so that is 2 pi m over q B, okay. So, that is the pitch of the helix and so this is exactly integrating out the equation of motion and that is what we did here, okay. Let me do another problem. And so this will give us an idea of this free body diagram.

And as a representative example, let us do a system of pulleys, which is called as the Atwoods machine. So this goes to explain the problem that we just did goes to explain how we integrate the equation of motion and find the trajectory of the particle, okay. So, we have been, you know, fortunate enough to integrate this equation of motion because there was no potential other than, you know, we just integrated the force twice and calculated, you know, the trajectory. And let us do application of free body diagram. So we take a system of pulleys and in particular this called as a Atwoods machine. Let me try to draw it neatly. In fact, some of these problems the drawing is important.



$\rightarrow$ Helix.

Time period : $T = \dfrac{2\pi}{\omega_c}$.

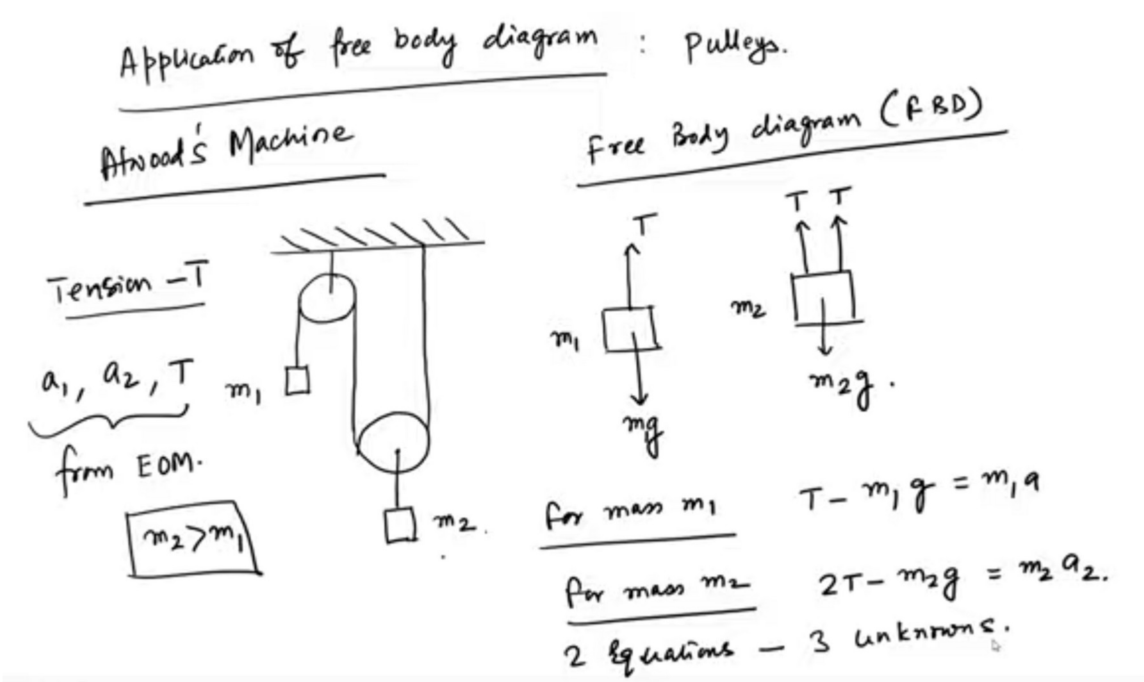
pitch of the helix (n)

$$n = v_z T = v_z \frac{2\pi}{\omega_c} = v_z \frac{2\pi m}{qB}.$$

So this is a pulley here. And these are all massless pulleys and they have frictionless these ropes. And this one goes like this and there is another pulley that is there and there is a mass that's hanging from this pulley and this pulley is connected here. Okay, so this is called as the Atwood's machine. Let me sort of draw these masses. This is mass m1 and this is mass m2 and each one of the tension is T. So, it is basically the same string. So, it has to have a tension T. Okay. And what we do is that in order to analyze this problem, we have to know what are the accelerations of masses m1 and m2 and what are their relations.

And the unknowns are, of course, a1, a2, that is acceleration of mass 1, acceleration of mass 2 and the tension, which we have to find from the equation of motion. These are simple problems. I mean, as I said, there is the same string. So the tension is equal everywhere. So let me write down the free body diagram so that we know that we can write down these. You know the forces that are acting on the body. So, what we do is that we separate out the body and draw the forces that are acting on the body. So, this is m1, okay. So, this is m1. So, there is a tension that is acting upward and there is a weight that is acting downward. So, this is the free body diagram. In short, we call it FBD.

For mass 2, you have two such tensions acting which are same T and T and the only force that is acting downward. So, I should write it as m1 g and this is m2 g, Okay. So, these are the free body diagrams for each one of those. So, for mass m1 let me write the Newton's second law So, we have T minus m1 g equal to m1 a, okay. So, what happens is that this mass is going upward and this is coming downward and you can actually like here implicitly m2 is greater than m1 which can be achieve otherwise you just say m1 g minus T equal to m1 a. And for mass m2, one has 2T minus m2 g. So, this is equal to m2 a2. So, you know, this is assumed that I made a mistake here. So, T minus m1 g.



So, T is upward and m1 g is downward. So, it is fine. And we are assuming that we are writing down the equation of motion exactly in the same way. So, we are not assuming really which direction they are moving at this moment. But we will do that. So, we have two equations here and there are three unknowns and that is uncomfortable to solve, but we of course, have another condition which is very important. So, we now say that if m2 goes up by d. In fact, this is important, you have to realize that both actually go up in this particular situation. If m2 goes up by distance d, then the string length to conserve the string length,

to conserve the string length, we m1 goes down by 2d. So, let me explain this here. So, we assume that this is going up by a distance d. Now, this has to come down by a distance 2d and so this is the so thus you know for a general case that is you do not need to talk about d or anything. So, for a general case, basically the displacements of the two are related by y1 equal to minus 2 y2 where y1 is the displacement of one body and y2 is the displacement of the other mass that is y1 is of m1 and y2 is of m2. So, either you take m1 going down and m2 going up or you take m1 going up and m2 going down, but the relationships between the displacements will be that m1 will have a displacement which is twice that of m2 and in the opposite direction.

If m_2 goes up by 'd', to conserve the string length.
m_1 goes down by '2d'

For a general case, $y_1 = -2y_2$

$$a_1 = -2a_2. \qquad (3).$$

$$\left.\begin{aligned} a_1 &= g\,\frac{2m_2 - 4m_1}{4m_1 + m_2} \\ a_2 &= g\,\frac{2m_1 - m_2}{4m_1 + m_2} \\ T &= \frac{3m_1 m_2 g}{4m_1 + m_2} \end{aligned}\right\} \text{Solutions}$$

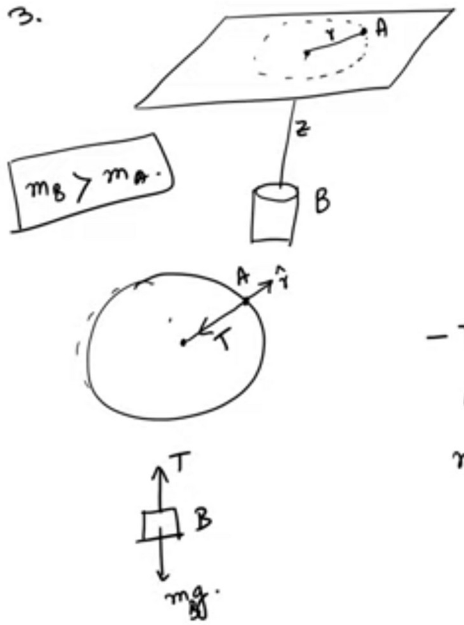
That tells you the acceleration of the two objects is in the opposite direction. So now this is another equation that you have here in addition to these 1, 2 and 3. So, this is how the accelerations are related and now one can easily solve for these equations to get g and 2m2 minus 4m1 divided by 4m1 plus m2 and a2 is equal to g 2m1 minus m2 divided by 4m1 plus m2. and T is equal to 3 m1 m2 g divided by 4 m1 plus m2 and these are the solutions of the problem, okay. So, that is the solution for the Atwood machine and then when there are two such pulleys which are, you know, coupled like this, one can solve them there. Let me do another problem. Let me show this here. So, this is problem number 3. So, let me, so this is 2 and 1. So, 3 and we have a table like this, okay.

And we have a sort of a rope which you know passes through a hole there and it is connected to a mass like this and this there is a small mass here. and that goes in a circular orbit. So, there is a plane, so this is B and this mass is called A and this is a flat table, it is a part of the table that you see, there is a small hole there and there is a string that passes through the hole. And, this mass actually is attached there and, you can assume that m B is greater than m A and so on. And, so this,, string is massless and it is, it is executing a circular motion there, Okay. So, we want to draw the free body diagram for, for both these things and, for A.

Okay, so let me draw a full circle there. So this is A and so this is the r cap direction. Now we are solving it in cylindrical coordinates or rather plane polar coordinates. And so there is this mass which is here would undergo which is executing a circular motion. So there is a tension that is there. So this is for the mass A. This is for A. And so there is a tension that is there. And this tension is supported by the acceleration or the radial acceleration, because that's why it's executing a force that is or rather the motion that is along the circular plane. Okay. And what about B? B is simple. You have, so this is B, and so there is a T there, and then there is a mg there, Okay? So, which is, let us call it as a mB g, Okay? So, how do we write the velocity in plane polar coordinates? So, we write it as r dot r cap plus r theta dot theta cap so r is equal to r r cap. Acceleration is written as r double dot minus r theta dot square that's r cap plus.

This r theta double dot plus 2 r dot theta dot theta cap. So, that is the component of position velocity and acceleration in the plane polar coordinates. So, this is the radial acceleration which now supports these coordinates. So, there is a tension which is minus T because you measure always along the outward drawn normal and this is equal to mA and r double dot minus r theta dot square. So, that is the radial part and there is nothing for the tangential part. So, this is mA and r theta double dot plus 2 r theta dot square. r dot theta dot is equal to 0. So, this is equation 1 and this is equation 2 and m B z double dot is nothing but equal to m B z minus T and because we have committed ourselves to m B greater than m A. So, this is m B z double dot equal to m B sorry this is not z this is g the acceleration due to gravity. So these are the three equations that you have to solve but you know this is say the R radius of this and this is the hanging part that is why we have written m B Z double dot and so R plus z that remains constant that is equal to L.

3.



$$\vec{r} = r\hat{r}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{radial}}\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}.$$

$$-T = m_A(\ddot{r} - r\dot{\theta}^2) \quad (1)$$

$$0 = m_A(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \quad (2)$$

$$m_B\ddot{z} = m_B g - T \quad (3).$$

$$r + z = l. \qquad l: \text{Constant}. \quad (4)$$

$$\ddot{r} = -\ddot{z}$$

That tells you that there is a relationship between R double dot and Z double dot which are related by this because L is equal to constant. So R double dot equal to minus Z double dot. So we get another equation which should aid us in solving this. So the negative sign, of course, tells you that if A moves inward, B moves down. So one can solve for the Z double dot, which is the acceleration of the mass B. So, this is equal to mA r theta dot square divided by mA plus mB. So, at t equal to 0, so immediately after B is released, r is equal to r0. and theta dot is equal to omega 0. So, this was kept in a static condition and then you let this mass go through.

So, what will happen is that this mass will come down and the mass B will come down and mass A will have a shrinking orbit. So, this is what it means. So, that tells you that Z double dot at t equal to 0 is equal to mB g minus mA r0 omega naught square. So, that is the initial thing and mB plus, sorry, mA plus mB, yes, Okay. So, now, it tells you that Z double dot is greater than 0, so B comes down. If it is this, then of course, B goes up. I mean, with mB greater than mA, that cannot happen. And if Z double dot equal to 0, so it is at t equal to 0. So, that can happen, of course, if you say that Z double dot is 0, that is the acceleration, initial acceleration is negative. Then, of course, the B goes up and if it is equal to 0, then there is of course, there is no acceleration. So, this also the stationary basically. So, this depends upon the initial conditions of the mass B, the acceleration how it is you know, it is released etc.

$$\ddot{z} = \frac{m_B g - m_A r \dot{\theta}^2}{m_A + m_B}.$$

At $t=0$, immediately after B is released,

$$r = r_0, \quad \dot{\theta} = \omega_0.$$

$$\ddot{z}(0) = \frac{m_B g - m_A r_0 \omega_0^2}{m_A + m_B}.$$

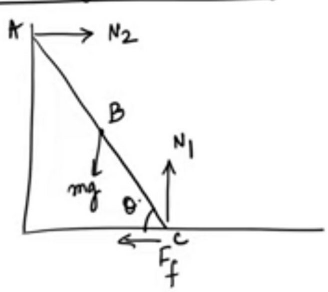
$$\boxed{\begin{array}{ll} \ddot{z}(0) > 0, & \text{B comes down} \\ \ddot{z}(0) < 0 & \text{B goes up.} \\ \ddot{z}(0) = 0 & \text{stationary.} \end{array}}$$

that will depend upon this and also the of course, the relative magnitude of these all these things okay. So, in fact, this I am only doing a very small, you know, subset of problems to give you ideas of what you should do and how you should, you know, go ahead and solve various problems. And let me do one last problem for now. And is basically this will again a different problem and where you have a basic talk about a leaning ladder. And we have a wall in which a ladder is kept like this, Okay, and it is making an angle theta. The ladder can fall down, the ladder may not fall down depending on the friction that is there. So this is that friction that you know kind of takes place. There is a weight or rather the normal reaction that will happen and there is also a normal reaction from this surface which is N2

The ladder has length L. And if it's a uniform ladder, then the weight acts at, you know, this from this point. And this is, say, for example, F, F, which is a force of friction. So F, this is force of friction. And N1 and N2 are normal reactions. So, what are the things that we have? We have if it has to be in equilibrium, there should be no vertical force, this has to be 0, so this is 1, some of all the vertical forces should vanish. Sum of the horizontal forces should also vanish if it is in equilibrium and there would be no torque and so, Tau is torque if it is stationary then the sum of the torque is 0. So, 1 gives N1 equal to mg. See, that is the only vertical things, vertical forces acting N1 upward and Mg downward. 2 gives N2 is equal to F f.

See N2 is acting on towards the right and FF that is the force of friction is acting towards left. And what does 3 give? That is the moment which is N2 into L sine theta. So, we are taking the moment about this point let us call it as B. say A and C are the two extremities. So, N2 into L sin theta because theta is the angle. So, N2 is this multiplied by the product of this L by 2 that is there. You are taking the moment about C. So, we take the moment about C. So, N2 into L sin theta equal to m g into L by 2 cos theta. So, that tells us that N2 is equal to m g by 2 tan theta equal to F of f. So, let us call this as equation 4. this is equation 5 and this as equation 6 and then of course, because F of F that is frictional force is equal to N2.

4. Leaning ladder



F_f : force of friction.

N_1, N_2 Normal reaction.

$\sum F_{vert} = 0$ (1)

$\sum F_{horizontal} = 0$ (2)

$\sum \tau$ (3) $\quad \tau$: torque.

(4)

(5)

(1) gives $N_1 = mg$

(2) $N_2 = F_f$

Moment about C (3) $N_2 (l \sin\theta) = mg \left(\frac{l}{2}\right) \cos\theta$. (6)

$$N_2 = \frac{mg}{2\tan\theta} = F_f$$

So, N2 is equal to mg by 2 sin theta. So, that tells us that if F. The frictional force F sub F is less than mu N1, which is equal to mu m g, then the condition that it will stay there is mg over 2 tan theta, this is less than equal to mu m g. So, a tan theta is equal to or rather it should be greater than equal to 1 over 2 mu. So, this is the condition for no slipping okay. So, if you align the ladder by making an angle which the tan theta of the angle or tan of the angle rather. tan of the angle is greater than 1 by 2 mu. One usually has an idea about the friction depending on the surface. And we sort of, you know, do an eye estimation that this should be the angle.

$$F_f \leq \mu N_1 = \mu mg.$$

$$\frac{mg}{2\tan\theta} \leq \mu mg$$

$$\tan\theta \geq \frac{1}{2\mu}.$$

This is the condition for <u>NO Slipping</u>.

If we keep it, then it won't slip. And this is what it's coming from there, that there are three equations, two coming from the force and one coming from the torque. And balancing them, it tells that this should be the condition for no slipping. So, we wanted in this particular problem that what is the condition for no slipping for this case of leaning ladder against a wall. We will stop here for today and carry on with more discussions of classical mechanics on the coming day. Thank you. Amen.