Topology and Condensed Matter Physics

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Lecture – 09

Quantum Hall effect

I have described the experiment, the Hall effect experiment in the lab and the way it is being carried out in a typical undergraduate lab. Let me now do the calculations of the Hall effect, the classical Hall effect as introduced by Edwin Hall in 1879.

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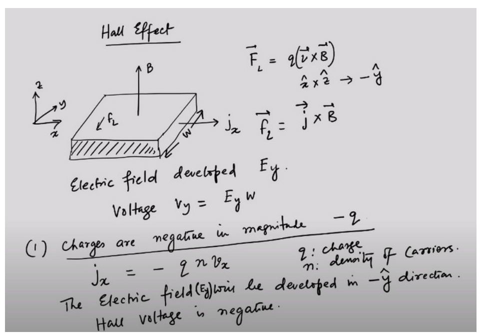

It is just classical Hall effect and it is just based on moving charges in a magnetic field. So, they face a Lorentz force and let me make this diagram a little clear so that you understand. So, this is a thin sample. I am showing a bit of thickness just for visual understanding. So, we will plot this, show the axis as X and this is the Y axis and of course the Z axis is this. It was shown slightly differently just that we show the Hall voltage being collected in an experiment. So, the Y axis is towards the direction that is shown that is a positive Y axis, positive X axis is towards the right and Z axis is upward and one applies a magnetic field here along the Z axis. So, this is the experiment and then one actually sends a current density in this direction because it is in this X direction and because the magnetic field is in the Z direction and there is a current flowing in the X direction.

So, the Lorentz force which is written as so let us call it as a FL just to make sure that it is low range. So, it is a Q and V cross B. Right now we are not worrying about the sign of Q which we will do that a while later. So, if V that is the electrons are moving along the X direction and the magnetic field is along the Z direction. So, this results in a force in the minus Y direction and the minus Y is towards this direction.

$$\vec{x} \times \vec{x} \rightarrow -\vec{y}$$

So, there is a Lorentz force acting in this direction and the charges will get accumulated in this surface of the sample. So, this is and let us say that this is a width that is W. So, the Lorentz force which we already wrote it is of the form which is more convenient to us it is a J cross B because V and or rather I should write it as a vector J not unit vector. So, it is a J cross B I mean so J is in the X direction we have said B is in the Z direction and so this lorentz force will act in the minus Y direction. So, is that shaded wall in which the charges build up and so there will be because of this charges being build up there will be electric field that will develop which we will call it as Ey.

$$\vec{F_L} = q(\vec{v} \times \vec{B})$$
$$\vec{F_L} = \vec{j} \times \vec{B}$$

Let us call that as Ey. So, this is in the of course in the Y direction and if W is the width that is shown there then that will result in a voltage because of this electric field let us call that as a Vy which will be Ey into W. So, that is a voltage and this voltage actually would the sign of the voltage rather would tell you about the nature or the properties of the charge carriers that are responsible here. Suppose the case 1 suppose the charges are negative in magnitude which are electrons. So, the charge is minus Q and in that case 1 has J x is equal to minus Q and n, n being the density of the carriers and because it is the charges are flowing along the X direction it is minus Q and Vx. So, Q is of course the magnitude of the charge and n is the density of carriers.

$$V_y = E_y W$$
$$j_x = -qnv_x$$

Alright so, we have when the charges are negative. So, this is say case 1 and then we have you know in that case the electric field that will be developed will be in the minus Y direction. And so, the Hall voltage because the electric field which is Ey is in the negative Y direction. So, Hall voltage is negative. So, this gives a negative Hall voltage will show what are the implications of this.

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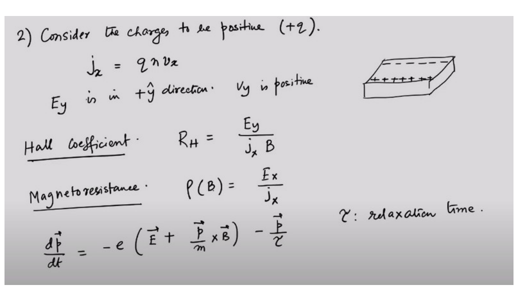

$$j_x = qnv_x$$

Now consider the charges to be positive which means they are plus Q. And if that happens then the J x that is there then that is equal to Q into n into Vx and so Vx is of course flowing in the direction in the positive X direction and let me show you that what happens to the charges in this particular case the first case that we have talked about. So, these are so, this is the edge and all the negative charges will you know. So, let me show it here all the negative charges will accumulate there, but then because of the charge neutrality of the sample this will the positive charges will get accumulated there.

Now what happens is that this cannot go on indefinitely because the Lorentz force will eventually be completely balanced by the force due to the electric field that develops in the Y direction. So, these motion of the charges will stop after all the free charges that are there they get accumulated. This is the direction of the Lorentz force and these are the accumulation of the charges. Similarly in this next case when you have these J x to be like this then the electric field will be positive is in the plus Y direction and if that is in the plus Y direction the Hall voltage will be also be positive. So, the Hall voltage is nothing but this Vy which we have discussed just in the last slide.

So, this Vy is positive and in that case what happens is the same picture if we go to and show them there in asimilar geometry. So, one has a positive charges that are accumulated there and the negative charges will accumulate there again for the charge neutrality of the sample. So, this is what happens when you have positive and negative charges in metals mostly one finds that there are electrons. So, the charge carriers are negatively charged and one get the Hall voltage to be negative. So, Vy to be negative.

So, now what are the quantities let us do a little calculation on the quantities that we actually measure and the quantities that we measure which has been told earlier is called as a Hall coefficient. And the Hall coefficient is defined as RH divided by Ey which we have defined then Jx we have defined that as well multiplied by the B and this B is the magnetic field that is a constant magnetic field that is applied in the Z direction perpendicular to the plane of the sample. And the charges are mainly concentrated in the 2 dimensional plane. So, before we talk about the Hall effect I mean let us talk about the magnetoresistance that is the resistance that the sample or the system develops because of the presence of a magnetic field. So, the magnetoresistance which is the longitude is also called a longitudinal resistance.

In fact for most of our discussion we will call this as a longitudinal resistance and this as a function of H it is Ex versus Jx, Ex is the electric field that is developed along the X direction and Jx is the corresponding current that is flowing in the sample. Now this of course is not a function of B let us write it as instead of writing it as H let us write it as B however it is not a function of B it is independent of B. So, it really does not matter that what B one applies and eventually when the equilibrium is established then the Lorentz force gets balanced by this transverse electric field and that is why there is no dependence on the magnitude of the magnetic field here. And so how do we write down the equation of motion for this case where there is a current flowing in the X direction and then the magnetic field in the Z direction. We have to write down the classical equation of motion which is nothing but it is there in electrodynamics excepting that now I am writing for the charge I am writing it for the electrons and this is E and it is a P it is actually a V cross B but V can be written as P over M, P is the momentum and then cross B.

Now this is not all and this is of course you would write it down just a charged particle which is moving in free vacuum but however in a material where you are doing the experiment there has to be another term which gives rise to resistivity. See free electrons are good description of Fermi systems. However they do not yield resistivity unless you consider the Drude picture that the electrons are otherwise free the lot of free electrons present the electrons actually drift in the sample freely unless they collide with each other and the collisions will give rise to the resistivity. So this origin of the resistivity was told by Drude and he gave a few hypothesis which sort of strengthened his ideas of how one can get a resistive behavior of the metal because everything is resistive finally. And the power dissipated in a circuit is given by I square R or you know V square by R so there is a resistance there and the resistance in Drude's picture that comes from the collisions either they are frequent collisions or they are infrequent collisions depending on the size of the sample that you are talking about but there are collisions and these collisions would instantaneously change the velocity of the particle after the collision and it goes in a completely random direction which tells you the average velocity of the electrons after collision is equal to 0 and that the time that elapses between two such collision is known as the relaxation time.

$$R_H = \frac{E_y}{j_x B}$$

$$p(B) = \frac{E_x}{j_x}$$

So if you read Drude's theory of metals you will get all these description so till the collision occurs the electron drifts as free particle. The collision occurs the velocity of the electrons so you talk just concentrating on one electron the electron goes in a direction which is completely random and cannot be predicted and Drude also had brought up this notion of temperature and he said that the hotter regions of the sample would have more energetic electrons emerging out after collisions. So that information is also there because you have learnt in your school level that the resistance actually increases as you increase the temperature either linearly or otherwise I mean mostly we have studied linear behavior with temperature. So this is not enough we have to have a term which is like minus p over tau where of course p is the momentum and tau is called as a relaxation time. So what is relaxation time? The relaxation time is the time between two successive collisions.

$$\frac{d\vec{p}}{dt} = -e(\vec{E} + \frac{\vec{p}}{m} \times \vec{B}) - \frac{p}{\tau}$$

So the electrons would collide and then they would drift like a free particle till that electron finds another electron or impurity or defect or disorder to scatter with between these two events two collision events it drifts like a free particle and the time corresponding time is called as a relaxation time. So this is the equation of motion. Now the equation of motion will have to be you know at equilibrium there will be no force on the particle that is this exactly what we have said that once the equilibrium is established the Lorentz force exactly balances the force due to the electric field that develops in the y direction. So once that happens there will be no net field and the dp dt will go to 0 because dp dt according to Newton's second law is the force. So if that is equal to 0 we can write down for the x and y components.

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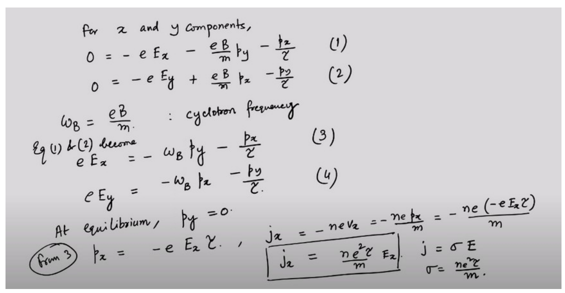

So for x and y components okay one can write down a 0 equal to minus e Ex minus Eb by m this is py I am writing now this equation the equation that you have seen here in terms of components you have e having both Ex and Ey the momentum of the particle has both components Px and Py once again I remind you that this electronic motion is confined in a plane. So P is not the vector P is not Px Py Pz let me write it here that this is equal to Ex and Ey Ex is in the direction of the current motion of the current and the Px of course will have to be Px and Py and you already know that B is in the it is a constant field in the z direction and so on okay alright. So this is equating the x and y component this is about the x and this is about the y so I write dPx dt for 1 and then the dPy dt and then individually putting them equal to 0 so this is Ey okay sorry there is a term that I missed here so it is a Px over tau the relaxation time and then there is a Eb over m Px and Py over tau if you read some books which do not use SI notation you will find a C in the denominator here okay we do not write that so we are writing in SI unit so that that C will not be there and if you now define a quantity called as omega B it is equal to Eb over m okay. Now this I write it as omega B but lot of texts actually write it as omega C because it is called as a cyclotron frequency and if you remember what cyclotron frequency is in presence of a magnetic field the charge particle actually will undergo a rotational motion okay and this rotational motion will correspond to an angular frequency and this cyclotron frequency is exactly the angular frequency that one is talking about so the frequency will increase linearly with B. So as you increase the magnitude of omega B will increase which means the cyclotron frequency increases alright.

$$0 = -eE_x - \frac{eB}{m}p_y - \frac{p_x}{\tau} \qquad \text{(Equation 1)}$$

$$0 = -eE_y + \frac{eB}{m}p_x - \frac{p_y}{\tau} \qquad \text{(Equation 2)}$$

So once you get this what happens is that we can write down you know simplify this equation and we can write down these equations as Eex equal to minus omega B Py so let us call that as equation 1 and 2 so equation 1 and 2 become omega B Py minus Px over tau the for the y component that is from 2 this is equal to minus omega B Px minus Py by tau okay. So this looks like a kind of coupled equation where Ex depends on both Py and Px and Ey also depends on both Px and Py okay.

$$eE_x = -\omega_B p_y - \frac{p_x}{\tau} \qquad \text{(Equation 3)}$$

$$eE_y = -\omega_B p_x - \frac{p_y}{\tau} \qquad \text{(Equation 4)}$$

Now physical consideration of the system would definitely yield that in the steady state the Py cannot go on indefinitely that is the Py will have to go to 0 because this is the y direction let me go back to that original figure and okay. So this is the figure so your Py which is the momentum in this direction will have to stop once the equilibrium is established because the charges cannot go out of the sample okay. And it is also in this direction the charges like either you know positive charges will accumulate at the front shaded region and a negative or the at the back the other opposite end or the vice versa but in any case the Py will have to go to 0 in at equilibrium.

$$p_y = 0 \;\; p_x = -eE_x\tau \;\; , j_x = -nev_x = \frac{-nep_x}{m}$$

So at equilibrium Py is equal to 0 so if you put that then what you get is that so Px is equal to minus Ex tau okay this Px that comes from the first equation so let us call it a 3 equation number 3 and then equation number 4 so this is like a from 3 you get this. So as a result you get J which is nothing but Ne into P by M because it is NeV that is how the current density is defined in terms of the electronic motion that is the velocity of the electrons so this is equal to minus NeVx and this is equal to minus NePx over M and if you substitute that then it becomes minus Ne and Px is nothing but minus Eex tau this capital E is the electric field the component of the electric field and so on okay. So this gives you Jx equal to Ne square tau by M and Ex okay so this is the Jx and this is a comforting equation and the reason is that that this is exactly nothing but it comes from Drude's theory or it is a restatement of Ohm's law which tells you that J which is a current density can be written as sigma into E okay and where sigma is the conductivity and conductivity is actually a tensor in our case it is a 2 by 2 matrix is a tensor of rank 2. So here the J is in the direction of the electric field so Jx in the direction of Ex and so on and this sigma is called as the is Ne square tau by M it is a known formula for conductivity of metals. So the conductivity that is sigma depends on N it depends on the charge the square of the charge rather the relaxation time tau and M okay.

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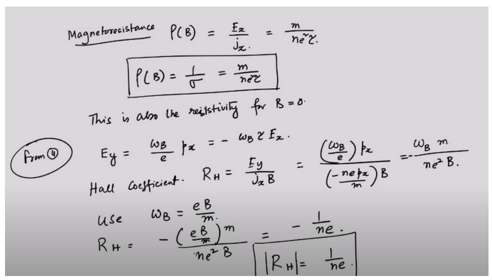

So this looks fine and if we go one step ahead and try to calculate rho which we have said so let us write so the magneto resistance which is nothing but rho because it is in presence of B I am writing it but of course there will be no B that is there so rho is equal to Ex by Jx okay. Now this makes sense because rho is inverse of sigma here rho is called as a resistivity okay so that is the resistivity that we are talking about and that is equal to M by Ne square tau and so this is of course independent of so rho is rho of B is inverse of sigma and it is given by M by Ne square tau okay. And this is also a known result and this is the magneto resistance that is resistance in presence of the magnetic field though you do not see the magnetic field for the reason that I told you that at equilibrium all these properties are being calculated and no matter what the value of the magnetic field is the resistivity in the direction of the current will always be this or so to say that this is the same as DC resistivity or resistivity when B equal to 0 let us not talk about DC. So this is also the resistivity for B equal to 0 okay and this is of course as I said that is independent of B. Now let me show you the second equation which we have not used much or fourth equation rather let us see what that gives.

$$\rho(B) = \frac{E_x}{j_x} = \frac{m}{ne\tau}$$
$$\rho B = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

So from 4 what one gets is that the Ey is equal to omega B over E into Px equal to minus omega B tau Ex so your Ey so that gives you Ey equal to this. So the hall coefficient which we have defined earlier the hall coefficient is defined as RH I am redefining it here it is equal to Ey divided by Jx into B the magnetic field and this is nothing but omega B by E into Px and divided by minus Ne Px divided by M into B and if you simplify this, this comes out to be so you have omega B Px M and with a minus sign let us put a minus sign here and you have a Ne square the Px will cancel and it is Ne square and B okay.

$$E_y = \frac{\omega_B}{e} p_x = -\omega_B \tau E_x$$

$$R_H = \frac{E_y}{j_x B} = \frac{(\frac{\omega_B}{e}) p_x}{(\frac{-n e p_x}{m}) B} = -\frac{\omega_B m}{n e^2 B}$$

So that is the hall coefficient so in the hall coefficient if we use omega B which is what we have that is the cyclotron frequency which is equal to Eb over M then your RH becomes equal to a minus Eb over M into M and then you have there is a M here M is was in the denominator so the M will be there. So there is this and then you have a Ne square B and this is equal to nothing but a minus 1 over Ne which is a result that is usually obtained in the lab. So I can write this as the magnitude is equal to 1 over Ne and this tells you that hall resistance or the hall coefficient rather here the hall coefficient is a constant and it does not depend upon B and the reason is same because we are talking about the at equilibrium the effect of magnetic field does not come.

$$\omega_B = \frac{eB}{m}$$

$$R_H = -\frac{(\frac{eB}{m}) m}{n e^2 B} = -\frac{1}{ne}$$

$$|R_H| = \frac{1}{ne}$$

So this is pretty much what one learns in the classical hall effect why we are doing it is the following that will be introducing quantum hall effect and at several places will have to fall back on the classical version of the hall effect and show that how this is different. In fact this is one of the main differences of the hall resistivity which here it is constant that is it does not depend upon B but there of course as B changes the hall resistivity the resistance that changes. So here it is independent of that. And of course it is also independent of the relaxation time so it really that tau was hanging around at various places but it does not make an entry into the hall coefficient the expression for the hall coefficient which means that it really does not matter what tau x is. So what is tau x physically is that as I told that it is the time that elapses between two successive collisions. So now the tau is large if the collisions are too infrequent that is the collisions rarely occur then tau is large and in a heavily disordered sample tau will be small and the electron will undergo a collision with another electron or defect or impurity etc too often. But here this result does not depend upon on the tau.

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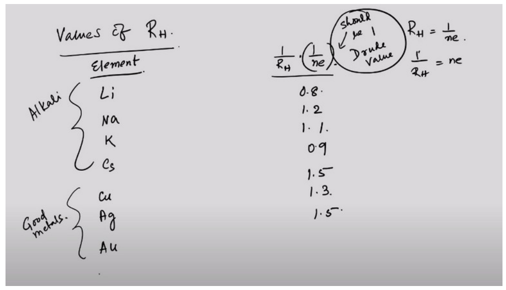

So just to give you order of magnitude for RH or the values of RH for typical metals you will be told in your undergraduate course that it finds out the carrier density and the sign of the carriers for semiconductors but it does not matter you can do the experiment on metals as well and the result will be clear because the mainly the charge carriers are the electrons that is what emerges out. So let me show some this is the element and what you get is you get a 1 over RH into Ne. So that tells you because RH is equal to 1 over Ne then 1 over RH is Ne and the Ne that is there so what you do is that you find this quantity and this quantity should be equal to 1.

So this is should be 1 that is the Drude value rather. But because of a number of reasons actually in experiments it deviates slightly from 1 and that is what we want to show. It can depend upon the temperature of the sample. So if you are not at very low temperature then it can deviate or if you are not at you know if you are at too low value of B it can deviate as well but as you know go to lower temperature and larger magnetic field it gets closer to that.

So for example lithium it has a value 0.8 then you know sodium it has a value 1.2 which is symmetrically placed then we have potassium it is 1.1 and so on. So a cesium it is a value exactly it is 0.9 and so on. So these are the alkali metals and then you have a copper which is known to be a good metal it is 1.5 then silver which is known as the best metal that is known. So this is 1.3 gold is for example it is a 1.5 again and let me give you some examples. So these are good metals maybe aluminum also but so and there are these Be, Mg etc. aluminum, aluminum comes here it has a value B has a value minus 0.2 so it deviates quite a bit the magnesium is minus 0.4 and this aluminum is minus 0.4, 0.3 or 0.4. So these are the values this of course the sign is negative and the current is carried by charges which are positive signs. So this is what we have seen. So these are called as holes as opposed to the electrons which is what we have seen. So with this introduction or rather this description of the classical Hall effect let us try to go towards the quantum Hall effect. This discussion will be very thorough and quite elaborate. So right now just where we are and then we want to understand that what happens in a quantum Hall effect.

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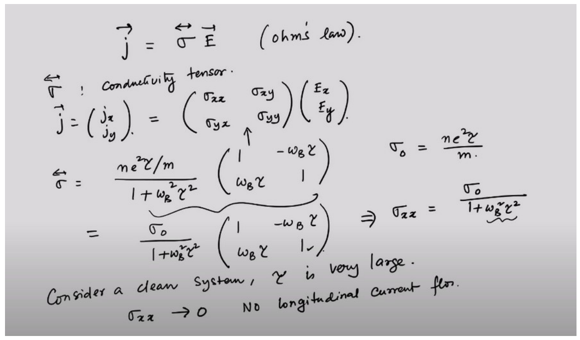

Let me sort of write down the same Drude equation or the Ohm's law. This is also called as Ohm's law it is J equal to sigma is nothing but V equal to IR because you can multiply J by the area which will give you I and then this E will be equal to V and so on and then sigma is an inverse of the resistance and so on. So this is nothing but the Ohm's law. So if you consider that you have a J that is only confined in the XY plane then and this sigma is actually the called as the conductivity tensor and sometimes we will write with a double arrow double headed arrow there but even if we do not always understand that this is tensor.

$$\vec{j} = \vec{\sigma}\vec{E}$$

So this is a conductivity tensor. Alright so once you know these things are clear so J has two components which are like JX and JY. So if we write this so our sigma will be like sigma XX sigma XY sigma YX we have to be a little careful sigma XY and sigma YX may not be same and in this particular case it is definitely not same. In fact they differ by a sign and I will give you a proof that Y in two dimensional electrons when it's subjected to a transverse magnetic field or a perpendicular magnetic field why necessarily the conductivity tensor or the resistivity tensor has to be anti-symmetric which means the sigma XY is equal to minus sigma YX, Okay and this is very typical property of electrons in a magnetic field.

$$\vec{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

So this is equal to EX and EY. So this if we actually go to this equations that we have seen here these equations these 1, 2, 3, 4 etcetera then we it is not difficult to write down that this sigma tensor is nothing but it's Ne square tau over m and divided by 1 plus omega b square tau square 1 and minus omega b tau and omega b tau and equal to 1. You see I said that they differ the off diagonal elements differ by a sign which is what is coming here as well take it as a small exercise and try to work out the relationship between the components of JX, JY and EX and EY. Okay so they are connected by this relation so when you write JX and JY and you put this sigma that I have written here in this slot and then they would smoothly connect with the EX and EY that you have seen. Now I told you that this sigma 0 let's call this as sigma 0 you can call it as sigma Drude also this is the Drude result this is Ne square tau by m. So this is equal to sigma 0 divided by 1 by omega b square tau square and 1 minus omega b tau and so on.

$$\vec{\sigma} = \frac{\frac{ne^2\tau}{m}}{1+\omega_B^2+\tau^2}\begin{pmatrix} 1 & -\omega_B\tau \\ \omega_B\tau & 1 \end{pmatrix}$$

$$= \frac{\sigma_0}{1+\omega_B^2+\tau^2}\begin{pmatrix} 1 & -\omega_B\tau \\ \omega_B\tau & 1 \end{pmatrix}$$

$$Thus \quad \sigma_{xx} = \frac{\sigma_0}{1+\omega_B^2+\tau^2}$$

Okay omega b tau and omega b tau and 1. Okay now what happens is that if you have a very clean system now we are farther you know making assumptions which will lead to some very interesting results these assumptions are needed if you make the system to be clean so consider a clean system. And what do you mean by clean system. The clean system means there are no impurities or defects or disorder in the problem in which case your tau will become very large. Okay this we have told earlier as well that the relaxation time in the event there are very infrequent collisions or almost no collisions tau becomes very large and when tau becomes very large this quantity will become equal to you know your from here what one gets is that the sigma xx is equal to sigma 0 divided by 1 plus omega b square tau square. Right because this element is 1 and that will get multiplied with what is there outside the matrix. So this sigma 0 divided by 1 plus omega b square tau square as same as sigma yy but then let us not talk about sigma yy because you are sending current in the x direction.

Okay the sigma yy is there this element is there but this element is we are not sending a current in that direction when we will be doing that then this thing will make sense but then this also exists. Okay now if tau is very large your sigma xx will go to 0. Okay now there is something very interesting happens so sigma xx will go to 0 because the tau becomes very large and you can neglect one in front of this second term. So when you can do that and you have an infinity in the denominator then of course sigma xx goes to 0. Okay and that tells you that no longitudinal current flows in the flows I mean basically there is no longitudinal current because the conductivity has vanished if the system is clean. Okay but otherwise of course this is just a sort of assumption that you are taking a clean system where tau is made to vanish tau is made to be very large in which case sigma xx vanishes.

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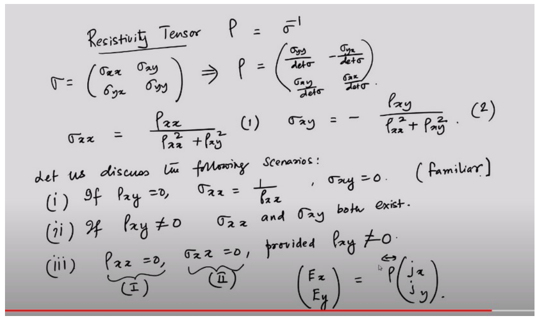

Okay alright but in general sigma xx will be a non-zero in a realistic case where the tau is not infinitely large it could be large but not infinitely large so that it exists and in that case if we want resistivity tensor. Okay which is written by rho and you have to be careful because if you need to get this you need to invert this and this is not an inversion that is 1 by sigma xx or something. It is sigma is a 2 by 2 matrix which is what we have said. You have to calculate y by taking so this will be like sigma yy divided by the determinant of sigma and then this will be like sigma yx and so on and then determinant so it is basically the cofactor divided by the determinant and so on.

$$\rho = \sigma^{-1}$$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\Rightarrow \rho = \begin{pmatrix} \frac{\sigma_{yy}}{\det \sigma} & -\frac{\sigma_{yx}}{\det \sigma} \\ \frac{\sigma_{xy}}{\det \sigma} & \frac{\sigma_{xx}}{\det \sigma} \end{pmatrix}$$

So this is like sigma xy I think there will be a sign somewhere and so on so forth. Okay so this is sigma xx by determinant of sigma. So this will give you the rho matrix or the resistivity tensor which is what we have called it. So the relationship between the sigma xx it's a 2 by 2 problem which you can easily you know invert it by taking the cofactor and dividing it by the determinant and in that case the relationship between xx and the rho x sigma xx and the rho xx are like this.

So this is the denominator that I have been talking about. Yeah the denominator which is determinant of sigma so this is that. Okay so this is sigma xx and the sigma xy these two are important because the other two will follow automatically when we there is a sign change that occurs here and that is that should be taken into account. So this is equal to minus rho xy divided by the same denominator which is coming from the determinant of the sigma matrix. So this is rho xy square.

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$
$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

So this is a relationship between the sigma xx and rho xx. Now you see that if you do not have a rho xy then of course sigma xx is 1 by rho xx. So that then if you do not have the off diagonal elements then you can always write the elements to be just simply you take the inverse of the elements and then you will be fine. But now we have very importantly the this off diagonal elements which actually talk about the Hall resistivity because you send current this rho xy or sigma xy are related to the Hall resistivity. You send current in the x direction and you measure the conductivity or the resistivity in the y direction.

So that is what is most important for us. So that cannot be neglected. Okay so if we have this let us you know discuss the following scenarios. So number ,1 if rho xy equal to 0 which is what I said then sigma xx equal to 1 by rho xx which you can see from the equation this one. Let us call that this as we have been calling equations.

$$if \;\; \rho_{xx} = 0, \;\; \sigma_{xx} = \frac{1}{\rho_{xx}}, \quad \sigma_{xy} = 0$$
$$if \;\; \rho_{xy} \neq 0 \;\; \sigma_{xx} \;\; and \;\; \sigma_{xy} exist$$
$$\rho_{xx} = 0 \;\; \sigma_{xx} = 0, \;\; \rho_{xy} \neq 0$$

So now let us call it as 1 and 2. Okay which are not same as the 1 and 2 that we have discussed earlier. Alright and of course your sigma xy equal to 0 because sigma xy is rho xy in the numerator and because rho xy equal to 0 then sigma xy equal to 0 as well and this is a familiar thing. Okay so this is a familiar scenario. Alright so what is the second scenario? The second scenario is that if the rho xy is not equal to 0 now this interesting if the rho xy is not equal to 0 then sigma xx and sigma xy both exist. Okay because xy is not equal to 0 so you have both of them to be existing.

Now as a special case consider that rho xx equal to 0 which tells you that sigma xx is also equal to 0 because rho xx is in the numerator of the sigma xx that will happen if the denominator completely does not vanish. That means provided rho xy is not equal to 0. You see that is coming from 1 here you have assumed that rho xx equal to 0 that will make the sigma xx equal to 0 but the sigma xx can only become equal to 0 if it is not a 0 by 0 problem. 0 by 0 is it cannot be defined so we will have to at least assume that it is the rho xy is not equal to 0 and this is the crux of the problem because rho xy we do not want to vanish for the reason that that is the main you know measurable quantity for us.

And this is truly interesting for the reason that what does this mean? This means that it is a absolutely a sort of conduction without any resistance and that tells you that this is a the conductivity is completely equal to 0 so it is completely insulating so whether it is a perfect conductor or a perfect insulator.

This one condition tells you it is a perfect conductor because no longitudinal resistivity and this tells you that it is perfect insulator because there is no longitudinal conductivity. Can that happen? Is it a wrong result? It is not a wrong result it is a correct result and it can happen in presence of a magnetic field and two dimensions is also important here. And then we will have to sort of take it from here and we will just pose this problem for now and then we will come back to this scenario where you have I mean if that is the reason then what will happen is that you know this your Ex and Ey that is the components of the electric field this is equal to rho j so jx and jy.

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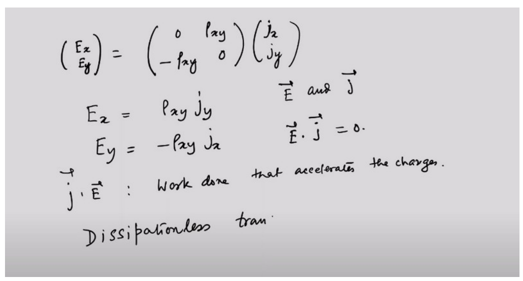

But what happens is that because of the rho taking this form one has that Ex and Ey where rho xx is equal to 0 so it is 0 rho xy minus rho xy and 0 and then so jx and jy. Now you see what happens is the following you have Ex this is equal to rho xy jy and Ey equal to minus rho xy jx.

$$E_x = \rho_{xy} j_y$$
$$E_y = -\rho_{xy} j_x$$
$$\vec{E}.\vec{j} = 0$$

If you just do the multiplication which tells you that E and j are perpendicular because Ex is proportional to jy or it is in the direction of jy and Ey is in the direction of jx which means that they are rotated with respect to each other. So, the electric field components are x component is in the y direction which is just a rotation by 90 degree and the other is the Ey is in the direction of minus jx which is again a rotation by 90 degree. So, that tells you that this is equal to 0 this is equal to 0 and if that happens that this if you remember that a j dot E is actually the work done that accelerates the charges and because E and j are mutually perpendicular this work done is equal to 0. So, this tells you that there is a very strange thing that is happening where the transport is completely dissipation less and this there is no work that is required in order to you know accelerate the charges. So, it just happens or it flows on its own. So, there is no work done that is required. So, this is that is why there is no dissipation and this all these things will be explained why it happens and why such a strange sort of properties that these electrons in presence of a magnetic field it has.

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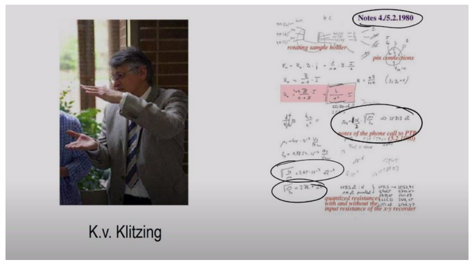

I will show you just some these are some these pictures of Klitzing who discovered in 1980 the quantum Hall effect. So, we are slowly coming to quantum Hall effect after those initial discussions and you see that this very interesting that the notes written by Klitzing on he said 4 slash 5 which means it is then the night of 4th and 5th February in 1980 and he writes that he found something which is extremely fundamental and then he keeps on writing these are these epsilon 0 by mu 0 or mu 0 by epsilon 0. So, this is having a dimension of resistance and there is a dimension of conductance.

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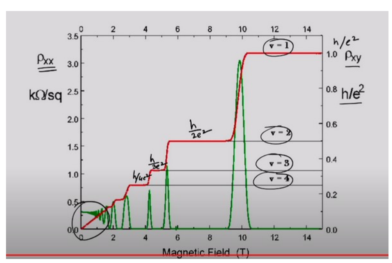

So, the first factor is the Hall resistivity and the green plot that you see is the longitudinal resistivity which is the magneto resistivity. You see the Hall actually shows a plateau. So, this is a plateau corresponding to mu equal to 1 there is a plateau corresponding to mu equal to 2 and mu equal to 3, 4 and so on and you would also see that when you are here you actually get a straight line for the red curve or even very close to B equal to 0 that is magnetic field to be very small you get that linear regime and that regime is the classical Hall effect. Here you see that the magnetic field has gone up to 15 Tesla, 15 Tesla is a very large magnetic field and these the Hall plateaus are quantized in unit of h over e square which means this value is h over 1 e square. So, this is h over just e square this is h over 2 e square this is h over 3 e square and h over 4 e square and so on so forth.

So, these are the plateaus and you also see that it is exactly what we are talking about on the last slide that you see that this green one which is a longitudinal resistivity which is rho xx and this is equal to 0 it remains 0 all the time this green thing excepting when there is a jump from one plateau to another it shows a spike. You see everywhere there is a plateau in the red curve is accompanied with a jump in the resistivity of the green curve. And these of course these things will have to be understood over time but at least the zeros of the magnitude resistance that you can see here that tells you the 0 and the jump it tells you that the system actually undergoes through a large number of you know phase transitions from metal perfect metal because when it is the longitudinal resistivity is 0 then it is a perfect metal and then when it jumps it becomes a perfect insulator again it is 0 again it is it jumps and so on so forth. So, the system undergoes through a series of metal to insulator transition this has not been seen anywhere and only happens for these electrons of free charges in presence of a magnetic field.

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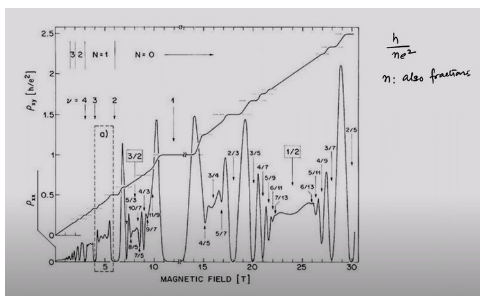

We will talk about that in the last slide that we show here is that later on there are three people Laughlin, Stormer and Sui they have shown that these resistivity do not occur only at this integer values of this h over n e square where n is an integer n can also be fractions.

So, the integer quantum Hall effect got a Nobel Prize cleansing and this also got a Nobel Prize where one has found out that there are several fractions you see that there is a 2 by 5, 3 by 7 probably close to 100 fractions have been discovered. However, the physics of the integer quantum Hall effect when n is an integer and when n is not an integer and is a fraction the physics is very different. So, this is called as fractional quantum Hall effect and the one that you saw here is called as the integer quantum Hall effect. We will mostly talk about integer quantum Hall effect but also will touch upon the fractional quantum Hall effect. Thank you.