Numerical Methods and Simulation
Techniques for Scientists And Engineers
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Lecture 11
Simpson's 1-3rd rule

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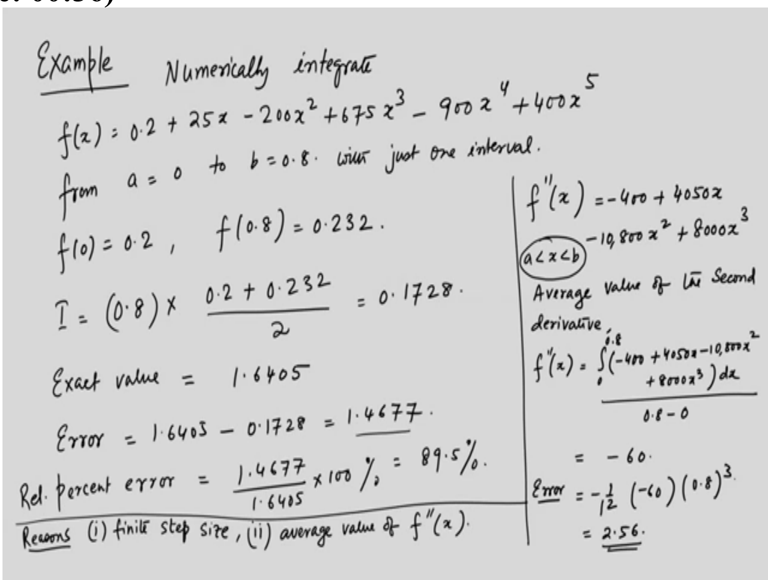

So, let us get ahead with a one more example of the trapezoidal rule, so suppose you have a function which you have to numerically integrate f of x equal to 0.2 + 25x - 200x square + 675 x cubed - 900x to the power 4 + 400x to the power 5, it is a polynomial that you have to numerically integrate from a lower limit a equal to 0 to an upper limit b equal to 0.8 with say just 1 interval which means this large polynomial is a 5th degree polynomial has to be replaced by a straight line okay.

And that is of course we know that it is a bad approximation but still we can integrate it and get a result. So, what is f of 0 it is equal to 0.2 and f of 0.8 is equal to 0.232, so the integral becomes 0.8 which is b - a which is called the h and the average value at the 2 extremities which is 0.232 divided by 2 and this comes out about 1.1728 if you do it carefully and the exact value is far, far away.

The exact value of this integral is 1.6405 okay, so you see that if this 5th degree polynomial is replaced by a straight line and you go ahead with doing the integral using the trapezoidal rule with just one interval that is you take the the point at the beginning at a equal at a and connect it to the point b which is 0.8 here by a straight line this is what you get all right. So, the error is a very visible and extremely large so the error is 1.6405 - 0.1728 so it is 1.4677, so more relevant quantities that the percentage.

Relative percentage error this is equal to 1.4677 divided by 1.6405 in to 100% and this is like almost 90%, 89.5% so that is the error if you do it okay. This is just to show you that trapezoidal rule is has to be carried over to multiple integrals and n equal to 4 and n equal to 6 and so on. We have done a problem earlier where we have compared the results between n equal to 2 and n equal to 4 and that showed that there is a significant improvement.

So, the message is that that if a trapezoidal rule has to be implemented you better take the number of intervals to be large okay, otherwise it is going to end up in this kind of an error large error. If you want to actually calculate the error, let us do it here that we know that the error is proportional to f double prime x which is the double derivative of f. So, this is equal to 400- 400+ 4050x - 10,800x square + 8,000x to the power x cube.

And so we have 2 so this x actually lies at a point between a and b where a equal to 0 here and b equal to 0.8 so we need to actually find out the average value of the second derivative. So, okay and this is equal to f double prime x just to make it clear that, so a less than x less than b okay this is the x lies between a and b and this is equal to 0 to 0.8 and then this - 400+ 4050x - 10,800x square and + 8,000x cubed and this into dx and divided by h which is b - a which is 0.8 – 0.

And if you calculate it becomes equal to 60 or rather – 60 so this the error is actually this was derived earlier so the air error is equal to – 1 over 12 into – 60 into h cube which is 0.8 whole cube and this is about 2.56 and you see that the actual error is 1.4677 and we are getting it from these from the estimate error estimate that we have done earlier which is proportional to the double derivative it comes out as 2.56.

Of course they are different but one important thing is that they are of same order of magnitude. So, what is the reason for such error, let write it here itself reasons for these discrepancy 1is that we have taken a finite step size which is as large as 0.8 here and the other error come from the average value of f double prime x okay.

So, here if you look at it we have calculated the average value by integrating the f double Prime expression over the full interval and then divided it by the interval so that is the average value of every double prime. And this thing is certainly incorrect because of the reason that you see that there are functions which are like x Square and x cube. So, for some values of x say greater than or between 0 and 1 which is the region of integration here.

You see x cube is much smaller than x so we have not attached the right weight of this term as compared to the first term that is linear in x. So, this cubic term in x of course is much smaller for the region that is under consideration and hence we are getting this discrepancy in the estimation of error okay. So, with this we are pretty comfortable with using the trapezoidal rule which is simply the you know using the polynomial the first degree polynomial.

And then the entire region of integration is actually converted into a trapezoid and one calculates the area of the trapezoid by taking the width and the average value of the height. Now this as we have seen earlier that this crude method of calculating the integral of a function can still be improved if we take several segments over which this trapezoidal rule or the trapezoidal formula is applied.

And in which case the error actually goes down so if you use a number of a large number of segments over which this trapezoidal formula would be applied then one 1can expect to get a much better result which is very close to the actual result. Let us see one more and probably the most popularly used or most widely used formula or the rather the formalism of doing integration.

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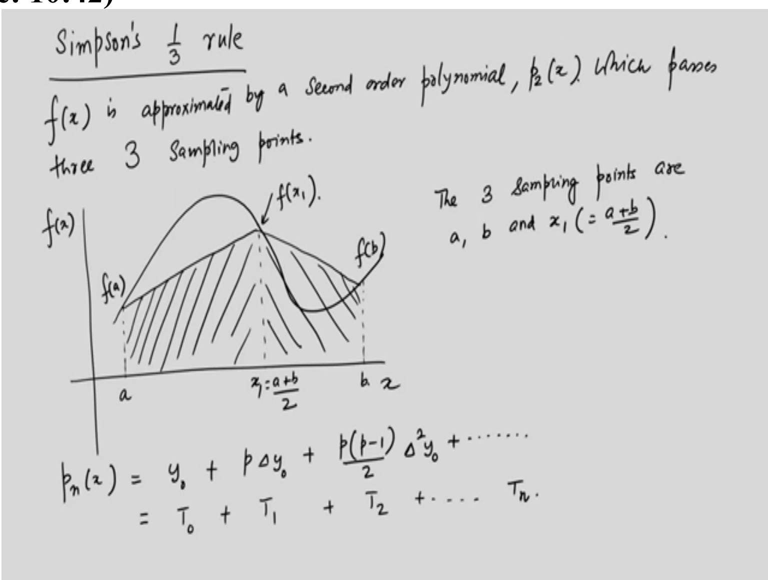

It is called as the Simpsons one-third rule and it is nothing but it just uses 2 point or rather 3 point instead of 2 point as they are in the trapezoidal rule. So, it uses 3 sort of second degree polynomial in order to do the integral so f of x is approximated by a second-order polynomial p 2x which of course passes through 3 sampling point now this which 3 is the question. So, let us show this graphically.

So, suppose you have okay so you have this as the function f of x versus x so this is your the initial point a and this is your final point b so it uses 3 point the 2 of them are a and b and the other one is actually an arithmetic mean of these 2 which means the midpoint of these a and b which is simply let us call it as x 1 which is equal to a + b by 2 okay. So, what it does is that it takes this trapezium and it takes this trapezium and calculate the area of each one of them okay.

So, this entire region is now replaced by this the sum of these 2 trapezoids or trapeziums and then one calculates this so this is a value f of a and there is a value f of b and this is a value which is f of x1 okay. Where x1 equal to a + b by 2 so the 3 sampling points are a, b and x 1 which is equal to a + b by 2 okay and it uses a second-order polynomial by these 3 sampling point which is p2.

So just for the benefit of the readers let me write down the second-order polynomial or rather in eighth order polynomial so the nth order polynomial looks like it is a y 0+ p Delta y 0+ p into p – 1 divided by 2 delta square y 0 and so on ok. So, which is we can write it as say T 0+ T 1+ T 2 and so on. And then all the way up to you know say T n. So, that is the second-order polynomial sorry this is the nth order polynomial and we are only going to consider the first 3 terms that is T 0 T 1 and T 2 which are written above.

So, we are going to do an integral using these 3 point which is f of a f of x1and f of p so a delta y 0 which is a slope will be calculated using 2 of these points and the Delta square which is the second derivative will be calculated involving all the 3 points. So, let us see what these the integral becomes.

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Retaining the first 3 terms for the Simpson's $\frac{1}{3}$ rule,

$$I = \int_a^b p_2(x)\,dx = \int_a^b (T_0 + T_1 + T_2)\,dx.$$

$$= \int_a^b T_0\,dx + \int_a^b T_1\,dx + \int_a^b T_2\,dx = I_1 + I_2 + I_3.$$

$$I_1 = \int_a^b y_0\,dx \;;\; I_2 = \int_a^b p\,\Delta y_0\,dx \;;\; I_3 = \int_a^b \frac{p(p-1)}{2}\Delta^2 y_0\,dx.$$

$$dx = h\,dp. \quad (x = x_0 + ph) \quad \Big\} \; h = \frac{b-a}{2}. \quad p = \frac{(x-x_0)}{h}. \quad \text{So. } x_0 = a,\; x_1 = b.$$

At $x = a$, $p = \frac{a - x_0}{h} = 0$.

At $x = b$, $p = \frac{b-a}{(b-a)/2} = 2$.

$$I_1 = \int_0^2 y_0\,h\,dp = 2hy_0 \;;$$

$$I_2 = \int_0^2 \Delta y_0\,p\,h\,dp = 2h\Delta y_0 \;; \quad I_3 = \int_0^2 \Delta^2 y_0 \frac{p(p-1)}{2}\,h\,dp = \frac{h}{3}\Delta^2 y_0.$$

So, retaining the first 3 terms for The Simpsons 1/3rd rule, so I which is a value of the integral is nothing but p 2x dx and it is between a and b which is nothing but equal to a to b and at T 0+ T 1+ T 2 dx so this can be written as a to b T 0 dx + a to b T1 dx + a to b T 2 dx and let us call them as I1+ I2+ I3 so that is the those are the 3 integrals these contributions will have to be combined in order to get the Simpsons 1/3 rule formula.

So I 1 is nothing but y 0 dx between a and b and similarly I1equal to a p delta 0 dx between a and b and the I2 becomes equal to a this I 2 and this I 3 this I 2 and this I 3 this is a and b and p into p - 1divided by 2 and so this is a delta 0 we call it a delta 0 sorry what delta 0 is delta y 0 pardon me, so this delta y 0 and this delta square y 0 and dx ok. So, this is pretty much that we have to do now remember that dx is equal to h dp because x equal to x 0+ p h.

So, this comes from x equal to x 0+ ph we are going to again show that and of course this h is equal to b - a by 2 because this was b - a in the trapezoidal rule now it is b - a by 2 because we are using 2 segments. Now you see it is important to note that a p is equal to from this formula here p is equal to x – x 0 over h, so x 0 equal to a and x 1equal to b now h is equal to b - h is of course b - a by 2 as it is written there.

So at x equal to a ok so see what we are trying to do is that we are trying to change this integral from x this now over x 2 will change it to the integral over p, so we need to know that if x goes from a to b what would p go from that is when we replace this dx by dp then we need to know what the integral you know what is the limit what are the limits of the integral. So, this is equal to a – x 0 by h now a is equal to x 0 so this is p is equal to 0, so at x equal to a p equal to 0 ok.

So, this is important and at x equal to b okay so p is equal to b - x 0 is of course a so this is b - now h is equal to b - a by 2 so this is equal to 2. So, at x equal to b p equal to 2 so when we replace these integrals from dx to h tp we will have to write from 0 to 2 that is the main idea. So, my I 1 becomes equal to 0 to 2 y 0 h dp which is equal to 2h y 0 okay. Similarly I 2 would be 0 to 2 delta y 0 h there is a ph dp which is equal to 2h Delta y 0 ok and I will write it here this thing.

So, I 3 it is equal to 0 to 2 Delta square y 0 and there is p into p - 1by 2 and then there is a h and then there is a dp ok. If you do this integral because now it is over p so it is a p square - p by 2 and then you have a term which is p cube and all that so this becomes equal to and if you when you put 0 to 2 it becomes equal to h by 3 Delta square y 0 ok. So, let me just point out these values of the integrals see y 0 is the value of the function at the initial point x 0 which is equal to a, delta y 0 is the slope calculated at the x 0.

And similarly this is the curvature or the double derivative which is evaluated there so now how do we calculate all these and get a form for the integral.

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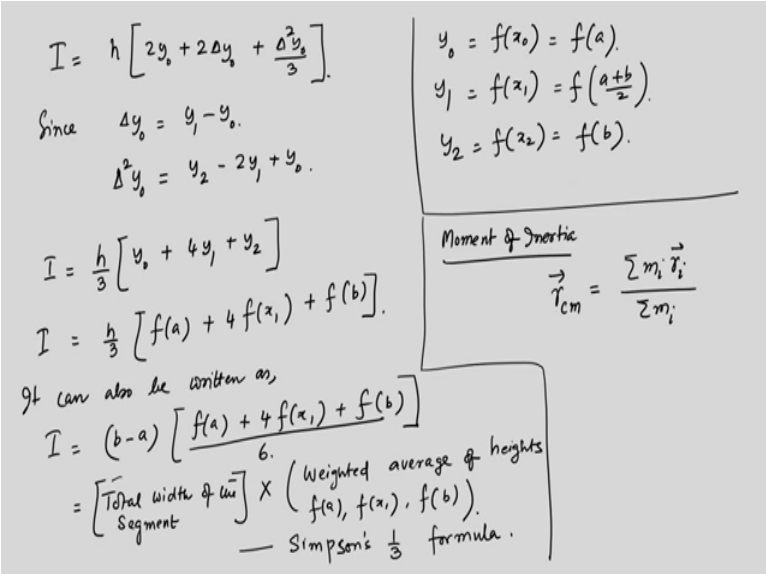

So, our I becomes equal to h 2 y 0+ 2 Delta y 0 + delta square y 0 over 3 all right, so this is a value of this integral so since Delta y 0 is nothing but y 1- y 0 so basically this is the value of the function or this is the difference between the values at x equal to x 1 and x equal to x 0. And similarly your Delta square why not this is from that the second-order formula for the double derivative that we have learned is y 2 involving 3 point - y 1+ y 0 ok.

So, y 2 is the value of the function so just to remind you that y 1 ok, let us start from y 0 is nothing but f of x 0 this is equal to f at a, what is y 1 its f at x1 which is f at a + b by 2 and what is y – it is f at x2 which is f at b ok. So, these are the things that are there. So your, I becomes equal to h over 3 and y 0+ 4y 1+ y 2 ok. So, this is after simplifying and putting in all this so all these things are put there and y 0 y 1 and y 2 have these meaning and we simply write it as h over 3 f of a + 4 f x1+ f of b.

So, this is a nice formula which says that you see you take the value of the function at the left extremity once take the value of the function of the right extremity once and at the middle point you weighted by a factor of 4 ok and this h over 3 is the so, h is the interval which is here it is the total interval divided by 2 so it is b - a divided by 2. So, this can be also be written as I equal to b - a f of a + 4 f x1+ f of b divided by 6 ok.

So, this is the total width of the segment and you multiply it by the weighted average of heights f a f x1 f b okay. So, if this word weighted average is not cleared then let me give you an example say in the usual you know semester systems that we have in the various institutions and so on you have a grading system which is a letter grades which you know and the letter grades actually correspond to points.

I mean these are also there in CBSCN and so on so one gets a letter grade and the letter grades has some point in the scale of 0 to 10 and if one gets b say b has a weight of 8 which means 8 out of 10 and in a course which has a credit say it is given by the credit is given by say 10. So, one would get a whole point for that would be 8 into 10 and so on. So, you have 5 subjects which have all credits at 10 8 6 4 or you know 10 12 and so on.

And then one gets A in one course which means he gets a full 10 on 10 on that course and if that course has a weight of 4 then it is a 10 into 4it is 40a nd then another course which has a credit of 8 one gets a B, so it is 8 into 8 64 so the average will be 40+ 64 divided by the total number of credits in this particular case it will be 4+ 8 which is 12. We can also give you the moment of inertia example.

So, you know the how the moment of inertia is calculated which is a weighted average. So, the moment of inertia or the position of the moment of inertia is given by you know it is m i are I so let us say it is a vector divided by sum over m i okay. So, what it means is that if there is a large mass as compared to all other masses the moment of inertia or the position of the moment of inertia would be located closer to the bigger mass as compared to the other masses okay.

And similarly if there are you know all masses equally weighted then the and the RCM will lie somewhere in the middle. So, this is that weighted average we are waiting once the left most point we are waiting once the right most point and we are waiting the middle point which is the mean or the arithmetic mean of the 2 end point we are waiting it 4 times. So, that is the height so this heights basically because this is on the y scale so the these are the weighted average of the heights.

So this is the formula which is called as the Simpsons one-third formula all right. So, this is this can be used easily as I told you that the only difference between the trapezoidal rule and this rule is that it has it includes 3 point for calculating and integral rather than 2 points. And now of course you can make a compound this Simpsons one-third rule which means that you can actually split the entire range of integration into several segments and each one of the segments in each one of the segments you can apply this formula.

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Error analysis of the Simpson's $\frac{1}{3}$ rule.

$$E^T_{simp} = \int_a^b T_3\, dx = \frac{f'''(\theta)}{6} \int_0^2 p(p-1)(p-2)\, h\, dp. \qquad a < \theta < b.$$

$$= \frac{f'''(\theta)}{6} \left[\frac{p^4}{4} - p^3 + p^2 \right]_0^2 = 0.$$

$$\left(\frac{16}{4} - 8 + 4 = 0 \right)$$

$$f'''(\theta) = f^{(3)}(\theta)$$

$$f''''(\theta) = f^{(4)}(\theta)$$

$$E^T_{simp} = \int_a^b T^4\, dx = \frac{f''''(\theta)}{4!} \int_0^2 p(p-1)(p-2)(p-3)\, h\, dp.$$

$$= \frac{h f''''(\theta)}{24} \left[\frac{p^5}{5} - \frac{6p^4}{4} + \frac{11p^3}{3} - \frac{6p^2}{2} \right]_0^2$$

$$= -\frac{h f''''(\theta)}{90}$$

So, let us see the error associated with it so once again remind you that this is basically we are talking about the truncation error. So, we have taken 2 terms of the polynomial or the rather the first 3 terms of the polynomial, second-degree polynomial the first 3 terms have been taken and we are leaving the term from the fourth term onwards. So, this would have or induce error into this Simpsons one-third rule formula.

So we can write it as so T for truncation and let us just write simp for the Simpson so it is between this a and b and in tune with our ongoing notation let us write T 3 because T 0 T 1 and T 2 have already been accounted for ok. And if you write it carefully then this becomes equal to f triple prime and let me write some theta ok and I will tell you what theta is and then you have a 0 2 to p into p – 1 into p - 2h dp ok, where theta is a variable that lies between a and b ok and this is that third order expression that we have.

So, if we have this because if you remember that we have taken expressions up to the second derivative so the third derivative is missed out so it is f triple prime theta divided by 6 and we have a p to the power 4 by 4 - p cubed + p squared and 0 to 2 because the range of p is from 0 to 2 but you see that if you put 2 here it becomes 2 to the power 4 is 16 by 4- 8+ 4 which is equal to 0 ok. So, this is the speciality of this Simpsons 1/3rule that the leading order truncation term that contributes 0.

So this is equal to 0 okay, so there is no error at this level so the error must be at a level which is 1more than that so the truncation error has to one has to go beyond so usually we have not seen such accidental cancellations and because of this cancellation Simpsons 1/3 rule is very accurate or it is much more accurate than the trapezoidal rule which has from the second term onwards the error starts coming in.

So, this is equal to then I a to be and T to the power 4 dx and this is nothing but f writing it slightly cumbersome notation but it means that there are 4 times derivatives being taken. So, if you have n of them so you can write it you know I mean you can use an alternate rotation for f this theta as f 3 theta and so on. so, f 4 theta can be written as f I mean so anyway we will go ahead with this notation at this moment.

And so this is equal to this and then there is a 4 factorial 4 factor, so this 6 is actually coming from the 3 factorial, so 4 factorial is nothing but okay we will write it and the next step we can write it there so it is a 0 to 2and it is a p into p – 1 into p into p - 2 - 3h dp and this one can be written as h f this fourth derivative divided by 24 and this is like p to the power 5 by 5- 6P to the power 4by 4+ 11p cube by 3- 6p Square by 2 this and then from 0 to 2 this gives a finite number which is equal to a - h f this by theta divided by 90 ok.

So, this is a number which is or this an error which comes as the for the derivative and divided by or rather multiplied by h over 90.

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Since $dx = h dp$.

$\frac{dx}{dp} = h \Rightarrow f''''(\theta) \simeq h^4$.

$$\left| E^{T}_{simp} \right| = \frac{h^5 f''''(\theta)}{90}.$$

Example

$$I = \int_{-1}^{1} e^x dx = \frac{h}{3}\left[f(a) + f(b) + 4 f(x_1) \right]$$

$h = \frac{1-(-1)}{2} = 1$, $f(x_1) = f\left(\frac{a+b}{2}\right) = f(0) = 1 = e^0$

$I = \frac{e^{-1} + 4e^0 + e^1}{3} = \underline{2.36205}$; Exact value $= 2.35040$.

Much better than $n=4$ trapezoidal rule (2.39917).

Since dx is equal to h dp dx by dp is equal to h so that gives that f for theta is order of h 4 so E symp and let us just write that the truncated one but the magnitude of that is h 5 by 4 and theta divided by 90 okay. So, it is a because of this h to the power 5 the error is going to be less it is just going to be a small error there. Let us see how small and how we can you know use this error estimation formula to calculate the error.

So, let us take an example I equal to – 1 to + 1 exponential x dx it is equal to h by 3 f of a + f of b + 4 f of x 1 okay I just change the order so h is equal to a - b - a by 2 so h is equal to b is 1 and this by 2 which is equal to 1+ 1 by 2which is equal to 1 f of x 1 is equal to f of a + b by 2 so this is equal to a + b is equal to 0 so this is f 0 which is equal to that is a value of the integrand which is 1 okay so that is fine so this is equal to 1or we can write it as e to the power 0 if you like so the integral is equal to e to the power - 1+ 4e to the power 0+ e to the power 1divided by 3which gives a value which is 2.36205.

So, this is the value that we get from just 2 segments Simpsons 1/3 rule. So, the exact value of this it is equal to 2.35040 so this is if you look at it this value here is much better than then n equal to 4 trapezoidal rule which gave a value which is 2.35040 no so this gives 2.39917 so this is the exact value. This is a value from just 2 segments Simpsons 1/3rule and this is a value for the 4 n equal to 4 trapezoidal rule okay.

So, you see that just this 2 segment Simpsons 1/3 rule is itself much better than that then the n equal to 4 trapezoidal rule let us see another example ok.

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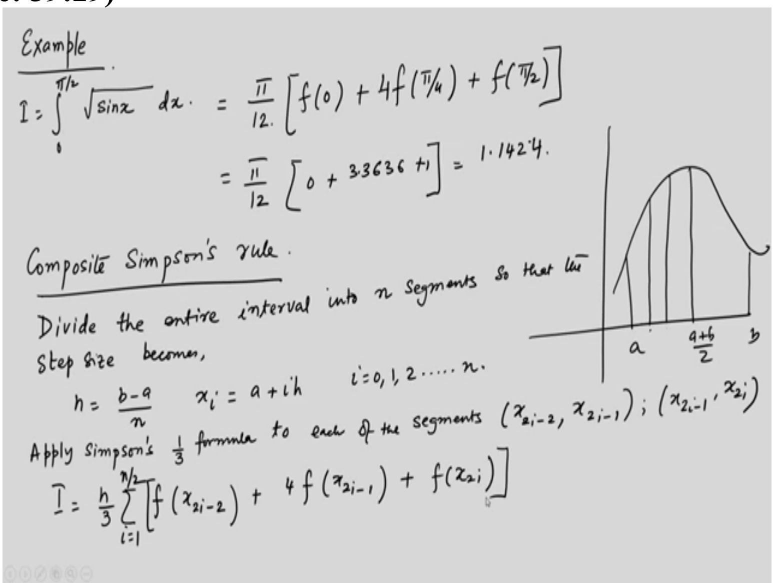

So, it is I equal to 0 to PI over 2 and root over of sine x and dx okay so we will just do 2 segments and then we will see that how you know it can be increased by a multiple or a composite Simpsons rule. So, then this is equal to PI over 12 okay because so it is 0t o PI by 2 so PI by 2 - 0 divided by 2 is 8 and then it is h by 3 just reminding of the formula that we have used it is h by 3.

So, h is b - a by 2 which is PI over 4 and then divided by 3 is PI over 12 so it is PI over 12 and the value at the extreme end taken once the value at the middle now the middle of pipe between 0 and PI by 2 comes the PI by 4 so we have a f PI by 4 and + f pi by 2 and this one is equal to PI over 12 it is 0+3.3636+1 and this can be you know simplified to get which is 1.14224 and so on ok.You should also check this because there could be just a slip of you know calculation from my part and should see everything thoroughly.

So, this is what 1gets from this from this 2 segment Simpsons 1/3 rule. Now let us see what is a composite Simpsons rule. So, what I mean is that the entire interval between a and b will be divided into n segments n could be a number of your choice could be 4 could be 6. And on each 1of those segments the 1/3 rule formula would apply okay. So, you will have for a small segment you use a 3 point formula to estimate the value of the integral.

Then you go to the next segment use the value of this so just to let you give you an idea of this we have done something like this where the 1 point here 1 point here. Those are the 2 extreme points we have taken point here which is the mean of these so it is a b, a + b by 2 just a little you know sloppy drawing from my parcel to do it better okay. But now you want to actually split them into several such things.

And use these ones as the a and this as the final value the rightmost value that is b and then take another value which is in between that that is an arithmetic mean of that then use this formula go to the next one again use this as a this as b and then you know use a point in the middle use this one-third formula and go ahead and sum up all these integrals okay. Or so that is called as a composite Simpsons rule.

So, the idea is to divide the entire interval into n segments so that the step size h equal to b - a by n where of course your x i is a + i h where i is equal to 0123 and so on okay. So, we can apply Simpsons 1/3 formula to each of the segments okay and the segments are x 2y -2x 2y -1 and x 2i - 1x 2i and so on okay. So, which allows us to write this as h over 3 sum over I equal to 1to N by 2so if you have n divisions you have n by 2 segments okay.

So, that is why the sum goes over to 1to n by 2 and if it is the first 1that is taken once 2i – 2 that is the leftmost point and then there is a 4 times the middle point which is x 2i - 1 + f x 2i and so on. So, this is the thing and this can be written as on a fresh page.

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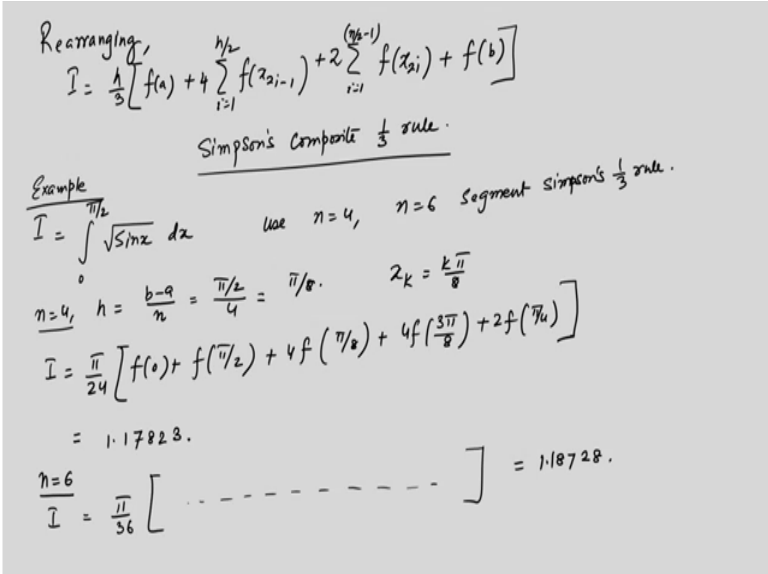

So, if you rearranged then you are so h over 3 f of a + 4 times I equal to 1to n by 2 f of x 2i - 1+ 2 i equal to 1to n by 2- 1f x this is x, x 2i and then f of b ok. So, this is the Simpsons a composite 1/3 ok. So, let me see a quick example of this so again if you want h or rather I equal to there is an example so I equal to 0 to PI by 2 root over sine x dx which we have done using just a 2 component or 2 segment formula.

So, now you use n equal to 4 and n equal to 6 segment Simpsons 1/3 rule, so your n is equal to 4 so h is equal to b - a by n this is equal to PI by 2 divided by 4 which is PI over 8 and so on and then one can calculate it I equal to PI over 24 and f of 0+ f of PI by 2 we are simply using this so your xk equal to k PI over 8, so it is 4 PI by 8+ 4 f 3 PI by 8+ 2 f PI over 4 and so on and this is equal to if you do it if you put this things carefully and do this f 0 is sine root over of sine 0, so sine 0 is 0 of course and then if it is PI by 2 sine PI by 2 is 1.

So, root over of 1is 1and so on so I am sort of skipping those intermediate steps this comes out as 1.17823. So, this is for n equal to 4 if you simply go to the n equal to 6 it becomes equal to its like PI over 36 and then I let you write all these terms for n equal to 6 there will be a large number of terms and this comes out as 1.18728 okay. So, this is so basically this is very, very close to the exact result.

And so a Simpsons 1/3 rule simply uses a second-order polynomial and builds up a formula from the width of the interval multiplied by the weighted average of the heights trapezoidal rule simply uses the average height of the that is the fa + fb by 2 that is the average height of the functions whereas this uses an average weighted average with a point of the middle and the point of the middle has a weight that is 4 times than the other 2 extreme points.

And here we see that how we can actually use n segment formula to improve the accuracy. So, the whole interval is actually split into n segments and we have here we of course have a even number of segments because we are taking one point at the middle and there is the only difference that it has with the 3/8 formula besides that the 3/8 formula may have slightly better accuracy.

But nevertheless Simpsons one-third rule is more used and we actually see, so this you fill it up and we actually see that the same sums one-third rule itself is very accurate and it is much better than the trapezoidal rule. And the reason is not too difficult to understand that you have taken a second-order polynomial which is a you have retained terms up to quadratic order. So, that the accuracy is improved ok.

So, this is the main idea behind doing these integrals will shortly discuss more methods but before we wind up for now let me write down the Simpson's 3/8 rule that is not going to be used but it is important for you to know for an academic interest.

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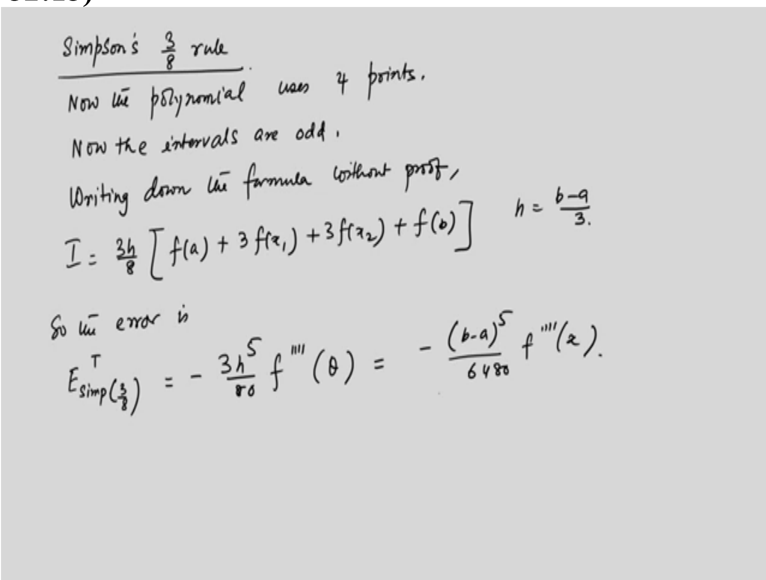

That there is a Simpson’s 3/8 rule which simply uses 4 points, so, now the polynomial uses 4 points instead of 3 okay. So, this is the thing and then and now the intervals are odd that is if you use 2 points in between then you have actually 3 intervals. So this is the thing so we will write down the formula without proof it is fa + 3 f of x 1+ 3 f of x 2+ f of b h equal to b - a by 3. So, the error is actually slightly better I mean slightly less error is E Simpsons and I will write in a bracket 3 by 8 and it is really the truncation error.

This is equal to - 3h over 5 by 80 and now I have again there are 4 so this is that theta that we were talking about between a and b. So, this if we convert this everything into h and all that it becomes equal to - h to the power 5 and so or let us just write b - a whole to the power 5 and

6480 and this if 4 times derivative this thing. So, if you remember will use so this number the denominator was 90 or not 90 in fact we have not written it.

So, this denominator was in fact so let me not confuse here, so if you write this thing so the denominator is actually 80 over 3 which is you know number which is so we have gotten slightly different denominator let us put it this way let us not compare and so on. So, this the accuracy is given by this anyway we are not going to discuss this in great details we will keep using the Simpsons 1/3 rule which is more you know sort of friendly and more accurate rather reasonably accurate for calculating numerical integration.