Introduction to LASER
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Lecture - 28
Ultimate Linewidth of a Laser

Welcome to this MOOC on LASERs. Today, we will see an important property called the Ultimate Linewidth of a Laser.

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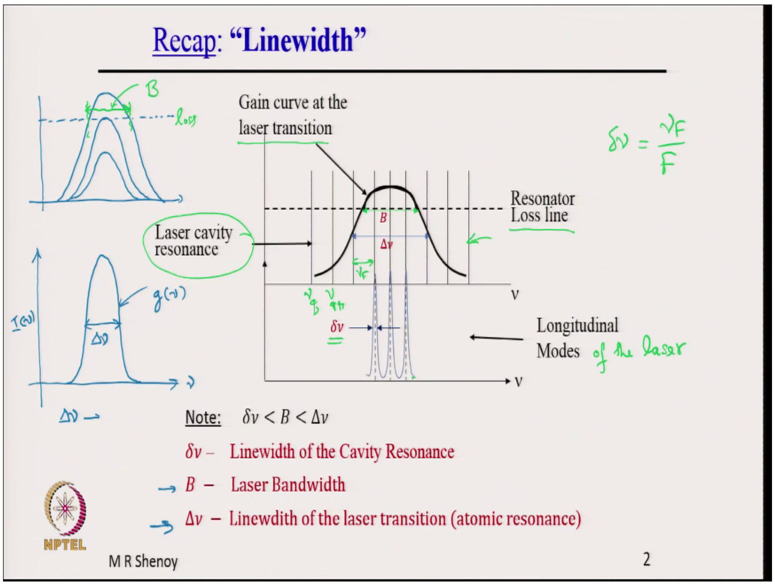

Now, a very quick recap of the linewidth. So, in this diagram, we have shown the three different linewidths that we have encountered so far. So, recall that if we have an atomic resonance, so if we have an atomic resonance, then the atomic resonance is correct. So, this is nu versus intensity of nu – I nu. Then the intensity distribution of a source corresponds to the atomic resonance which is characterized by g nu or the line shape function.

So, this could be for example, the spontaneous emission spectrum of a source, so that is why I have written intensity as a function of nu. And the linewidth of this is proportional to the linewidth of g of nu, and therefore, this is what we have called as delta nu. Delta nu is the linewidth of the laser transition any of the atomic resonances. This is the first linewidth that we have discussed about.

And we have seen the various line broadening mechanisms, and we have got expressions for delta of nu. So, various expressions Lorentzian, Gaussian and so on which give us the expression for delta nu. The second one is the laser bandwidth B, so that is illustrated here.

Delta nu is the full width at half maximum of the gain curve or the spontaneous emission spectrum. The second one is the laser bandwidth that is we know that if we pump a laser then depending on the pumping rate, the gain curve moves up and up for different pumping rates. And when the gain curve exceeds, the loss line a very quick recap.

So, this is the loss line. Then for all the frequencies in this range for all frequencies in this rage the gain is greater than the loss. And the laser can oscillate at frequencies provided it is allowed by the cavity resonance. So, this bandwidth, this is called the bandwidth, laser bandwidth B.

Laser bandwidth is the range of frequencies for which the gain in the amplifying medium is greater than loss in the resonator. So, this is the loss line. So, that is what is shown here again, we come back. So, this is the gain curve at the laser transition. So, gain curve at the laser transition. And this is the resonator loss line. Therefore, B here corresponds to the laser bandwidth. So, laser bandwidth is the range of frequencies for which the gain is greater than or equal to the resonator loss.

Now, then we saw that the vertical lines show the position of the cavity resonances. So, laser cavity resonance here. So, the vertical lines indicate the frequencies at which there are cavity resonances. And those cavity resonances, those frequencies for which gain is greater than loss can lace or can resonate and build up inside the resonator drawing energy from the laser medium.

And they form the longitudinal modes of the laser of the laser. If there were no gain medium, then if we had only a passive resonator that is the optical resonator, then all these frequencies, for example, if I call this as nu q, nu q plus 1, these are the various frequencies which can build up inside a resonator.

But only three frequencies in this diagram have gain greater than loss. And therefore, they can lace or laser action can occur at these three frequencies. Note that, delta nu here is the linewidth of these cavity resonances. We have already got an expression for delta nu. Delta nu is equal to nu F by F. Nu F is the free spectral range that is the separation between these.

So, this is nu F, nu F. And for high finesse resonators delta nu is equal to nu F by F. So, these are the three different linewidths that we have seen. Now, today we are discussing about the laser linewidth. What is this laser line? The actual linewidth of the laser. So, let us see further.

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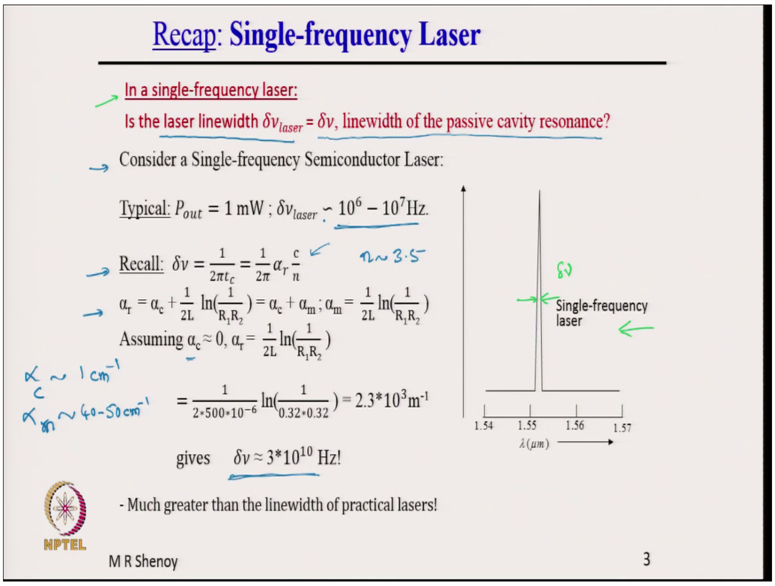

Now, in a single frequency laser, so let us recall is the laser linewidth delta nu laser equal to delta nu which is the linewidth of the passive resonance. So, look back here. In this particular laser, there are three longitudinal modes possible. And single frequency laser by definition has only one longitudinal mode and a transverse mode as well. And the spectrum is shown here. A single frequency laser has one longitudinal mode oscillating with a linewidth; its linewidth is delta nu here.

So, this is the linewidth of the single frequency laser which means the full width at half maximum. Delta nu is the linewidth of the passive cavity. So, note that delta nu is the linewidth of the passive cavity that is the resonator only without the medium. And we are asking the question if we have a single frequency laser which has only one longitudinal mode, then, is the linewidth of the laser equal to the linewidth of the passive cavity resonance?

Let us see this. Consider a single frequency semiconductor laser. Some typical values let us say P out is equal to 1 milli Watt, and delta nu laser that is the laser linewidth typically for single frequency lasers such as distributed feedback lasers or vertical cavity surface emitting lasers typically have linewidth of about 10 to the power of 6 to 10 to the power of 7 Hertz that is about 1 to 10 mega Hertz.

Now, recall that delta nu is equal to 1 by 2 pi t c which is equal to 1 by 2 pi into alpha r resonator loss coefficient into c by n. So, alpha r is equal to alpha c plus 1 by 2L into ln 1 by R 1 R 2. We have derived all these expressions which we call as alpha c plus alpha m, alpha m standing for the second term which is primarily determined by the reflectivities of the mirror.

Now, for a semiconductor laser, a very good quality semiconductor laser, usually alpha c is much smaller compared to the alpha m. Typical numbers of alpha c is of the order of 1 centimeter inverse here; 1 to 5 centimeter inverse. And alpha r is typically alpha m the mirror reflectivity is typically of the order of 40 to 50 centimeter. These are typical number. So, note that alpha c is much smaller for a semiconductor laser. And therefore, I have assumed it to be 0. And alpha r, therefore, is equal to 1 by 2L into ln 1 by R 1 R 2.

So, we substitute the numbers 2 times length. Length is typically 300 to 500 micrometer. So, 500 micrometer I have assumed, and ln the reflectivity of the cleaved ends of a semiconductor laser are 0.32 and 0.32 approximately. And this gives us a number 2.3 into 10 to the power of 3 meter inverse, so that is alpha r.

And that gives if we substitute in this expression here c velocity of light n is typically 3.5, n is the refractive index of a semiconductor which is approximately 3.5. If we substitute, we get delta nu is equal to 3 into 10 to the power of 10 Hertz. What is this? This is the linewidth of a single cavity resonance or a single longitudinal mode of the passive cavity that is when there is no gain, which is much greater than the linewidth of practical lasers.

So, practical lasers I have shown here exhibit typically linewidth of 10 power 6 to 10 power 7 Hertz single frequency lasers. And delta nu the cavity resonance linewidth is much larger. Therefore, what is the linewidth of the laser, and what determines the linewidth of the laser?

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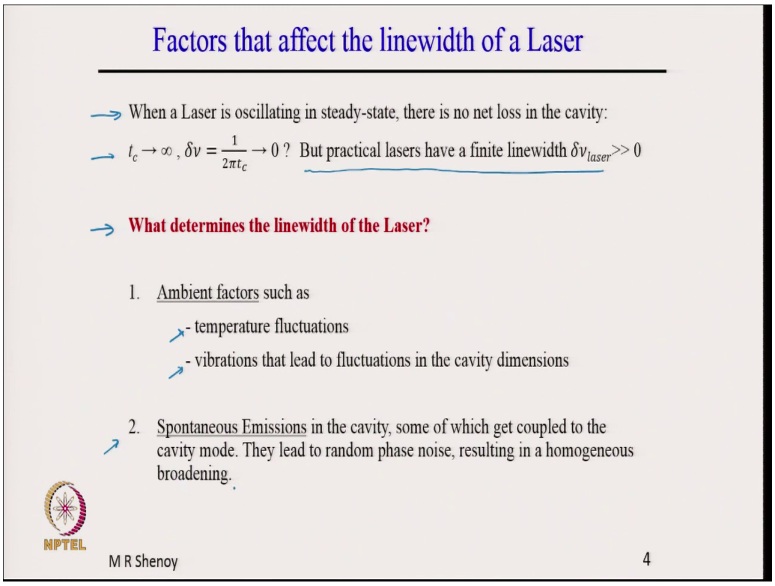

So, the question is when the laser is oscillating in steady state, there is no net loss in the cavity. Note that when the laser is oscillating in steady state, there is no net loss in the cavity because whatever loss was there in the resonator has been made up by the gain in the medium. And the lasing condition as we had seen is gain equal to loss.

Gain in the medium has to compensate for the loss of the resonator including the output power that we get. And therefore, there is no net loss when the laser is oscillated, that means, the cavity life time t c tends to infinity which means delta nu the linewidth equal to 1 by 2 pi t c we have derived all these expressions earlier tends to 0. But practical lasers have a finite linewidth.

There are two issues that we have discussed. One practical lasers have a linewidth which is much smaller than the linewidth of the cavity resonance. Delta nu is the linewidth of the cavity resonance. And delta nu laser is the actual linewidth of practical lasers. The second point that we see is delta nu should have been going to 0 if there is no net loss, but practical lasers have a finite linewidth delta nu laser which is much much greater than 0.

So, what determines the linewidth of the laser? There are two factors here. The first factor is ambient factors such as temperature fluctuations and vibrations that lead to fluctuations in the cavity dimensions. So, these fluctuations result in a fluctuation in the resonance frequency of the laser which is equivalent to having a finite linewidth of the laser.

The second point is more fundamental which is spontaneous emissions in the cavity some of which get coupled to the cavity mode. And they lead to random phase noise resulting in a homogeneous broadening, just like random collisions which led to a homogeneous broadening line broadening delta nu. Similarly, the random phase noises which are added due to the spontaneous emission into the cavity mode results in a finite linewidth.

So, this is a fundamental limitation of the linewidth. And this is what we call as the ultimate linewidth, alright.

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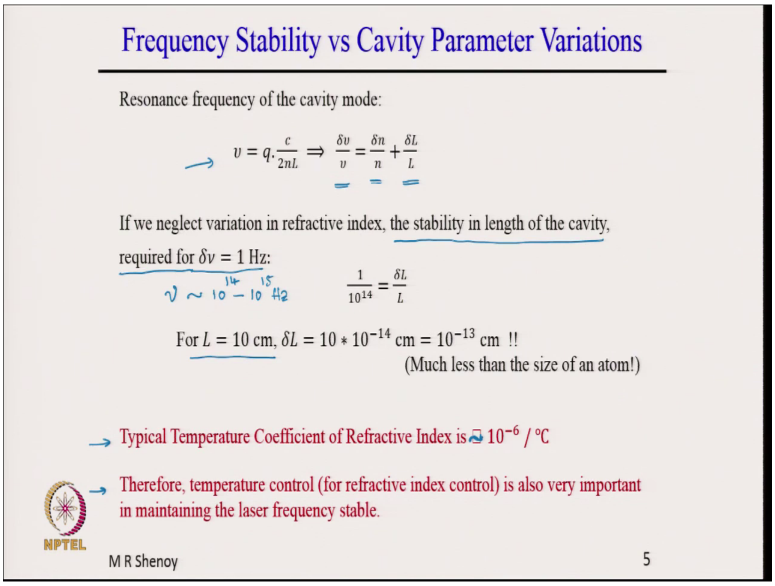

So, let us discuss this a little bit more, frequency stability versus cavity parameter variation. So, recall that the resonance frequency nu, this is frequency not velocity, resonance frequency nu is equal to q times c by 2nL. Therefore, if we write delta nu by nu that is if you differentiate both the sides, then we get it is equal to delta n by n plus delta L by L. The first term tells us about variation in the refractive index; this tells us about variation in the dimensions that is the length of the cavity.

What the expression tells is, what is the variation of the resonance frequency due to variations in the refractive index and variations in the dimension of the cavity. Now, first of all if we neglect the variation in the refractive index, for example, refractive index is a function of temperature, if we keep the temperature perfectly constant, then the refractive index is unlikely to vary for normal intensity levels. And therefore, we can neglect the first term on the right hand side. And we write delta nu by nu is equal to delta L by L.

So, what is the stability in length of the cavity required for a change in the resonance frequency that is delta nu equal to 1 Hertz. So, we substitute delta nu by nu. Nu is the resonance frequency which is of the order of 10 to the power of 14 to 10 to the power of 15 Hertz for visible light. So, 15 Hertz here that is for radiation optical radiation. And therefore, we have substituted nu which is equal to delta L by L.

So, if L is 10 centimeter the cavity length is 10 centimeter, then delta L will be equal to L divided by 10 to the power of 14 which comes out to be 10 to the power of minus 13 centimeters. See the number that is nuclear dimensions is much smaller than the size of an atom. The cavity length has to be maintained to an accuracy better than the size of an atom if you need to have a stability delta nu equal to or uncertainty delta nu equal to 1 Hertz for the resonance frequent alright.

The second point is typically we have temperature coefficient of refractive index for the materials which we used, for example, glass or nd yag rod. The coefficient is of the order of, so this is of the order of 10 to the power of minus 6 per degree centigrade. And therefore, temperature control is also very important because we have a delta n by n.

So, temperature control for keeping the refractive index constant is also very important in maintaining the laser frequency stable. So, from this expression, we have just picked up two points that is how the change in refractive index or fluctuations in refractive index value, or fluctuations in the cavity dimensions would lead to a corresponding delta nu. So, this is what we mean by ambient factors.

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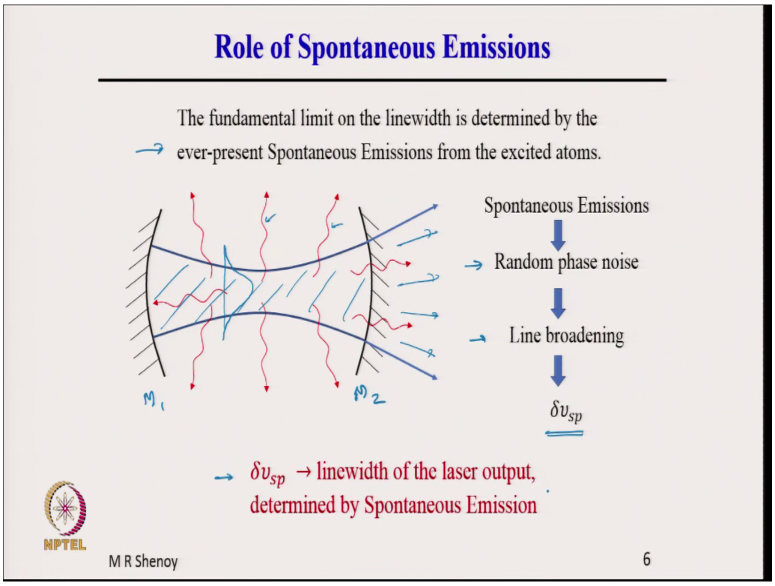

The second one is the role of spontaneous emissions which is more fundamental and it determines the ultimate linewidth of the laser. It is the fundamental limit on the linewidth is determined by the ever present spontaneous emissions from the excited atoms. So, as I mentioned that the spontaneous emissions lead to random phase noise. What is illustrated here? These arrows indicate the emitted photon going out of the resonator.

This one here is the laser beam the actual laser beam which is building up which is resonating back and forth, and then a part of it is coming out from the output mirror. So, this is mirror M 1 this is mirror M 2, this is the output which is coming. Now, look at the spontaneous emissions, spontaneous emission is random and he is also random in direction.

So, it is given out in all directions. Some of which may also add into the cavity mode, this is the cavity mode. So, this is a let us say if it is a fundamental mode of a spherical mirror resonator, it will have a Gaussian distribution here. And the beam is going back and forth, building up all the while and compensating for the output which is going out.

Now, some spontaneous emissions can also be added to the cavity mode if they are given in phase or if they are given in the same direction. Therefore, when it comes out here, the output will contain some amount of spontaneous emission which is random, which has no phase correlation with the stimulated emission output which is coming from mirror M 2. And that is what is indicated here.

The spontaneous emissions add random phase noise to the output which leads to a line broadening and corresponding to that the linewidth is denoted as delta nu sp. So, the delta nu sp is the linewidth of the laser output determined by spontaneous emission. Let us discuss this more.

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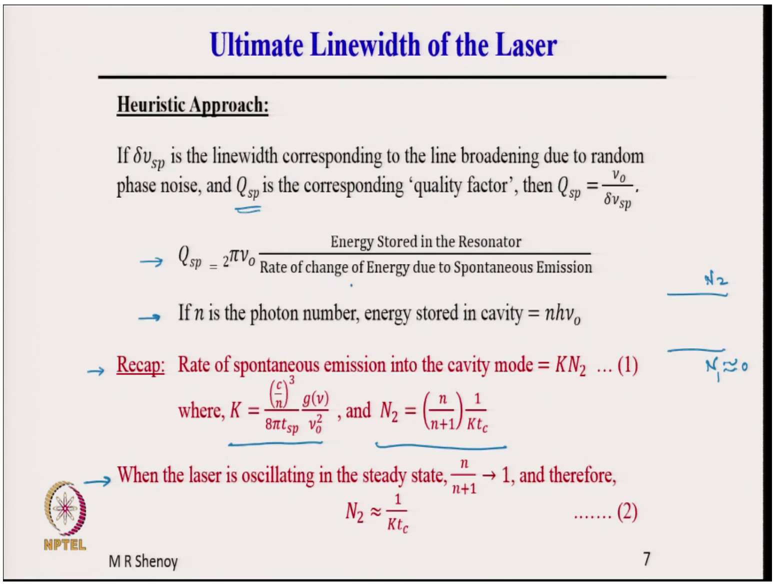

There is a heuristic approach presented here to get an expression for the delta nu sp. It is a heuristic approach means it is to stimulate interest to find out the rigorous way of solving this. So, its a if delta nu sp is the linewidth corresponding to the line broadening due to random phase noise, and if Q sp is the corresponding quality factor then we know that Q sp can be written as nu 0 that is the cavity resonance frequency divided by delta nu sp that is the linewidth corresponding to the spontaneous emissions.

And therefore, we define Q sp as 2 pi nu 0 into energy stored in the resonator this is the definition of quality factor Q divided by rate of change of energy, now we are saying Q sp is because of spontaneous emission, and therefore, rate of change of energy due to spontaneous emission. If n is the number of photons, then the energy stored in the cavity is n h nu 0.

A very quick recap we had these expressions and derivations when we did variation of laser power around threshold. Please check that lecture variation of laser power around threshold. We had derived that the rate of spontaneous emission into the cavity mode is equal to K times N 2, where N 2 is the number of atoms in the excited stage.

So, we considered an equivalent 2 level systems from a 4 level system we had made an equivalent simplified 2 level system with N 1 nearly equal to 0. And therefore, the rate of spontaneous emission in the cavity mode is given by K times N 2, where K is this expression here. And N 2 was shown to be equal to n by n plus 1 into 1 by Kt c.

When the laser is oscillating in steady state typical number of photons n is 10 to the power of 10, 10 to the power of 12 and so on. And therefore, n by n plus 1 is almost equal to 1. And therefore, N 2 is nearly equal to 1 by Kt c equation 2.

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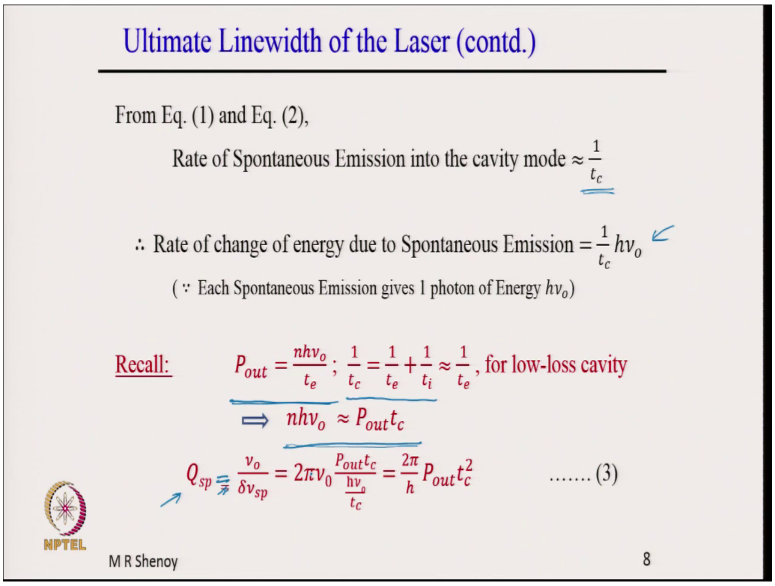

From 1 and 2, we see that the rate of spontaneous emission into the cavity mode is 1 by t c 1 and 2. So, substitute for N 2 here, K, K cancels. And therefore, rate of spontaneous emission into the cavity is 1 by t c. And therefore, the rate of change of energy due to spontaneous emission, so that is what we need rate of change of energy due to spontaneous emission in the definition in the denominator we have rate of change of energy due to spontaneous emission.

So, we have first found out rate of change of spontaneous emission. And therefore, rate of change of energy due to spontaneous emission is that 1 by t c multiplied by energy of 1 photon, each spontaneous emission gives 1 photon of energy h nu 0 into the cavity mode.

Again recall that the output power is given by n h nu 0 by t e, where n is the number of photons in the cavity, nu 0 is the resonance frequency, and t e is the time constant associated with external coupling. We had also had an expression 1 by t c is equal to 1 by t e plus 1 by t

i. So, the second term is due to intrinsic losses in the cavity. And the first term 1 by t e is a time constant associated with output coupling.

As I have already mentioned usually 1 by t i is much smaller. And therefore, it can be neglected in comparison to 1 by t e, just as I explained that alpha c was neglected in comparison to alpha m. And therefore, we write this as of the order of 1 by t e, nearly equal to 1 by t e. And therefore, P out is equal to 1 by t e here. So, we have taken 1 by t c nearly equal to 1 by t e which means t e equal to t c.

And therefore, if we replace this t e by t c, please remember this is a heuristic approach n h nu 0 is equal to P out into t c. Now, we substitute in the definition of Q sp which is equal to this equal sign nu 0 by delta nu sp which is equal to 2 pi nu 0 into P out into t c because for n h nu 0 we have substituted P out into t c divided by h nu 0 by t c that is the rate of change of energy from here, and that gives 2 pi h into P out into t c square; nu 0, nu 0 cancels. Let me call this as equation 3.

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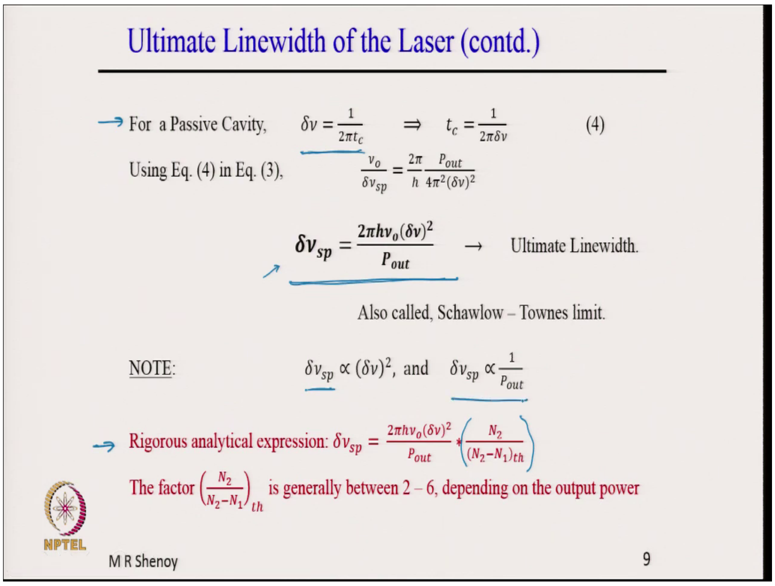

For a passive cavity, the linewidth of the cavity resonances delta nu are given by the expression 1 by 2 pi t c which means t c is equal to 1 by 2 pi delta nu. Substitute this in equation 3 and we get nu 0 by delta nu sp is equal to 2 pi h into P out. And for t c, we have substituted this t c was in the numerator t c square. So, 1 by 4 pi square into delta nu square. In other words, delta nu sp is given by an expression like this. This is the ultimate linewidth of a laser which is determined by spontaneous emission.

This is also called the Schawlow-Townes limit because they were the ones who gave it given expression for the first time or modified Schawlow-Townes limit. But the important point to note is delta nu sp is proportional to delta nu square that is the linewidth the limiting linewidth is proportional to square of the linewidth of the cavity resonances.

Therefore, smaller the delta nu, smaller will be delta nu sp. Therefore, indeed cavity linewidth determines the linewidth of the laser. So, although practical laser linewidth is not equal to the linewidth of the cavity resonance, it is one of the factors which determines the linewidth is the cavity resonance linewidth of a cavity resonance.

So, note that delta nu sp is proportional to delta nu square. And more importantly delta nu sp is inversely proportional to P out. What does this mean? This means that if the output power is increased delta nu sp the fundamental limit would reduce. What is indicated here is a rigorous analytical approach classical approach would give 2 pi h nu 0 into delta nu square by P out into a factor N 2 divided by N 2 minus N 1 threshold, just an additional factor.

This factor is generally between 2 to 6 depending on the output power. Larger the output power N 2 will be more. Note that N 2 minus N 1 threshold remains constant no matter what the output power is, but N 2 would increase and correspondingly N 1 would also increase because the difference has to remain constant.

Therefore, depending on the output power, this factor here the factor may be anywhere from 2 to 6. In other words, the correct analytical expression gives the same order as the expression that we have got by a heuristic approach.

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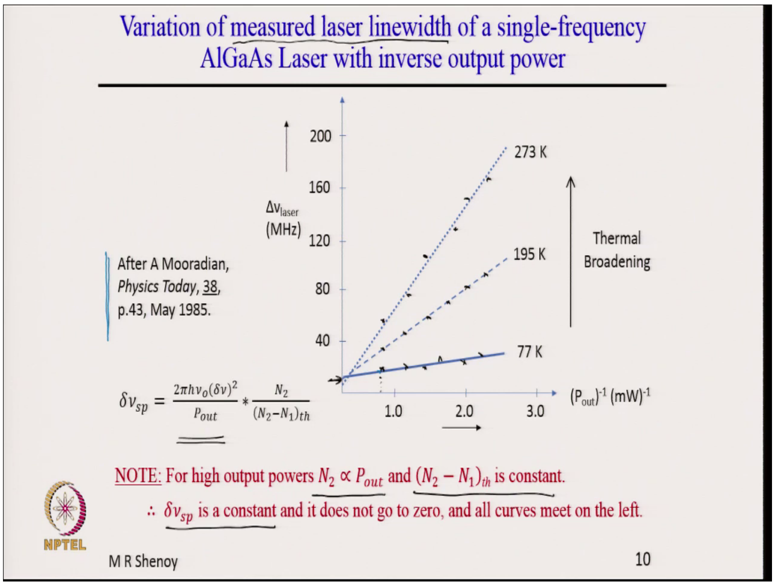

Now, what is shown in the graph is a test of this whether this delta nu sp is inversely proportional to P out. So, if we plot the linewidth of a laser, so what is shown here this is an experimental result from Mooradian which is reported here. So, it is very interesting paper gives much more details. So, what is shown is the linewidth of the laser measured linewidth of the laser – its a single frequency aluminium gallium arsenide laser semiconductor laser.

As a function of the inverse output power P out inverse because then we should get a straight line. And indeed qualitatively I have shown it as straight lines, but in the actual paper you would see the measured points as well. The measured points are given. Let me show here. The measured points are very, very close.

I am just qualitatively showing in all cases it is measured at a three different steady state temperature of the semiconductor laser. In all cases the measured points where I am qualitatively putting the points here just to indicate that these are actually the measured laser linewidth. So, variation of the measured laser linewidth of a semiconductor laser – single frequency semiconductor laser with input power which really proves this point that the linewidth is inversely proportional to P out.

Note that for high output powers, N 2 is proportional to P out. So, this will be a factor into P out, and then P out, P out cancels. And therefore, when P out becomes very large 1 by P out will tend to 0 here, but we should have had delta nu sp going to 0 according to this expression, but delta nu sp does not go to 0, but it saturates or it comes to some finite value because N 2 is proportional to P out.

Therefore, P out, P out cancels, and anyhow N 2 minus N 1 is constant. And therefore, it comes down to a constant value. So, at high output powers N 2 is and this is constant. And therefore, delta nu sp is a constant. It does not go to 0 and all curves almost meet when they are extrapolated.

Please see that the points beyond this because this corresponds to extremely high powers very high power, only at very high powers 1 by P out is 0 and the extrapolated points are all come to almost to the same point in the measurement.

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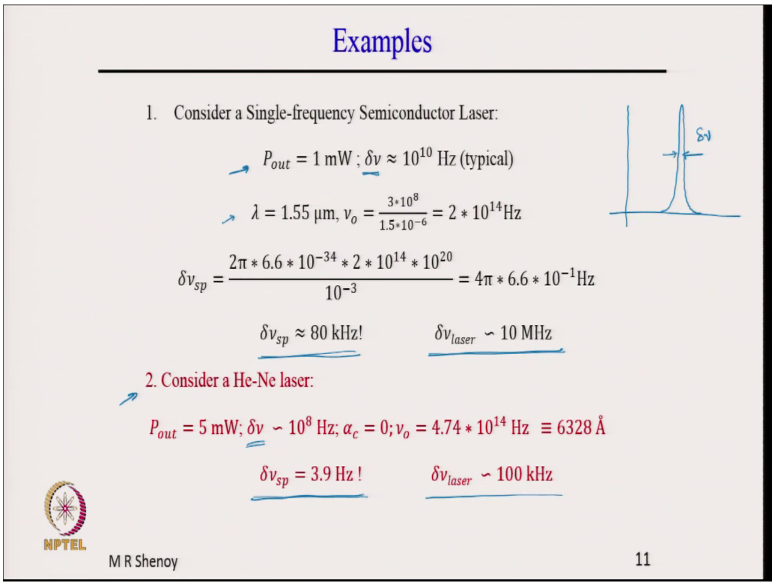

Now, let me put some numbers and take some examples. So, consider a single frequency semiconductor laser. So, the P out is typically 1 milli Watt. Assume that delta nu, I had calculated. This delta nu is always the linewidth of the cavity resonance. So, delta nu always refers to the linewidth of the cavity resonance. So, consistently we are used the notations so delta nu for this is equal to nu F by F which we had derived earlier.

So, delta nu that linewidth of the cavity resonance is typically this much. And for a laser at 1.55 micrometer, this is the frequency nu 0. So, if we substitute in the expression, this is the expression that we had derived, heuristically we had shown is 2 pi into h into 2 into 10 to the power of 14, this is the frequency into delta nu square delta nu is 10 power 10. So, this is delta nu square divided by 10 to the power of minus 3. What is this? There is 1 milli Watt power P out 10 to the power of minus 3.

So, we get this value here. So, if we multiply this, we get delta nu sp approximately 80 kilo Hertz. And we had already mentioned that the delta nu laser is typically 10 mega Hertz for a single frequency semiconductor laser. So, delta nu laser is of course much greater than delta nu sp.

If we consider a He-Ne laser here typical numbers again P out 5 milli Watt, delta nu for the cavity resonance is 10 power 8 Hertz, alpha c is approximately 0 – it is a gas laser – normally it is very close to 0. And this is the frequency for this wavelength 6328 Anstrong for the helium-neon laser. And delta nu sp comes out to be 3.9 Hertz. And practical single frequency lasers helium-neon lasers have a linewidth of tens of kilo Hertz typically of the order of 100 killo Hertz.

So, what is the point in these numbers? The numbers says that the delta nu sp – the theoretical limit is much smaller compared to those of the practical laser linewidth, but these are much smaller than the cavity resonance delta nu. So, the cavity resonance linewidth line width of the cavity resonances is much larger than the linewidth of practical lasers.

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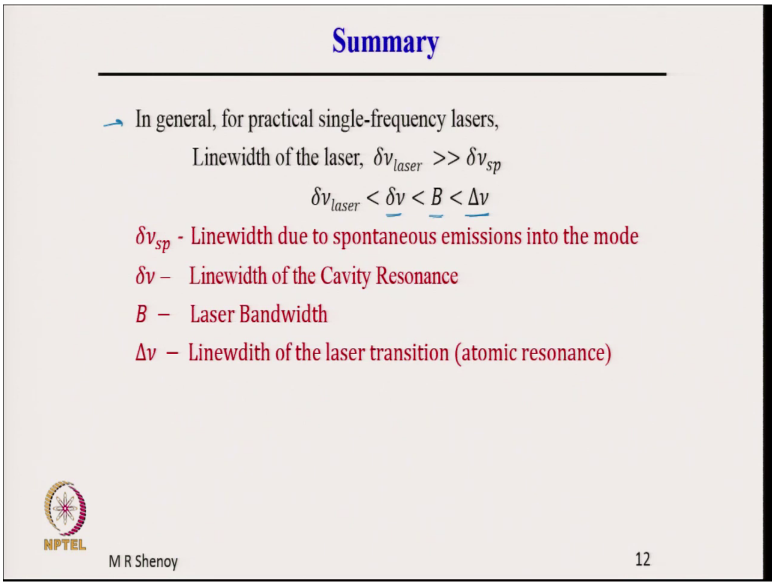

So, the summary is in general for practical single frequency lasers, the linewidth of the laser delta nu laser is much greater than delta nu sp. The limit determined by the spontaneous emission or the ultimate linewidth. Whereas, delta nu laser is smaller than delta nu due to cavity resonances which is again smaller than the amplifier bandwidth and is smaller than the linewidth of the transition.

So, delta nu is the linewidth of the laser transition or the atomic resonance. B is the laser bandwidth. This is the discussion that I shared in the first slide. And delta nu is the linewidth of the cavity resonances.

So, this we summarize the properties of the laser output. And in the next lecture, we will take up pulsed lasers.

Thank you.