Introduction to LASER

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Part – III: Optical Resonators
Lecture - 13
Resonance Frequencies

Welcome to this MOOC on LASERs. So, today we will see part-3 of the course, we have already seen in part-1, we saw interaction of radiation with matter; we discussed about the Einstein coefficients. And then we saw the condition for amplification by stimulated emission.

In part-2 then we worked out the laser rate equations and we obtained the various schemes possible schemes of pumping to obtain amplification. Now, today we will see part-3 optical resonators and in that the first talk would be on Resonance Frequencies.

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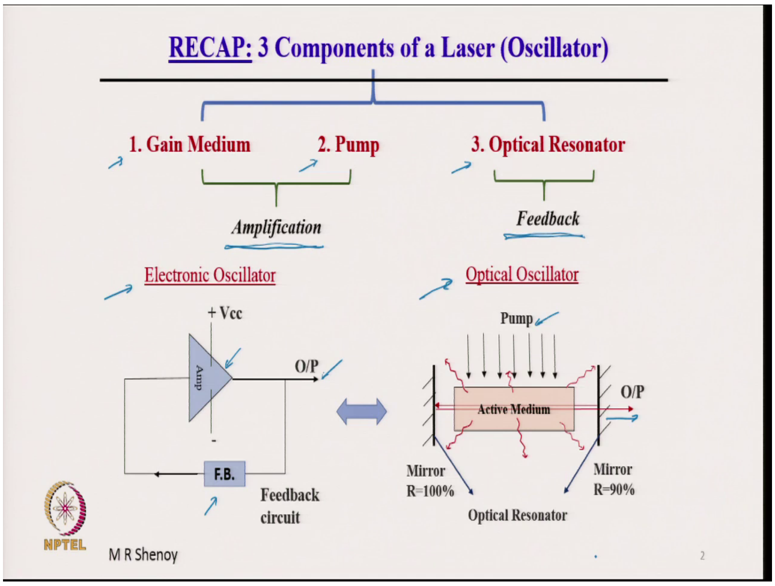

A very quick recap the 3 components of a laser are; the gain medium, the pump, and the optical oscillator or optical resonator. The gain medium when pumped appropriately provides amplification this is what we had seen in part-1 and 2. A gain medium which when pumped appropriately; appropriately here refers to a proper scheme to create population inversion.

And the optical resonator provides the necessary feedback to realize a coherent source which is the laser. Again a very quick recap of the equivalence between an electronic oscillator and an optical oscillator; electronic oscillator as a source of RF and optical oscillator as a source of light or source of radiation at the optical frequencies.

So, there is a active device here which is pumped or powered by a power supply. And there is a feedback circuit which provides feedback to the active device. And this results in a steady state oscillation and the output comes from here. So, output RF comes from here and depending on the feedback conditions, change in the feedback conditions, the output frequency can be varied.

Similarly, in the case of an optical oscillator, there is a optical source, oscillator here refers to the source. The active medium is pumped suitably and the resonator optical resonator which usually comprises of two mirrors – it may be plane mirrors or spherical mirrors provide the necessary feedback that is a part of the energy is fed back into the gain medium and a part goes out which forms the useful output power.

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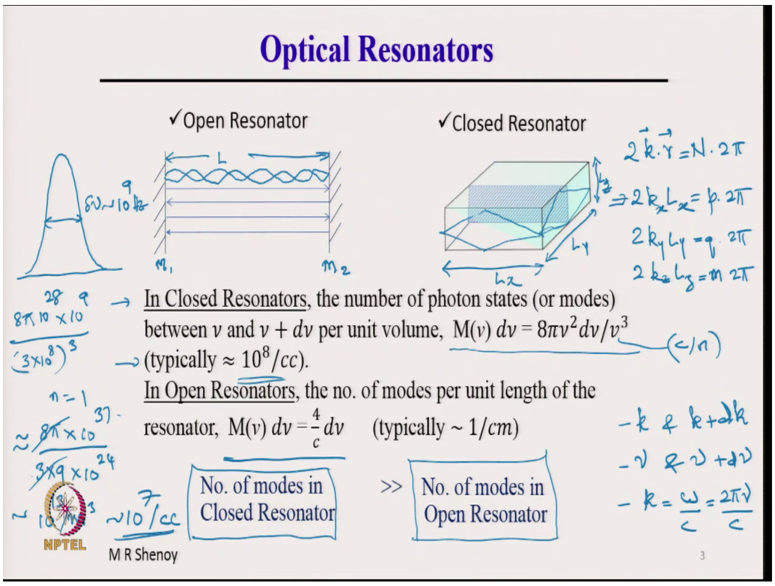

Optical resonators are broadly classified into two categories; open resonators and closed resonators. What is illustrated here as a open resonator? Comprises of two plane mirrors here of M 1 and M 2 with a separation L between them. And light can go back and forth if the plane mirrors are parallel, then the light which is perpendicular to the mirror can go back and forth and build up inside the resonator.

It is an open resonator in which only one type of path is possible. Looking at it from the ray point of view from a ray picture only one type of rays which are incident perpendicular to the mirror will resonate or will build up by going back and forth inside the resonator.

In a closed resonator, this is like a solid let us say a cube where different ray paths from a purely form a ray optics approach different ray paths are possible which can result in standing waves inside the resonator. For example here, if I were to show standing waves, the standing waves would comprise of corresponding to the rays which are going I am just illustrating.

In the case of a cubicle or a closed resonator, for example, a crystal, then we can have different types of paths ray paths. Here there is only one type of ray path, but here you can have many many types of ray paths. For example, there can be rays which if the two sides are reflecting, then the rays can go back and forth and form a complete loop like this.

The rays can also travel in this direction and form a complete loop. Every complete loop refers corresponds to a standing wave. There can be various types of ray paths one can think of. In other words, in a closed resonator the number of or the types of ray paths which are possible are many many, but that is a simplistic ray picture.

But actually in a closed resonator, the number of photon states or modes between frequencies nu and nu plus d nu per unit volume is given by M nu d nu is equal to 8 pi nu square d nu divided by v cube. So, this is v which is c by n, so v cube.

This can be derived the density of states or density of modes, this can be derived by first working out the standing wave condition which means you should have twice k dot r is equal to some integral N type is 2 pi. So, round trip k dot r, this implies you have 2 k x into L x where L x is the, so this if this is the x-direction, then this is L x, this is L y, and this is L z.

Then 2 k x each one of them this must be equal to p times 2 pi 2 times k y L y equal to q times 2 pi, where p, q and m or L are integers. So, 2 times k z L z is equal to let me use a integer m times 2 pi. Now, from this you can find out the density of states which is given by the number of states between k and k plus dk. So, k and k plus dk.

I am not giving the derivation here, but once you know the number of states per unit volume between k and k plus dk, you can determine the number of states between nu and nu plus d nu using the relation between k and so using the relation between k; k is equal to omega by c which is equal to 2 pi into nu by c. And therefore, using this, you can determine the number of states between nu and nu plus d nu.

If you work out this, then we get that the density of states m nu d nu comes out to be 8 pi nu square d nu divided by v cube. Typically, if we put some numbers for this, then we get about 10 to the power of 8 allowed states per unit volume. For example, if we take laser a laser material, then we know that this bandwidth over which amplification is possible. So, this delta nu is of the order of 10 to the power of 9 Hertz. Let us see let me put some numbers and work this out.

So, 8 pi into nu square light frequency may be 10 to the power of 14 Hertz. So, 10 to the power of 28 into d nu is 10 to the power of 9 divided by v cube c by n. So, c is 3 into 10 to the power of 8 c cube into n. Let me assume n is equal to 1. And then we can see that this will come out to be approximately, so this is per meter cube.

So, 8 pi into, so this is 8 pi into 10 to the power of 37 divided by 3 cube. So, that is 9 into 3, 3 into 9 into 10 to the power of 24. So, this will approximately cancel, because 3 is very close to pi and 9. So, this will cancel approximately. So, this is of the order of 37. So, 10 to the power of 13; 24 and 37 and therefore, this is of the order of 10 to the power of 13. This is meter, therefore, per meter cube; per meter cube.

And if you convert this, so this will into cc, this will come out to be approximately 10 to the power of 7 per cc. So, I have put here 10 to the power of 8 per cc depends on the actual this bandwidth may be 10 to the power of 10 Hertz. And in that case, this will come out to be 10 to the power of 8 per cc.

The point is the number of modes if you compare in open resonators, the number of modes per unit length open resonators are characterized by length only, closed resonator is like a material a piece of material which is characterized by volume that is why the density of modes is per unit volume.

When you have open resonator, then it is characterized by length and the number of modes per unit length of the resonator is given by an expression like this. And this will come out to be typically of the order of if one or that is a few modes per centimeter length. So, the point is the number of modes in a closed resonator is much much larger than the number of modes in a open resonator.

Why are we interested in this particular statement? The number of modes is large means the line width of the laser source would become relatively large. If the number of modes that is the allowed photon states are very few, that means, the corresponding frequency line width of the source would be very small. And we would like the laser to be as monochromatic as possible, and normally that is why laser resonators are usually open resonators.

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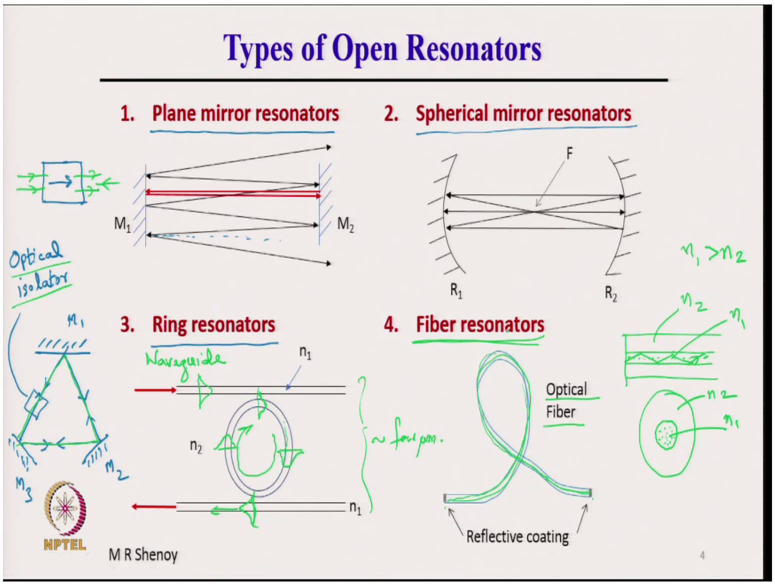

Types of open resonators. So, plane mirror resonator here, so which comprises of two plane mirrors M 1 and M 2. As you can see the red colored ray path which goes back and forth which is perpendicular to the two mirrors goes back and forth, rays travelling perpendicular to the mirror can get confined inside the resonator. Whereas, if a ray has a very small angle, even if it makes a very small inclination with the horizontal in this diagram, then after few round trips it will go out of the resonate.

If we take spherical mirror resonator on the other hand, then rays can be confined there are different ray paths which are possible. We will discuss this in a little bit more detail. So, a spherical mirror resonator comprises of two spherical mirrors usually concave mirrors, but one can also have one convex and one concave mirror to form stable resonators or where the rays can remain confined and build up with time.

There are ring resonators, ring configuration. One of the configuration is shown here where light enters. So, this is a; this is a waveguide configuration which means this is a planar waveguide here ok. Let me show a simpler ring resonator; a simpler ring resonator would comprise of three mirrors, for example.

Ring resonator basically means, so this is these are mirrors rays which is reflected here, reflected here, and then comes back. You can see that the ray path takes a complete round trip like this. It is a ring resonator, because the ray path follows a ring path or a circular path or a closed loop path.

So, this is a simple ring resonator comprising of three mirrors M 1, M 2, M 3. Note that there are several advantages in going to a ring resonator, what I have shown is a ray path which is traveling clockwise. But one could also have a ray path which is travelling anticlockwise in this direction, it is also possible.

But there are situations or there are experiments where you need the ray to travel in only one direction. In that case, one prefers a ring resonator. And you can have optical diodes, so optical diodes to block the reverse path.

So, if you put an isolator, optical isolator, it is called optical isolator. Optical isolator is basically a diode where which permits light to pass in one direction. So, it is a symbol is arrow showing like this which means light entering let me take a different color. So, light entering from here would pass through would be allowed. But light which is coming back in this direction will be blocked, and that is an optical isolator.

So, if we put an optical isolator, it is possible for us to have ray paths only in one direction travelling like this that is an advantage of a ring resonator. But if you try to put an optical isolator here, then the rays cannot go back and forth and they cannot build up, whereas, here it is possible to have a ray building up in one direction, travelling in one direction that is the advantage of a ring resonator.

What is shown here is a waveguide based; so this is a waveguide. Those of you are not familiar waveguide; this is a waveguide ring resonator. A waveguide comprises of. So, let me show here the waveguide. Optical fiber is here which is also a waveguide. So, waveguide is a structure which has a high index region which is surrounded by low index regions. So, this is n 2, this is n 1. And if n 1 is greater than n 2, then light can get trapped inside and propagate by total internal reflection.

It could be of circular geometry, then we like a cylindrical optical fiber. Then so this is a fiber we have already seen erbium doped fire amplified amplifiers where we have discussed about this. So, this is refractive index n 1; this is refractive index n 2. It could also be in a thin film geometry where planar geometry, so as shown here. So, it could be in a planar geometry.

Now, what is shown here is light entering the waveguide here. So, this is the mode of the waveguide. Light getting coupled at this point, light get thin coupled to this ring which is also a waveguide, it gets coupled here. And then it travels here and light can build up in this ring resonator. So, it can build up in this. And also every time a small fraction can get decoupled here, and then it comes out like this. So, light is building up in this ring, and then a small part is also given out at this end.

So, this is the resonator ring resonator, this is for input and this is for output ring resonator configuration. This is in a planar structure or on a substrate on a silicon substrate you can realize such ring resonators of very small dimensions, the whole dimension this dimension could be of the order of few micrometers. So, really micro resonators can be realized. But in a bulk form, you could also form such ring resonators using three mirrors, and the fourth one is fiber resonator.

Although in this case light is confined to the fiber optical fiber whether it is in the waveguide or in the fiber light is confined, still these qualify as open resonators, because there is only one path light from this end, there is a mirror, there is a reflector, light would travel all along like this, come here and then go back, reflected all along like this. But there is only one path.

Whether you keep the fiber looped or whether you keep the fiber straight it is just from one mirror to the other mirror and back, there is no other path possible. There is only one path which is possible unlike the case of the; unlike the case of open resonator where various ray paths are possible various paths are possible for light to go back and forth that is why an optical fiber resonator also qualifies as an open resonator. So, these are the four broad categories of open resonators very interesting type of resonators.

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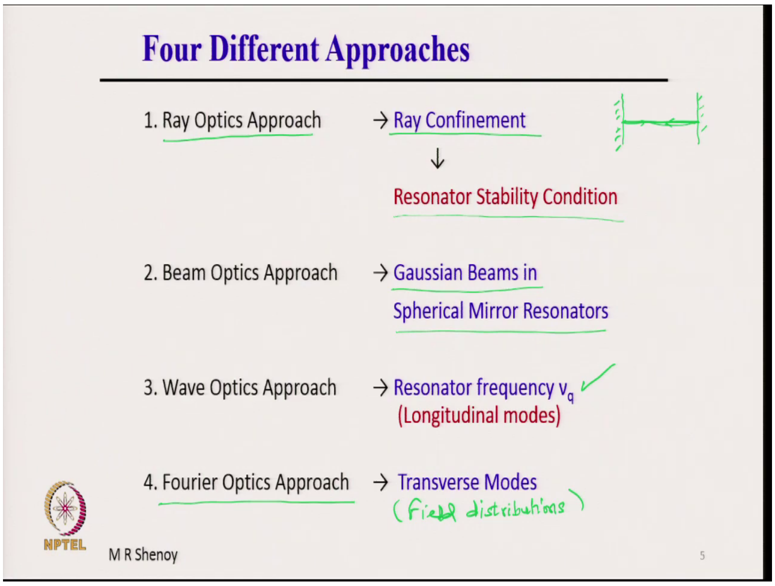

There are four different approaches which are used in the analysis of optical resonators. The first one is based on ray optics a ray optics approach, where resonance or rays resonate or build up if the rays are confined to the resonator or the resonance is characterized in terms of confinement of rays.

So, confinement of rays to the resonator which means if you had the plane mirror resonator, then here we are looking at only these rays which can build up. Any ray which goes at slight angle will go eventually will go out of the resonator. So, ray confinement means those rays which are confined to the resonator. Using the ray optics approach, we will obtain the resonator stability condition.

A second approach which is widely used is also called the beam optics approach. We will use the beam optics approach to determine the Gaussian beams in spherical mirror resonator. And the wave optics approach where propagation inside the resonator is treated in terms of waves which are going back and forth.

We will use this to determine the resonance frequencies or the longitudinal modes of the resonator. The transverse modes of the resonator or field distributions, these are the field distributions, allowed field distributions in the resonator distributions are usually determined using a Fourier optics approach.

So, in this course, I will briefly explain each one of the approaches depending on the convenience and ease one can use any one of the approaches. So, we will use the simplest approach to determine a certain parameter of the resonator or a certain characteristic of the resonator.

Accordingly using the ray optics approach, we will determine the stability condition, resonators are said to be stable if the rays are confined to the resonator. Then we will use the beam optics approach to determine the Gaussian beam of the resonator for spherical mirror resonators. And then using the Fourier optics approach, one can determine the transverse modes or field distributions of the resonator.

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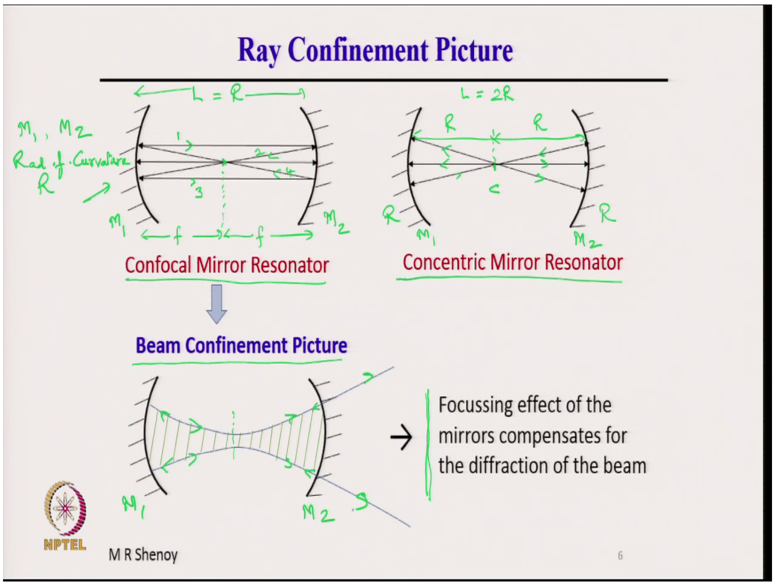

So, the ray confinement picture is illustrated here. In the ray confinement picture as you can see that what is shown here is called a confocal mirror resonator; confocal means the center point here that is the focal point of mirror M 1 and mirror M 2 coincide focal confocal same focus point of these two mirrors which means this distance here is f of the mirrors. So, this is f of each one of these mirrors – so confocal.

And therefore, for spherical mirror resonators f is equal to R by 2 that is where R is the radius of curvature. So, the separation here L is equal to radius of curvature of the mirror. So, if you have two, so M 1 and M 2 are identical that is spherical mirror resonators of radius of curvature R, so radius of curvature or ROC, radius of curvature R. In such a resonator, a particular type of rays confined R shown here.

For example, if a ray starts from here and goes parallel towards the other mirror, then any parallel ray which is incident on the spherical mirror will be rendered, so will be will pass through, after reflection will pass through the focal point. And from the focal point any ray which comes from the focal point and incident on the mirror will be rendered parallel.

So, this ray here path 1 travels towards mirror M 2; from here path 2, it travels towards mirror M 1. And from here again it travels path 3 to mirror M 2. And because it is a parallel ray which is travelling here it will be focused again through path 4 towards the focus. And then it comes back to its original point. In other words, all rays which travel like this will come back or they are indefinitely trapped inside the resonator, and that is called a ray confinement picture of the resonator.

If we have concentric mirror resonators, for example, we have already seen for the plane mirror resonator. Concentric means they have a common center of curvature that is the distance here for this mirror if radius of curvature is R, radius of curvature is R for the mirror M 1 and M 2, then the center of curvature C is here, then this separation is R, this separation is also R. And therefore, L is equal to 2 R, magnitude only we are talking of magnitude I will introduce the signs later.

Now, in this case, as you can see a ray which travels towards the center of curvature will any ray which comes from the center of curvature will be normal to the mirror and will be reflected back along the same path. So, any ray which is passing through the center of curvature will be reflected back along the same path, so that is what is being shown here.

In other words, the ray will get indefinitely trapped by going back and forth. So, this is called the ray confinement picture. So, the ray confinement picture which tells us that certain resonators can be stable resonators or which where the energy can build up or resonate. There is a beam confinement picture, where the beam is confined into the resonator. There is a spherical mirror resonator, spherical mirrors M 1 and M 2, a beam of light which is so this is a beam of light which is travelling.

So, let us say the beam travels from here. It slowly diffracts as it propagates. But when it reaches the mirror, the mirror refocuses it back. So, when it reaches the mirror, the mirror focuses it back. In other words, the beam can go back and forth, its diameter of the beam will change as it propagates back and forth. So, the diffraction effect of a finite beam is compensated by the focusing effect of the mirror, so that is what is stated here.

The focusing effect of the mirror compensates for the diffraction of the beam. And if the second mirror is partially reflecting, then a part of the energy goes out after every reflection.

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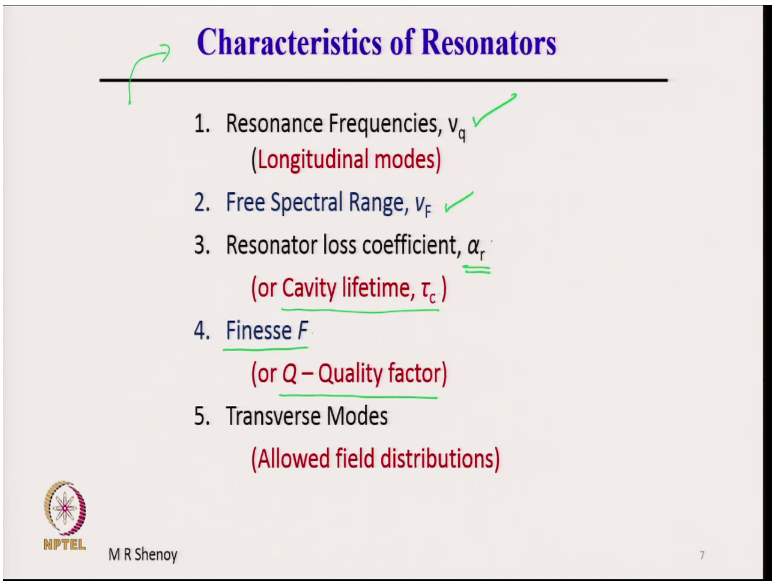

The characteristics of the laser resonator. I have listed here some characteristics, some important characteristics. First one the resonance frequency, a resonator means resonance frequency is characterized by resonance frequencies nu q. Resonance frequencies are also called longitudinal modes. We will see why it is called longitudinal modes. And there is a free spectral range nu f.

Resonators are also characterized by resonator loss coefficient alpha r or equivalently the cavity lifetime tau c, or equivalently finesse F, or equivalently the quality factor Q. The parameters alpha r, tau c, F and Q are all equivalent each one of them represents the loss in the resonator or characterized by the losses in the resonator.

A low loss resonator means it has a high finesse or a high quality factor. We will see this in detail. And finally, the transverse modes which represents allowed field distributions in the resonator.

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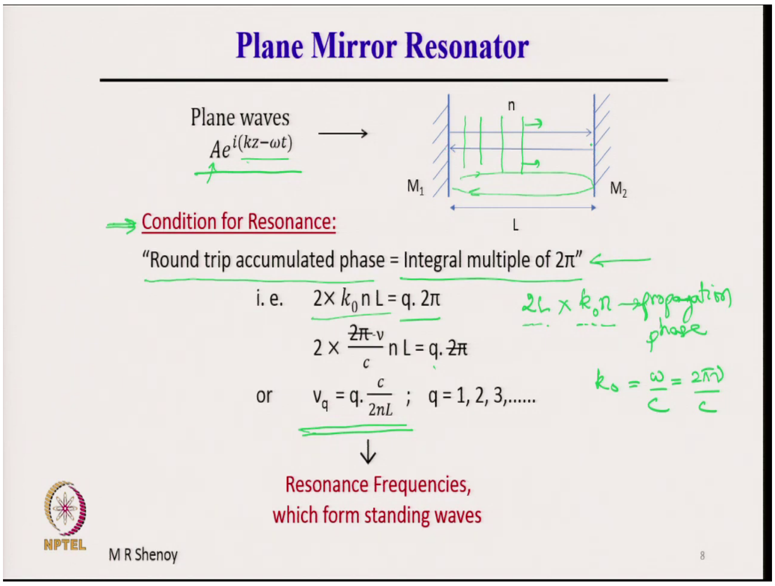

Now, let us look at the first characteristics that is the resonance frequencies. So, if we consider plane waves, this is a plane wave comprising this amplitude and a phase term k z minus omega t. So, a plane wave propagating, so let us look at the plane wave which is propagating here and getting reflected back from the other end.

Now, the condition for resonance; condition for resonance in resonators optical resonators is round trip accumulated phase must be equal to integral multiple of 2 pi, round trip accumulated phase equal to integral multiple of 2 pi. In this, this is true for any type of resonator.

Now, for the plane mirror resonator, the round trip is a distance covering 2 L that is back and forth. So, 2 L into the phase accumulated actually it is 2 L into multiplied by the phase constant k 0 into refractive index n of the medium, so that is what is written a twice k 0 into n L is the propagation phase.

So, this is the propagation phase. In propagating through one round trip, propagation phase; propagation phase is propagation distance multiplied by the phase constant or propagation constant k 0 into n. k 0 is the phase constant of free space and n is the refractive index. So, twice k 0 and L is equal to an integral multiple q is an integer, so integral multiple of 2 pi.

So, k 0 can be, k 0 is equal to omega by c which is equal to 2 pi into nu by c. One can also write the nu in terms of wavelength lambda, but this is fine. So, that, so 2 into 2 pi nu by c into n L, and this gives nu q. That is when the integer is qth order. So, this q is an integer. And this is called qth order resonance is given by nu q is equal to q times c by 2 n L, where q is equal to 1, 2, 3, 4 etcetera is an integer.

So, these are the resonance frequencies which form standing waves inside the resonator. When the round trip phase is an integral multiple of 2 pi, those frequencies which satisfy this condition will form standing waves inside the resonator.

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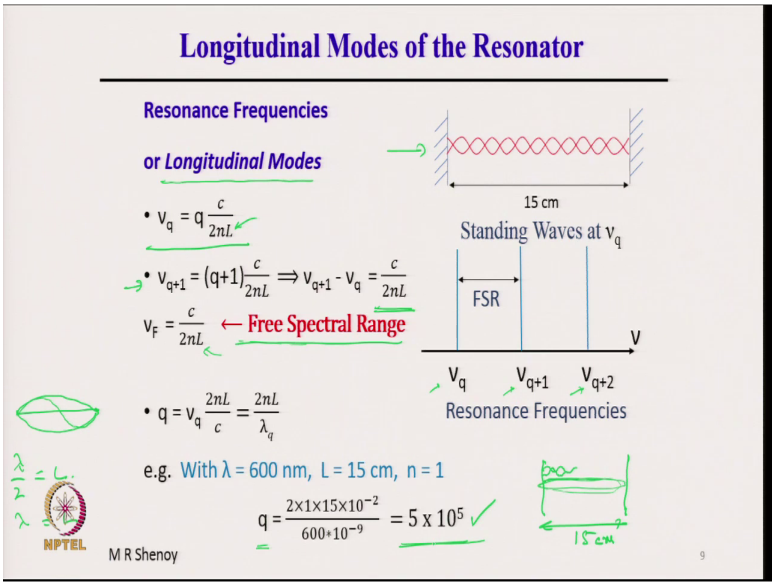

This illustrated here, so it is shown here. Resonance frequencies forming standing waves nu q is if nu q is equal to q times c by 2 n L, nu q plus 1 that is the next resonance. So, this is the qth order. If this is 100th one, then this is 101.

So, q plus 1 times c by 2 n L and therefore, nu q plus 1 minus nu q that is the separation. So, nu q here nu q plus 1, these are the resonance frequencies shown in the frequency axis. The separation between them is equal to c by 2 n L and this is called the free spectral range.

So, we have already seen two important characteristic of the resonator namely resonance frequencies nu q and the Free Spectral Range – FSR; nu f is equal to c by 2 n L. Note that the resonance frequency nu q is determined by the length of the resonator.

Similarly, nu f the free spectral range is determined by the length of the resonator rest are all constant. And therefore, it is these are also called longitudinal modes. Longitudinal modes or allowed solutions or resonance frequencies determined by the length of the resonator.

To get an idea about what kind of numbers are these q for the light waves. Normally, when we write q is equal to 1, 2, 3, etcetera, then we may try to imagine that it is q is equal to 1, 2, 3, etcetera, but in the case of light waves for practical resonators q will be a large number and that is why let us take an example here.

So, q therefore, is equal to just transpose. So, q is equal to nu q into 2 n L by c and the frequency is written as c by lambda and then we get 2 n L by lambda q. If we say that we are looking at radiation of wavelength lambda is equal to 600 nanometer, suppose 600 nanometer, so you have a resonator here, a resonator of 15 centimeter. So, 15 centimeter, and lambda at 600 nanometer is resonating or building up or forming standing waves here.

So, q what would be the number q? So, q represents the number of half wavelengths in this length, and lambda is equal to 600 nanometer, L is equal to 15 centimeter and n is equal to 1. If you substitute, note that q is 5 into 10 to the power of 5. In other words, there will be 5 into 10 to the power of 5 such loops. So, there will be. So, recall that if you take a string of length L, the first mode will be this, where the wavelength lambda is equal to or this is half the wavelength. So, lambda by 2 is equal to L.

If you take the next one, then lambda is equal to L or 2 times lambda by is equal to L. So, that is 2 times lambda by 2. So, such q, so if you take the next one, then the length will be divided into 3 such loops. And similarly in the case of light because the wavelength is very very small, q will come out to be large number. So, when we write nu q is equal to q into c by 2 n L, the q numbers in the case of light waves are very large numbers.

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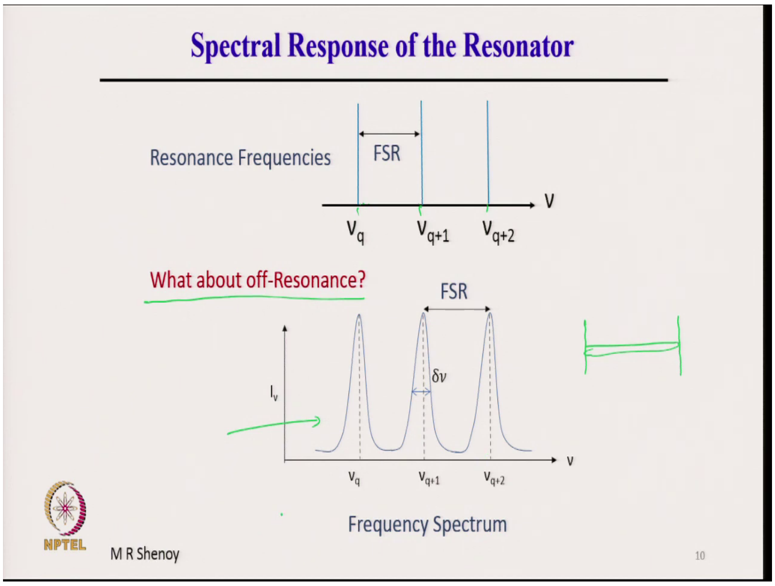

Finally, an important parameter or characteristic of a resonator is spectral response of the resonator. What does this mean? We will discuss this topic in detail in the next lecture. But what we need to see is this, the resonance frequencies represent frequencies which are building up inside the resonator. What about the frequency slightly away from the resonance? How much will they build up, or what will be the intensity of the frequencies which are off resonance? So, what about off resonance frequencies?

So, this will be given by the complete spectral response of the resonator which would look something like this that at the resonance frequency the intensity what is plotted is intensity versus frequency inside the resonator. So, at the resonance frequencies, the intensities will be maximum because they are perfectly building up at those frequencies. Energy is perfectly building up inside the resonator by going back and forth.

So, it is a complete, but frequencies which are slightly away, it is not suddenly zero like this. So, it also has a certain amount of intensity. And this is very important and that will be discussed in detail in the next class which is called the spectral response of the resonator.

Thank you.