Basic Quantum Mechanics

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Module No. # 07

Bra-Ket Algebra and Linear Harmonic Oscillator-II

Lecture No. # 5.

Coherent State &Relationship with the Classical Oscillator (Contd.)

In the previous lecture we were discussing the time evolution of the coherent state.

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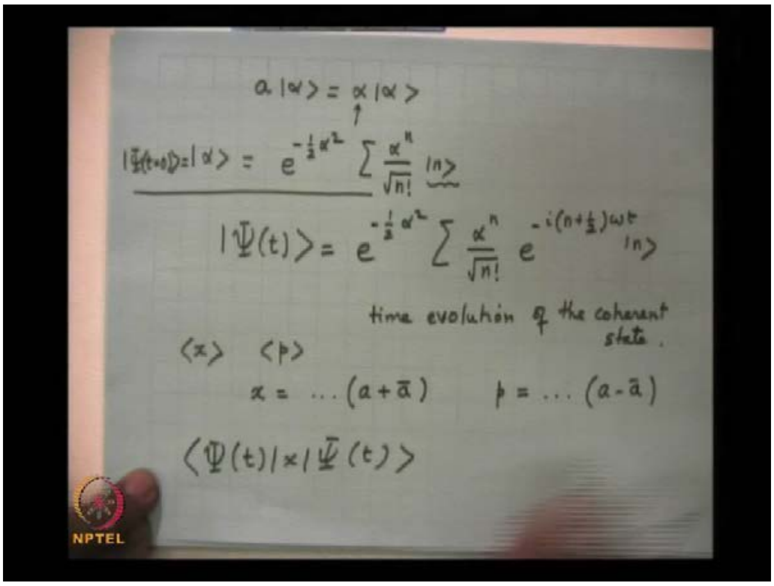

So, as we said that the coherent state were, the Eigen ket of the operator a and we had shown that alpha can take any complex value just for the sake of simplicity, we assume alpha to be real and we found that the normalized, Eigen kets was equal to a to the power of minus half alpha square summed alpha to the power of n factorial ket n.

Then we said that let the harmonic oscillator at t equal to 0 be in this state t equal to 0 let us suppose, it is in the coherent state then, how will the state evolve with time and the answer is that, if this is the state at t equal to 0 then psi of t at time t will be equal to since, these are the Eigen kets of the operator h E to the power of minus half alpha square summation alpha to the power of n factorial E to the power of minus i n plus half omega t ket n.

The above equation, this equation describes the time evolution of the coherent state. Now, for this state we wanted to we calculated, what is the expectation value of x and what would be the expectation value of p for that we express this, expressed x as a sum of a plus a bar and similarly, of p we express this as equal to something big multiplied by a minus a bar. Then, we evaluated the matrix elements that is we evaluated psi of t x psi of t.

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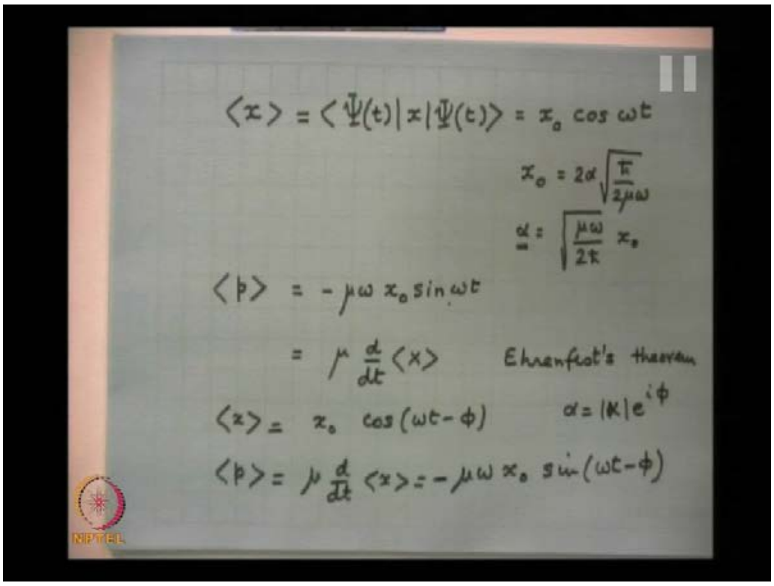

Similarly, we evaluated the expectation value of p and we finally, got the following results, that x was equal to x which is defined as equal to psi of t x psi of t this we showed equal to x naught cos omega t that x naught was equal to 2 alpha times h cross by 2 mu omega. Thus from this expression 1 can write down that alpha will be equal to mu omega by 2 h cross x naught. This is the Eigen value of the operator a then, we calculated the expectation value of p and we found that, this was equal to minus mu omega x naught sin omega t. When, we compare this equation with this we found that, this was equal to mu by d d t of the expectation values obeyed the classical equation of motion.

So, this as we know is the Ehrenfest's theorem that the expectation values, observe follow the classical set up equations. Now, I leave as an exercise that, if alpha was a complex quantity, if we had assumed alpha to be a complex quantity.

We still would have obtained the same result accepting that for x be here, instead of x naught, we would have got a phase factor I leave that as an exercise, where the phase factor is such, that alpha was equal to mod alpha psi mod alpha E to the power of i phi, but this relation would have been valid p is equal to mu d by d t x average, which is equal to minus mu omega x naught, but there will be a phase factor except for the phase factor, all the analysis will remain the same.

Now, we as we go back to my previous slide, we ask ourselves that, what is this coherent state, we say that this coherent state represents that of the classical oscillator. So, it is the super position of the Eigen states of the Hamiltonian then, we ask ourselves what is the typical n value.

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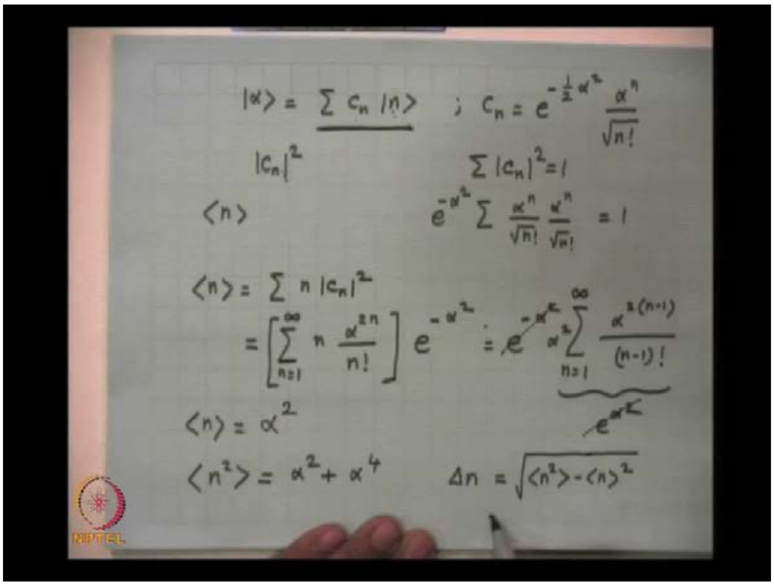

So, for that we had to calculate for this the expectation, you see we wrote down that ket alpha was equal to c n ket n. Where, c n is equal to E to the power of minus half alpha square alpha to the power of n by square root of n factorial. So, mod c n square represents the probability of finding it in the n-th state.

So, the classical oscillator is not in a particular energy state, it is in a super position of a very large number of states and that is what I want to make you understand, what are the numbers, that are involved how many states get excited is that just 1 energy state. Now, it is the super position of the states it is the super position of the state that the probability of finding in the n-th state is mod c n square.

So, how many number of states are get excited, we calculate the average value of n that gets excited. So, we of course from here, you can immediately show that mod c n square is of course1, because this state has to be found in 1 particular; the system has to be found in 1 particular state. So, you can immediately sum this mod c n square is E to the power of minus alpha square, and this will be alpha to the power of square root of n factorial alpha to the power of, if you square this, alpha to the power of n divided multiplied by n factorial.

So, this alpha to the power of 2 n by n factorial, that is E to the power of alpha square. So, that is 1 mod c n square is 1 and mod c n square represents the probability of finding, if the system is the coherent state the probability of finding. It will be n-th state what is the typical value of n let me calculate the average value of n. So, the average value of n will be just n mod c n square.

So, if I substitute that this will be n and I can take the alpha square outside, E to the power of n alpha to the power of 2 n by n factorial multiplied by E to the power of minus alpha square. So, this is n divided by n the first term is 0, because n equal to 0, this will be 0; this will be n equal to 1 to infinity here, I write down this, as E to the power of minus alpha square summation n equal to 1 to infinity and if I take, if I write this down, as alpha 2 to n minus 1.

So, there will be alpha square outside n minus 1 factorial, this is just I can write n minus 1 equal to n. This is just E to the power of alpha square, this term cancels out with this term and you will get finally, the result alpha square.

I leave as an exercise for you to show that the expectation value of n is alpha square exactly the similarly, way I can you can show that expectation value of n square is alpha square plus alpha to the power of 4. So, the spreads in n that is equal to I can define this as n square minus n average square, n square is equal to alpha square plus alpha 4 minus n average square that is alpha 4. So, this is equal to alpha and therefore, this is equal to square root of n factorial.

So, you have derived a very important result and we will discuss the physical significance of that in a moment.

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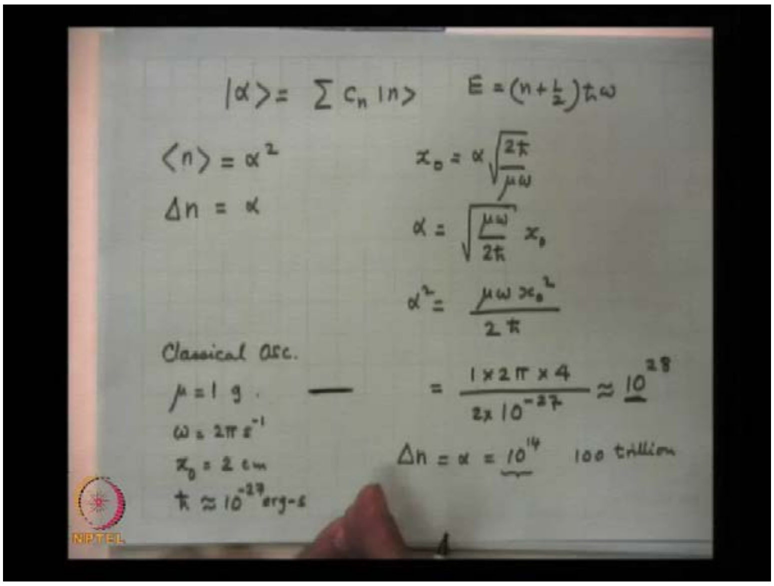

So, we have the coherent state is equal to c n ket n, I ask myself the question, what are the states that are getting excited? The average value of n is Alpha Square and the spread in the value of n is alpha.

Now, we had the expression for x naught as you recall x naught was equal to alpha under root of 2 h cross by mu omega. So, alpha was equal to under root of mu omega by 2 h cross x naught, alpha square was equal to mu omega x naught square by 2 h cross let me, take a classical oscillator of mass say 1 gram a time period the O S C pendulum the bob of the pendulum has mass of about 1 gram, it has a time period say 1 second.

So, omega is equal to 2 pi by t 2 pi second inverse x naught is let us suppose, the amplitude is 2 centimeter and h cross. We know this is about 10 to the power of minus 27 erg second, alpha square becomes 1 into 2 pi into 2 pi 2 square is 4 into 10 to the power of minus 27 into 2.

If, you work this out it is approximately equal to 10 to the power 28, the quantum numbers that, we are thinking of the average quantum number is 10 to the power of 28.

You could have said that, this I could have told you immediately, because the energy if I write as n plus half h cross omega then, the if I write E is equal to n plus half of h cross omega and I know that, the classical energy of the oscillator is half mu omega square x naught square.

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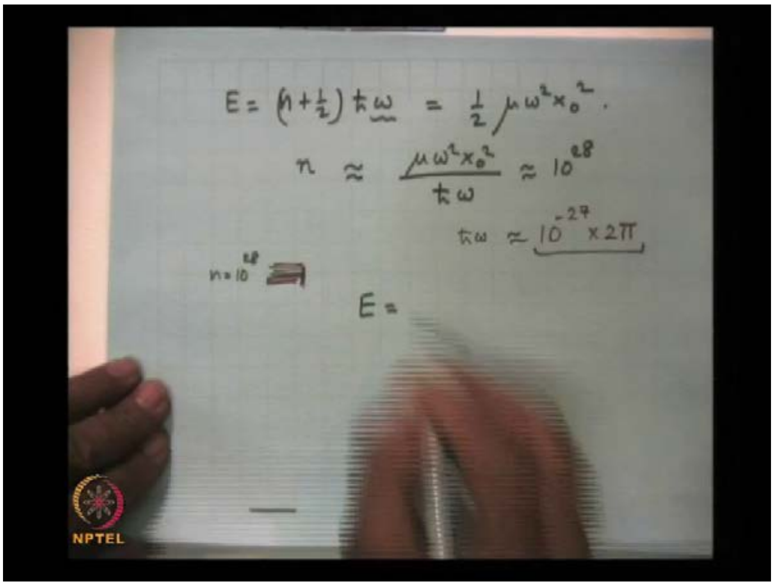

So, if omega is equal to 1 then, n the approximate value of n. You say, that the this is equal to mu omega square x naught square by h cross, if n is very large compare to h cross omega, and if you calculate. This you will find that, this will be 10 to the power of 28. So, the quantum numbers that are taking is a phenomenally large number, it is a phenomenally large number.

But the other thing that, I want to tell you are that, how many states get excited and if you calculate delta n, the spread in the value of n, that is alpha which is about 10 to the power 14. So, it is about hundred trillion states that get excited, but this number is extremely small, it is a trillion 100 trillion of this number. So, please appreciate what I am trying to say, if this is n equal to 0 then, this is n equal to 10 to the power of 10 to the 28.

Now, the energy levels are closely spaced here, that there are 10 to the power of 14 approximately 10 to the power 14 states, that get excited for the energy levels are closely spaced, what is the spacing the energy levels h cross omega.

So, h cross is about 10 to the power of minus 27 erg second and this is 2 pi erg. So, this is the energy, this is a very small amount of energy, although the quantum number are involved is large, this is my classical oscillator does it corresponds to a fixed energy. Now, it does not it corresponds to a 100 trillion number of super position of 100 trillion states, around the quantum number 10 to the power of 28, but this spread of energy is extremely small.

Let me, tell you 1 more thing this is slightly difficult, but once you understand then, this is the bohr corresponds principle, that in the limit of large quantum numbers, the system behaves like a classical system.

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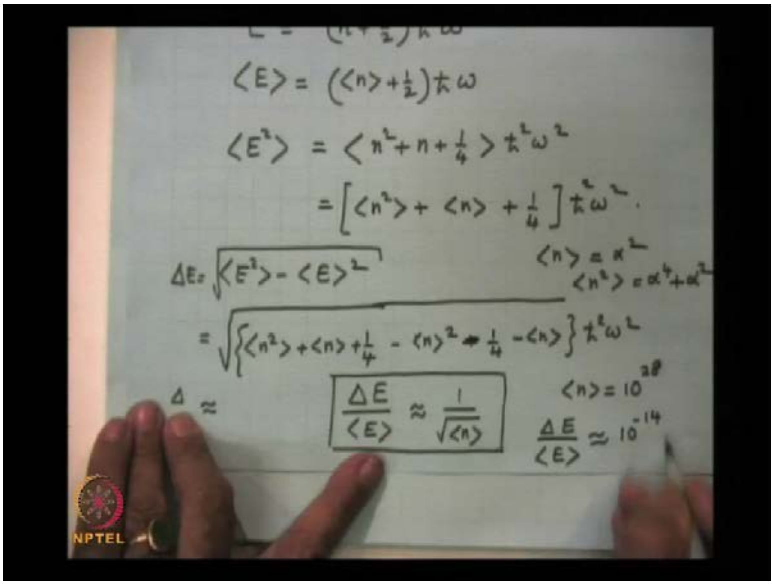

So, let me write down the expression for energy is equal to let me write it down in a fresh page, E is equal to n plus half h cross omega. So, the expectation value of E is equal to the expectation value of n plus half of course, half will be very small in comparison to that, but even then this is the expectation value of E, what is the spread in the value of E I say calculate E square.

So, E square will be average value of if I square this n square plus n plus quarter h cross square omega square. So, this will be n square average plus n average plus quarter h cross square omega square. Now, I had just shown that the average value of n was all first square.

So, E square the spread in the energy is will be E square minus E average whole square under the root. So, this will be if I take the square root E square will be n square plus n plus 1 by 4 of course, 1 by 4 I can neglect minus E average square minus n average square minus minus this square of that is plus minus 1 by 4 minus n whole thing cross square omega square.

Then, we have calculated that n was equal to alpha square and n square was equal to alpha 4 plus alpha square, I just substitute this and carry out and you will find that this will be equal to delta. You finally, we will obtain delta E by average value of E will be equal to 1 over square root of n. So, this will come out to be the spread in the energy will come out to be something.

So, if n is equal to 10 to the power of 28 then delta E by the expectation the spread in the E will be about 10 to the power of minus 14. So, the spread in the energy is extremely, small the spread in the energy is extremely, small let me give, you what I have trying to explain I have plotted here.

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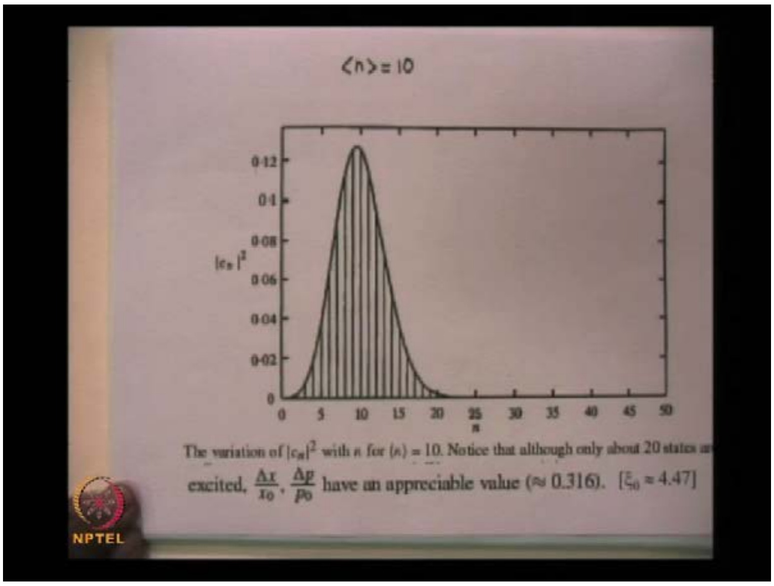

You see this is a coherent state for which the average value of n is 10. So, there are about 15 odd states which can get excited 15 discrete states and the vertical lines. I hope you can see them the vertical lines on this figure are the probabilities and if you add them up c 1 c 0 square plus c 1 square plus c 2 square plus c 4 3. So, on they will add up to 1.

So, for example, for c 10 squares is about 0 point 12. So, there is a 12 percentage probability, it is in the super post state. It does not correspond to a particular energy, there is a 12 chance that it will be in the n equal to 10th state.

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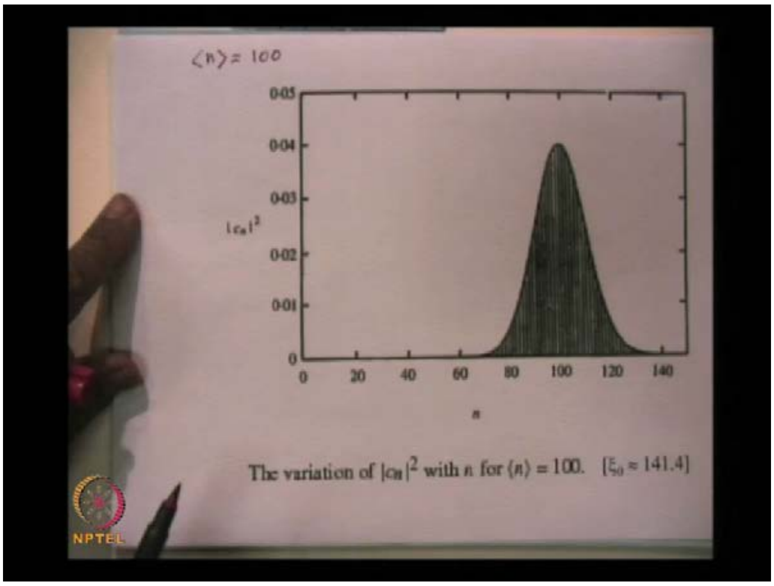

So, there are there is a certain spread now, when we have let us suppose, n equal to 100 then, there is about the average value of n is 100, but the spread is around 10 the spread is around 10.

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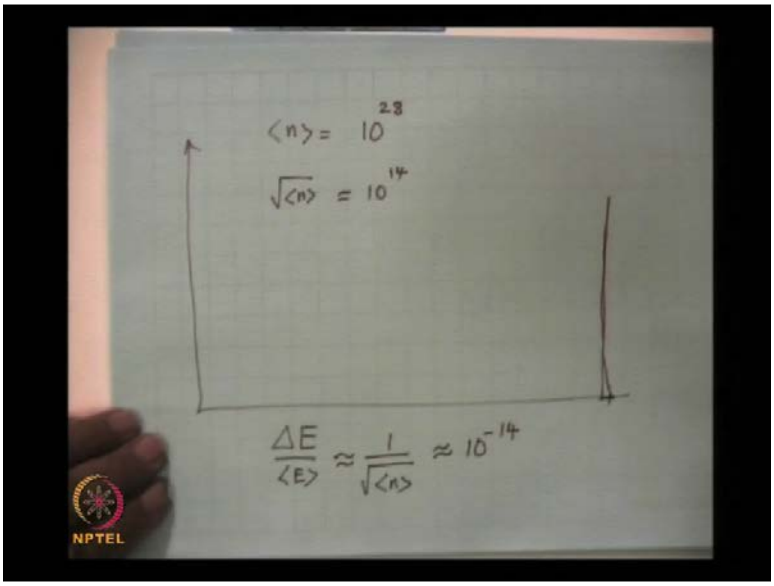

On the other hand, if n is of the order of 10 to the power of 28 then, the spread in the value of n is about 10 to the power of 14. So, that in this state, if you plot it will be a very thin line is spread in the E is extremely, small in comparison to the E. So, that is what a classical oscillator is delta E by E is approximately, very closely to square root of n. The average value of n and if my average value of n is 10 to the power of 28. Then, this will come out to be 10 to the power of minus 14.

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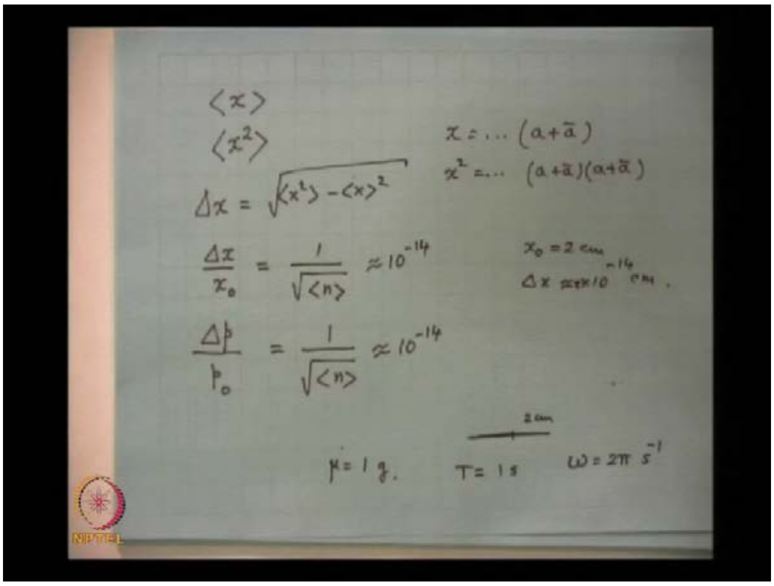

I will I leave this as an exercise for you that, we had calculated the average value of x, I suggest that, you calculate the value of x square also, because x if it is some quantity multiply by a plus a bar. Then x square will be some square of the quantity a plus a bar multiplied by a plus a bar. So, there will be four terms a plus a bar a plus a bar a plus a bar, it will just a little complicated, but very straight forward, and then you calculate delta x and that will be x square minus x average square under the root, and then you find that if you calculate delta x by del x naught the amplitude of the classical oscillator that will come out to be that will also come out to be n. And similarly, delta p by p naught will also come out to be 1 over square root of n, because these numbers are very large.

So, this was about 10 to the power 14, this is also 10 the power of minus 14 this is also 10 to the power of minus 14 for the classical oscillator is the uncertainty principle valid of course, yes the uncertainty principle is valid there is Everett uncertainty in the measurement of x as well as in the measurement of p, but delta x by x naught. If x naught is 2 centimeters then, delta x is equal to 2 into 10 to the power of minus 14 centimeter, it is a very small number.

So, this is the domain of classical mechanics where, the position can be determined to a fair amount of accuracy, because the value of the planck constant is extremely, small. So, when you look at a classical oscillator and you have a simple pendulum, which is oscillating back and 4th a simple oscillator, which is back and 4^{th} that is a suppose, the amplitude is 2 centimeters, the time period is 1 second. So, that the omega is equal to 2 pi second inverse then, mass of the oscillator is 1 gram, and then you ask yourself what does my quantum mechanics tells us, this is a coherent state.

It is a super position of a very large number, how much large 10 to the power of 14 state super position of a large number of energy states, but the quantum numbers. Which, predominant we are talking of very high quantum numbers 10 to the power of 14 state in the vicinity of the n equal to 10 to the power of 28

So, that the width of the energy is extremely; small is extremely, small that we presents the quantum mechanical description of the harmonic oscillator, and that is why we spend the, so much of time in the studying the time, evolution of the coherent state. Which happens to be an Eigen state of the oscillator with this, we come to the end of the linear harmonic oscillator where we have discussed the let me, summarize what we have all done in the theory of the linear harmonic oscillator.

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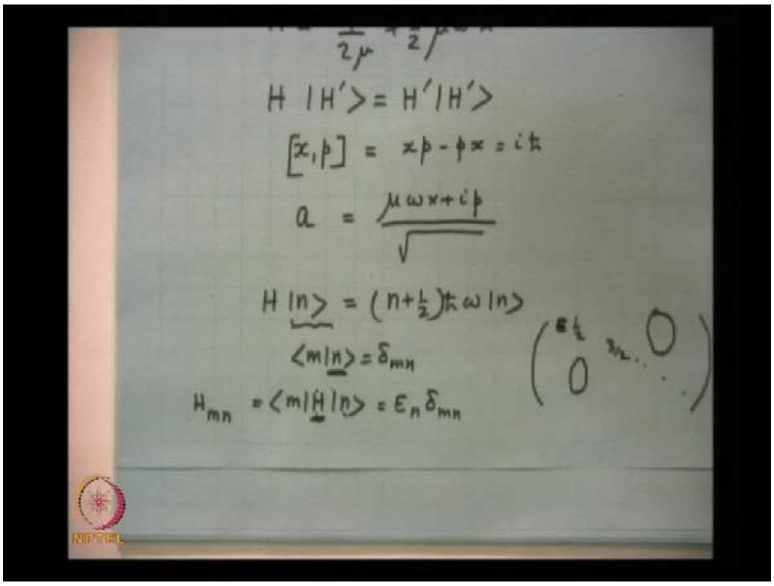

We started out with the fact that the Hamiltonian is p square by 2 mu plus half mu omega square x square our objective was to solve this Eigen value equation H prime get H prime.

The only thing that, we assume was the commutation relation that x p is commentated; x p is equal to x comma p is x p minus p x is equal to i H cross using, we then introduce 2 operators a and a bar a was equal to mu omega x plus i p divided by square root of 2 mu H cross omega and by doing operator algebra. Which is due to Dirac, we found that the Eigen kets are denoted by ket H n this is equal to n plus half H cross omega ket n and these form and complete set of orthonormal ket that m n is equal to delta m n. If I represent the matrix H in terms of these base taking, these as the base factor then, I will represent then in the form of square matrix whose 8 elements will be m h n. This is m n-th matrix element of the operator H.

In which the base vectors are the Eigen kets of H and since, these are Eigen kets of H. This is a diagonal matrix and you will have E 0, that is half H cross omega, 3 half H cross omega, 5 half H cross omega and the rest all this terms. This is the matrix representation of the Hamiltonian, when the base vectors are given by ket.

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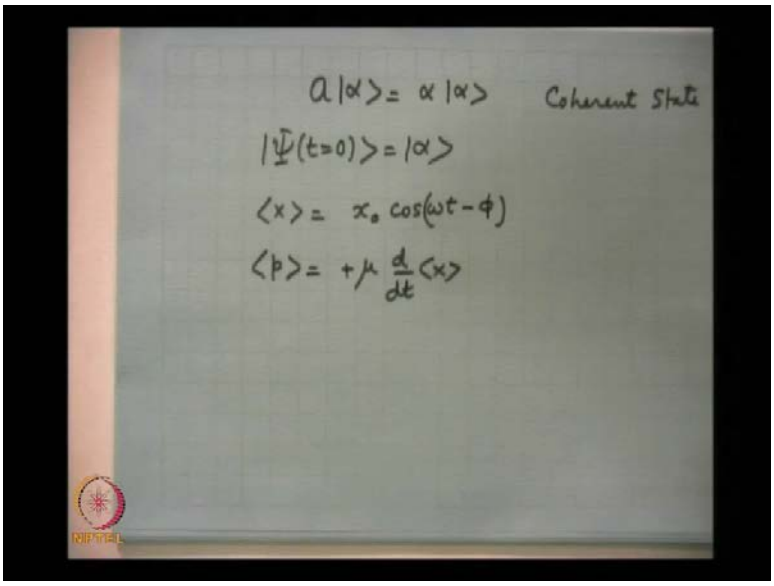

Similarly, I can make a representation of a a bar x and p so this is these are the matrix representation of the operator H then, what we did is we solved the Eigen value equation a ket alpha is equal to the Eigen kets of the operator alpha. Which, we call them as coherent state and we studied the time evolution that, if let us suppose, psi t equal to 0. If it was alpha then how will it evolve with time and we found that for such a state the expectation value of time oscillated. If a alpha is assumed to be real, if a alpha is assumed to be complex.

Then, this will be something like this and then we also found that p is equal to minus mu d by plus mu d by d t x this is the Ehrenfest theorem, and then we applied this results to a classical oscillator and we found that for a classical oscillator, extremely large quantum numbers get excited with a very small spread although the number of states, that get excited is very large.

But the spread is extremely, small to the quantum number itself and therefore, we can say in principle that the energy of the oscillator is very accurately determined the reason spread in the energy; the reason uncertainty in the measurement of the position, and there is also uncertainty in the measurement of momentum. So, that completes the harmonic oscillator.

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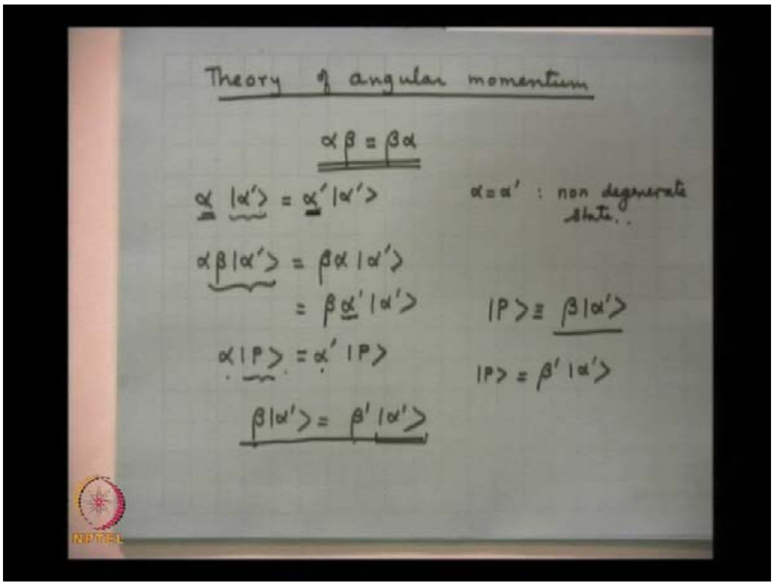

We now, go over to the theory of angular momentum the theory of angular momentum before, we do that what I would like to say is we will consider 2 operators, alpha and beta. Which commute with each other if 2 operator are such that, alpha beta is equal to beta alpha, then we say that the operator commute something like 2 square matrixes commuting with each other

Now, let ket alpha prime be a Eigen ket of the operator alpha belonging to a non degenerate Eigen value alpha prime, alpha equal to alpha prime represents a non degenerate state, that is there is only 1 Eigen value only 1 Eigen function belonging to that, Eigen value then we have that alpha beta. If I write this alpha beta operating on ket alpha prime, this is equal to, because alpha and beta commute with each other then, beta alpha get alpha prime.

But alpha operating on ket alpha prime is a number, this is equal to beta alpha prime ket alpha prime, because ket alpha prime is an Eigen ket of the operator, but this is a number. So, I can take it outside, I get alpha prime ket p where, ket p is defined to be equal to beta ket alpha prime. So, this is ket p beta operating on ket alpha prime is ket p, you have alpha ket p is equal to this.

So, we have found that if ket alpha prime is an Eigen ket of the operator alpha belonging to the Eigen value alpha prime, then ket p. Which is equal to beta ket alpha prime is also an Eigen ket of the operator alpha belonging to the same Eigen value alpha prime, but it is a non degenerate state.

So therefore, ket p lastly a multiple of ket alpha prime so beta alpha prime is equal to beta prime alpha prime we must remember that here alpha prime alpha is an operator alpha prime is an number, because this is an Eigen value equation.

This is an Eigen value equation ket alpha prime is an Eigen ket of the operator alpha belonging to the Eigen value alpha prime. So, alpha prime here is a number here, alpha prime is here alpha is an operator and now, once it is a number I can take it outside. This tells us that therefore, ket alpha prime is also, Eigen ket of the operator beta belonging to the Eigen value beta.

Thus, if you have 2 operators which commute, then the Eigen ket belonging to a non degenerate state will be a simultaneous ket I, further assert that if alpha and beta are observables then, as we have said earlier that, if alpha is an observable. It must be represented by L real operator. So it is Eigen values are real, it Eigen gets form a complete set of functions complete set of Eigen kets and if 2 operator commute, then there will be simultaneous complete set of simultaneous Eigen kets.

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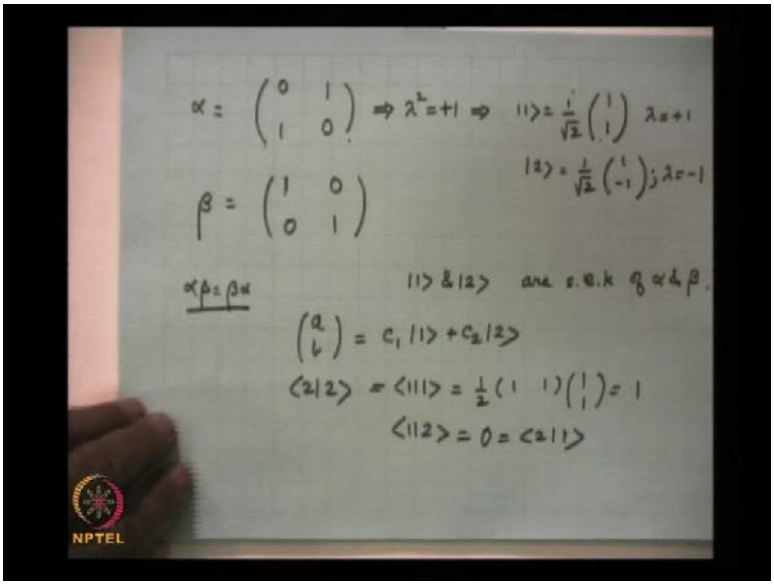

If you have difficulty in understanding let me, give you an example, consider, the 2 by 2 matrix 0 1 1 0 1 matrix say alpha here and beta is equal to 1 0 0 1. Now, I leave this as an exercise for, you to show that alpha beta is equal to beta alpha.

Now, alpha is a 2 by 2 matrix alpha is 2 Eigen values, this leads to lambda square is equal to 1 lambda is equal to plus minus 1 and I had we had done this before, that the 2 Eigen kets are say 1 and 2 1 is 1 over under root 2 1 1 1 over under root 2 is just normalization and the second Eigen ket is 1 over under root 2 1 minus 1.

Now, this is a unit matrix, this is a Eigen, this has degenerated Eigen values, but these are non these are Eigen kets belonging to non degenerated, Eigen values this is for lambda equal to plus 1. This, we had done before I am not doing it again, but if you have difficulty you can immediately, calculate the Eigen vectors of this hermitian matrix, and you will find that ket 1 and 2 are simultaneous Eigen kets simultaneous, Eigen kets of alpha and beta, because alpha beta is equal to beta alpha at any vector like, a b can always be represented as a linear combination of ket 1 plus ket 2

So, in the 2 dimensional spaces, they form a complete set of orthonormal. These are orthonormal that is bra 1 ket 1 this will be 1 over 2 1 1 1 1, this is 1 bra 1 ket 2 ,this will be 0 bra 2 ket, 1 will be 0 and bra 2 ket 2 will be also 1 now, let me take another example

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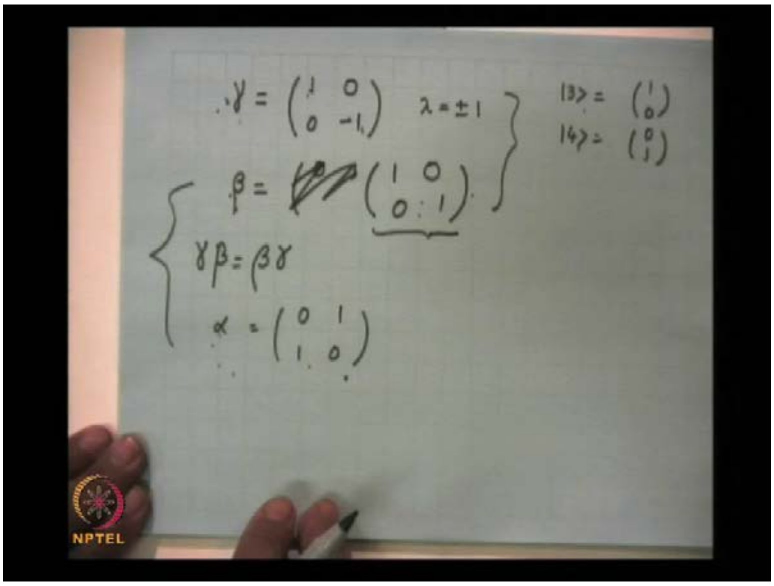

that I consider, say gamma in matrix which is 1 0 1 minus 1 and the we again consider, beta I a equal to 0 1 sorry sorry I am sorry I am sorry 1 1 0 0 beta is the same, but gamma is now, different and you can again show that gamma beta is equal to beta gamma.

Now, there are 2 non degenerate Eigen values 1 and minus 1 1 and minus 1 the Eigen vectors are again, the Eigen values are lambda is equal to plus 1 and minus 1. These are the diagonal elements here, and you show that, there are 2 Eigen kets now, 3 and 4 are right 1 is 1 0 and the other is 0 1 .These are the orthonormal Eigen kets of gamma.

Since, gamma and beta commute these are also Eigen kets of the operator beta. Now, I have 3 matrix alpha, which is equal to 0 1 1 0, alpha commutes with beta gamma commutes with beta, but alpha does not commute with beta. So, you can have simultaneous Eigen kets of beta, and gamma you can have simultaneous kets of alpha and beta, but you will never have simultaneous Eigen kets of alpha, and gamma that you had 2 people. There you both do not are not friend I am friendly to you, I am friendly to you then his this beta matrix commutes with both of them

So, you can ask simultaneous Eigen kets of beta and gamma beta commutes with alpha, you can have simultaneous Eigen state beta and alpha, but alpha does not commute with gamma therefore, you cannot have simultaneous Eigen kets of alpha and gamma.

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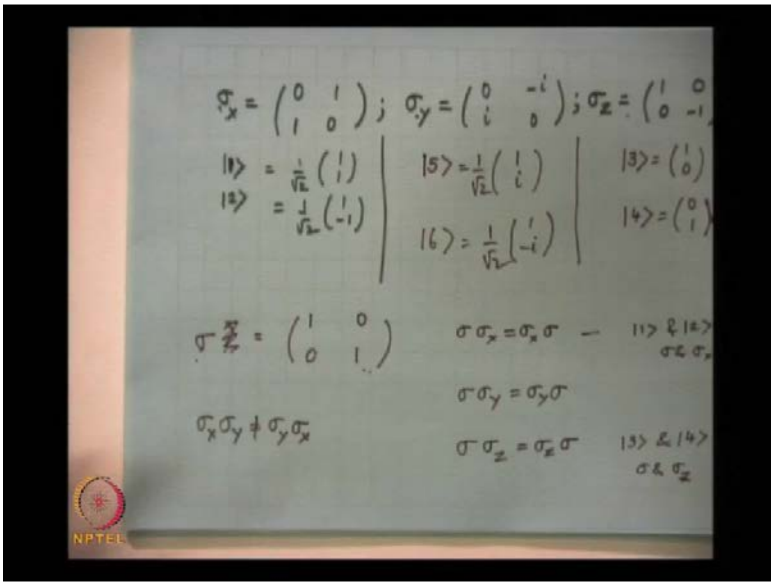

Let me, conclude this actually the matrixes that, we are considering are known as pauli matrixes pauli spin matrixes, some of you may have heard this the pauli spin matrix are sigma x is equal to 0 1 1 0 sigma y is equal to 0 minus i i 0 and sigma z is equal to 1 0 0 minus 1 I would request all of you to play around, these matrix and you show that of this the kets the Eigen kets I represent this by say x say 1 and 2, these are 1 1 and 1 minus 1 is a square root of 2 of this let me, write it down with red that, this is let us suppose, of this we had said that ket 3 and ket 4 was 1 0 and 0 1.

And here, I think 1 of them is say ket 5 this is equal to 1 i square root of 2 and ket 6 1 minus I something, like that I would like you to confirm this 1 2 3 4. I am sure this is also, I am sure I am almost first 2 points that, I have the unit matrix say say the say the say let me, write down a matrix sigma which is equal to 1 0 0 1.

Now, sigma commutes with sigma x or let me, just write it down the sigma sigma commutes with sigma x, that is sigma sigma x is equal to sigma x sigma sigma commutes with sigma y and sigma commutes with sigma z ket 1 and 2 are simultaneous, Eigen kets of sigma and sigma x ket 3 and 4 are simultaneous Eigen kets of sigma and sigma z and similarly, ket 5 and 6 are Eigen kets simultaneous, Eigen kets of sigma and sigma y.

So, sigma on the other hand sigma x does not commute with sigma y and I leave this is an exercise for, you to show that you can never have simultaneous, Eigen kets of sigma x and sigma y. You can never have simultaneous Eigen kets of sigma x and sigma z,but you will always have simultaneous, Eigen kets have simultaneous Eigen kets of sigma x and sigma y and 1 more thing that, you must have noticed that since, these are this is are degenerate this is degenerate Eigen value all Eigen kets of sigma x are Eigen kets of sigma.

But since, this is a degenerate case all Eigen ket of sigma are not Eigen ket of sigma x, but in the 2 dimensional space. You can always choose in the case of degenerate state a linear combination of the Eigen vector is also a possible Eigen vector also are possible Eigen vector, we had shown that, that if you have a degenerate state let us suppose,

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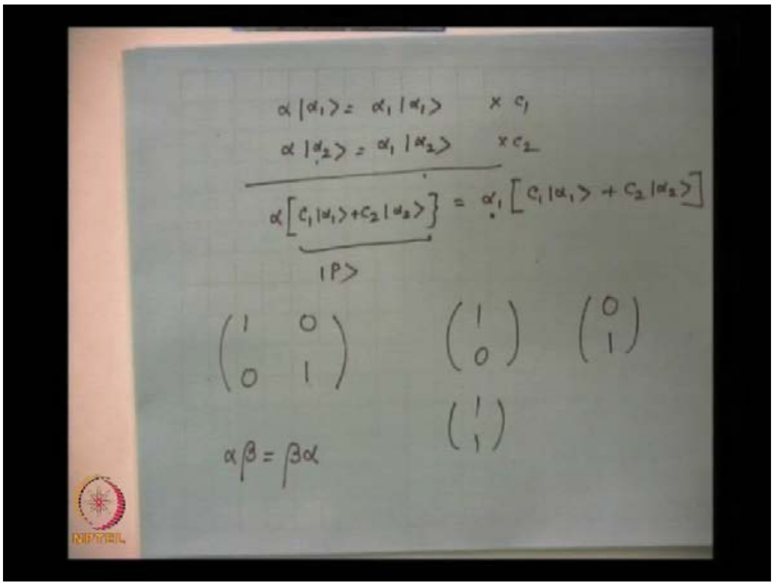

I have an operate alpha and there are 2 Eigen kets, alpha 1 is equal to alpha 1 alpha 1 and there is another ket alpha 2, which has the same Eigen value, then you multiply this by c 1 and you multiply this by c 2 and c 1 and c 2 are numbers. So, you get alpha into c 1 alpha 1 plus c 2 alpha 2 is equal to alpha 1 c 1 alpha 1 plus c 2 alpha 2. This is also this ket p is also an Eigen ket of the operator p alpha belonging to the same Eigen value.

So, for a degenerate state if, there are 2 linearly independent degenerate state the linear combination is also Eigen ket, in the case of in the case of the square matrix 1 0 0 1 there are 2 degenerate Eigen values, 1 0 is an Eigen vector 0 1 is an Eigen vector, but the sum of the 2 that is 1 1 is also an Eigen vector.

I hope I have able to make you understand that that when 2 observables commute alpha beta is equal to beta alpha you can always construct a simultaneous set or a complete set of simultaneous Eigen vectors. Now, we go over to the theory of the angular momentum then, we had discussed the just before, we had discussed the hydrogen atom problem. This, we define the angular momentum

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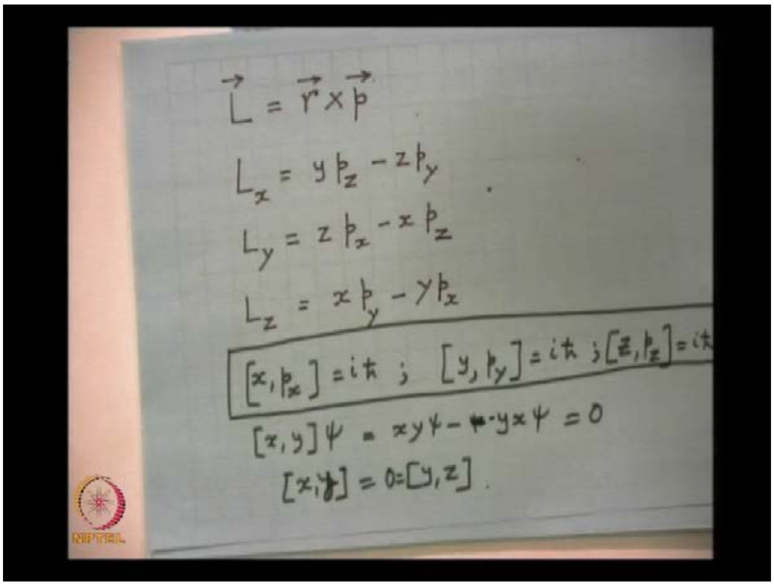

Vector as L is equal to r cross p so L x was equal to y p z minus z p y, and then the other three are given by L x L y z p x minus x p z and L z is equal to x p y minus y p x. Now, we took over the same definition, but now we will assume we will also take care assume the fact that x and p x do not commute in fact we had, this relation that comma p x was equal to i H cross and similarly, y comma p y is also equal to i H cross and z comma p z is also equal to i H cross.

But x and y commute that is x comma y operating on psi on a wave function be equal to x y psi minus psi x y sorry x y psi minus y x psi. Now, both of them are equal this is 0 so x comma y or y comma z they are all 0 not only that if I had x comma say p y.

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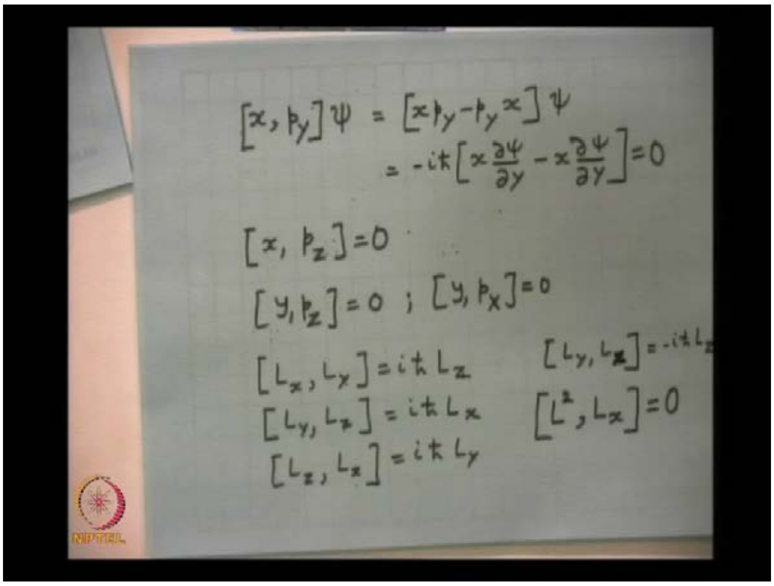

So, this as we had said that if, I represent this by a differential operator so this will be x p y minus p y x operating on psi. So, this will be minus i h cross x delta psi by delta y and since this is a partial differential with respect to x y x can be treated constant. So, this is x delta psi by delta y. So, this is 0 x comma p z is also 0 and similarly, y comma p z is also 0 and y comma p y is also 0.

Y, sorry y comma p y is not y, p x is also 0 all these matrix elements all the commutates are 0 excepting these, three in developing the formal theory of the angular momentum. We will just assume this and we will derive using this that the commutates of L x L y, we will show that this will be equal to i h cross L z and similarly, you can write them down in cyclic order L y L z this is equal to i h cross L x. You can see this and L z L x is equal to i h cross L y so these are all in cyclic order x y z y z x z x y and obviously therefore, for example, L y comma L x will be equal L y comma L x L y comma L x will be minus sign of that, these are now, i inverse cyclic order i h cross L x y x z 1 must remember, r this we will also show that L square commutes with L x and L square commutes with L y and L square commutes with L z, but L x L y L y L z and L z L x they do not commute with themselves therefore, you can have simultaneous Eigen vectors of L square and L x or of L square and L y or of L square and L z

So in the next lecture, we will discuss the simultaneous Eigen vectors of L square and L z thank you.