Neutron Scattering for Condensed Matter Studies
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Week 8: Lecture 21B

Keywords: Coherent scattering length, refractive index, magnetic scattering length
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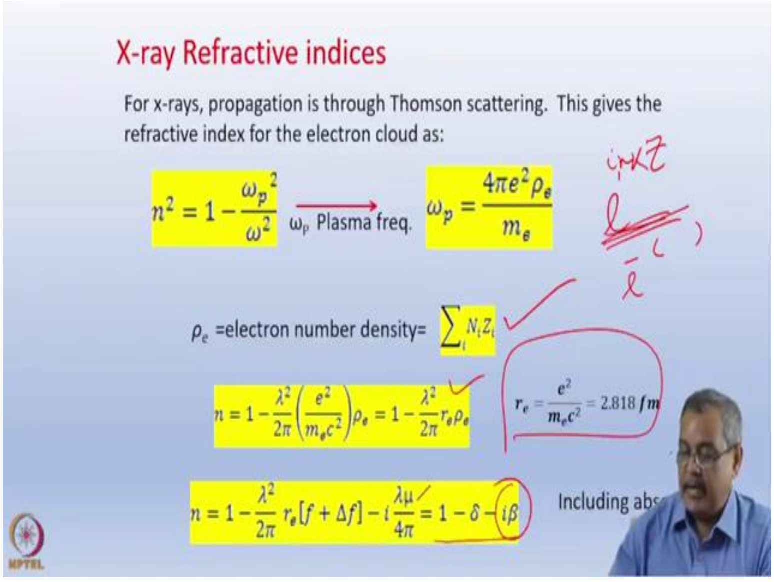

In the expression written here for refractive index (n), $\rho_e = \sum_i N_i Z_i$ is the electron number density, r_e is the classical electron radius. Since x-rays can be heavily absorbed depending on the medium, so if I introduce the absorption in a medium then refractive index can be written as, $n = 1 - \delta - i\beta$ where $i\beta$ is the absorption term. This term gives the absorption in the medium and reduction in the propagated wave in the medium. For example, a plane wave propagated in the medium is written as e^{inkZ} and in that medium if you have absorption, then this will give some term such as $e^{-(\dots)}$ which will give the reduction in the wave intensity propagating in a medium. This expression is for x-ray.

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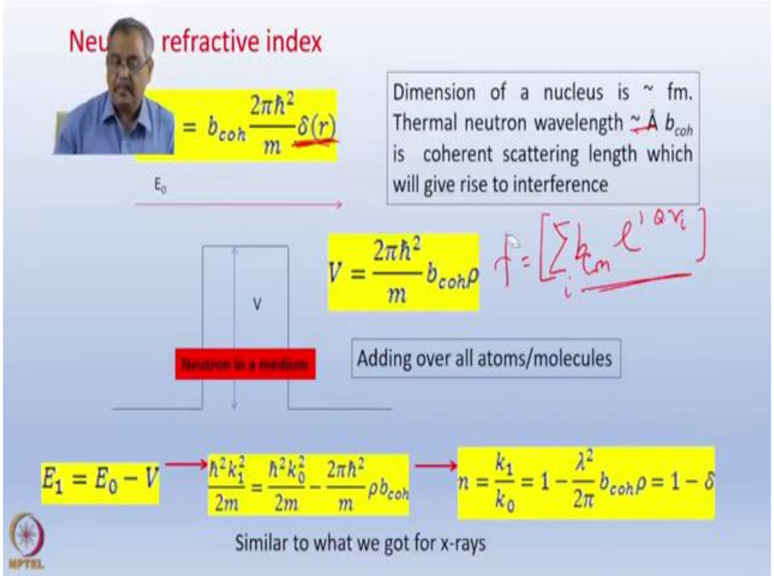

Now, let us come to neutron. We have discussed in our lectures in the basics, that the potential experience by a neutron from a single scatterer is given by, $V(r) = \frac{2\pi\hbar^2}{m} b_{coh}\delta(r)$, where b_{coh} is basically average coherent scattering length which gives us diffraction. This logic for δ function potential I had discussed earlier. I am repeating that the fundamentals of using $\delta(r)$ as the spatial extension of the potential is because the nuclear dimension is just femtometer and the thermal neutron wavelength is around Å.

If you remember, I wrote the term $\sum_i b_{coh}e^{iQ.r_i}$ which is proportional to the scattering amplitude.

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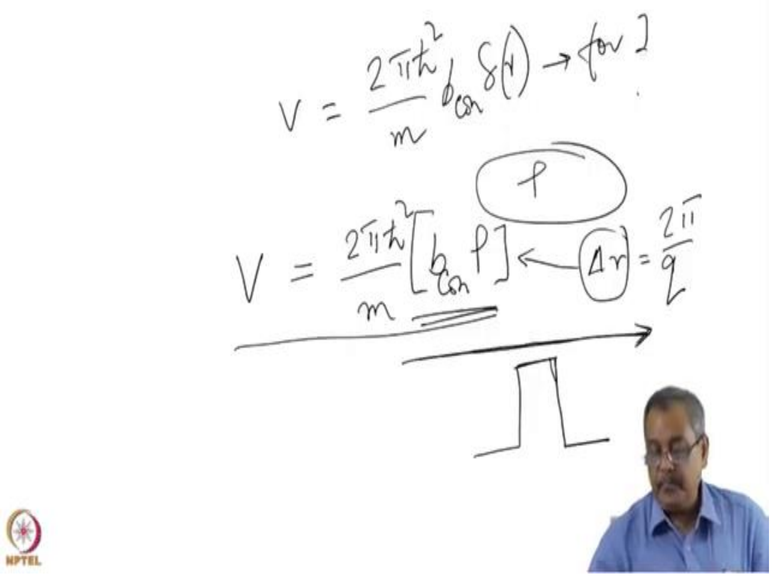

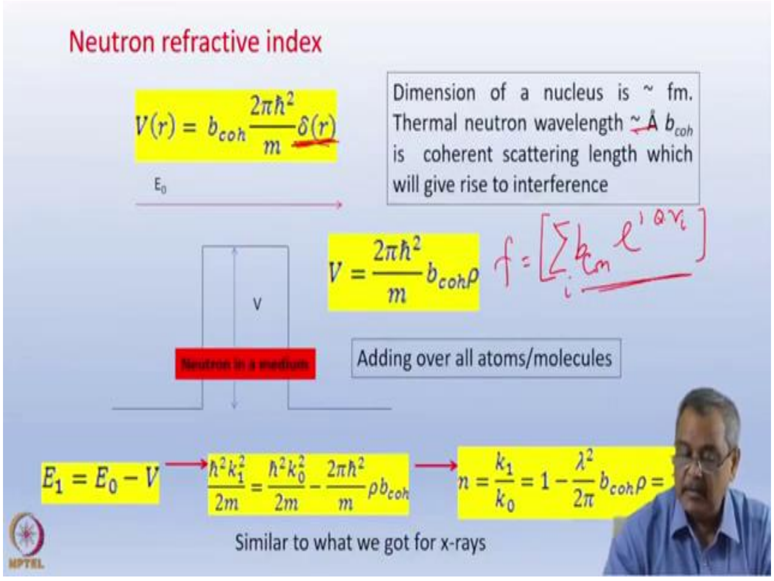

Now, I am talking about a medium with a certain density, ρ, means there are many scatterers per unit volume. In this case, potential can be written as, $V(r) = \frac{2\pi\hbar^2}{m} b_{coh}\rho$. When the neutron approaches this medium, it sees this potential. Now, please consider the fact that I am doing experiments at very low q. Here the inherent spatial resolution of the measurement, by uncertainty principle, is given by, $\Delta r = \frac{2\pi}{q}$ and this Δr allows me to use a volume density of scattering length for estimating potential of the medium for neutrons.

Now we have got a potential which depends on the density of the medium. So, as a neutron approaches a medium it sees a potential which is given by $\frac{2\pi\hbar^2}{m} b_{coh}\rho$. This is a step potential. Now, the neutron is traveling in the medium. This is how it looks like as shown in the figure.

So, with the density, I have this potential and the neutron is traveling in the medium. I show neutron energy is greater than this potential otherwise it cannot travel in the medium and we have to talk about tunneling, but here it is not tunneling, it is the apparent neutron energy *E'* in the medium.

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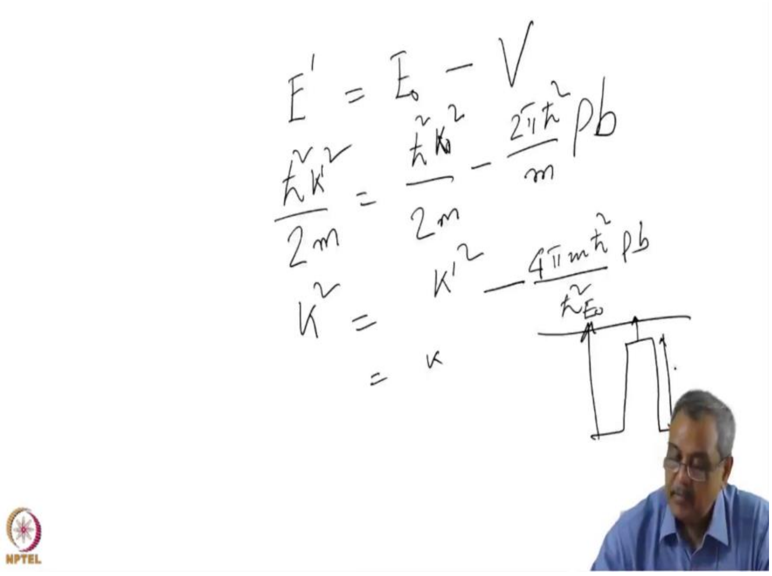
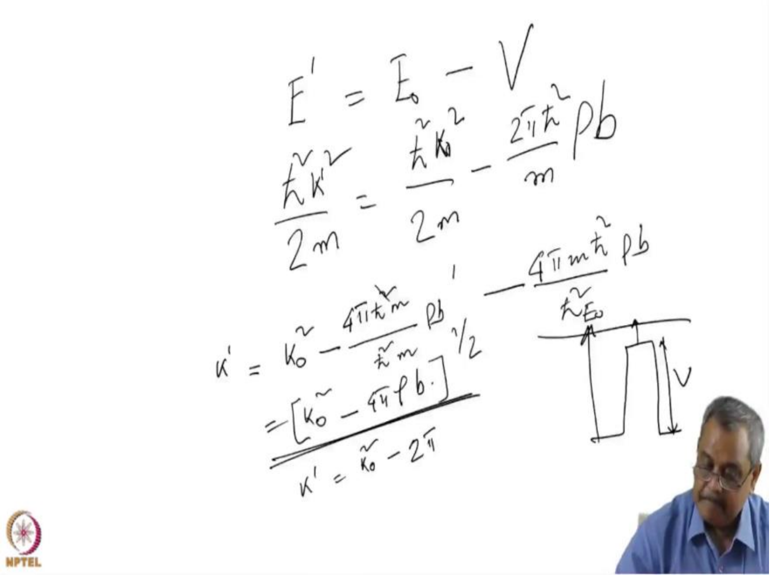
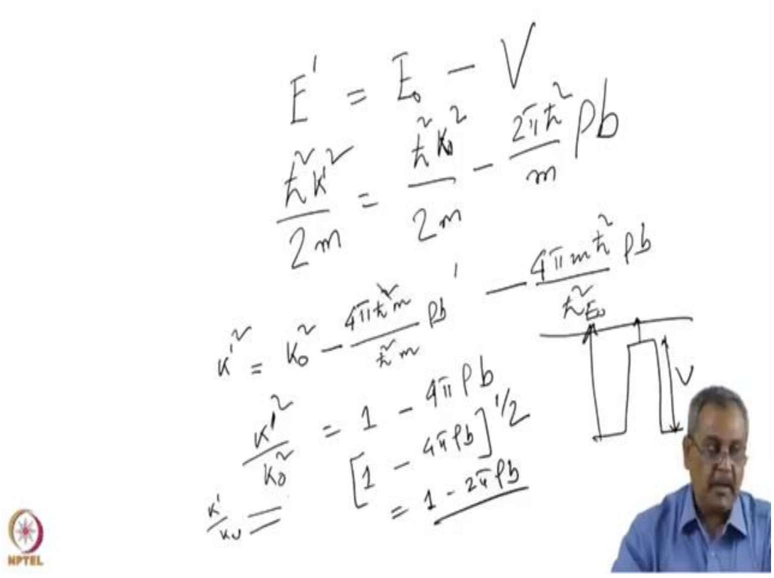
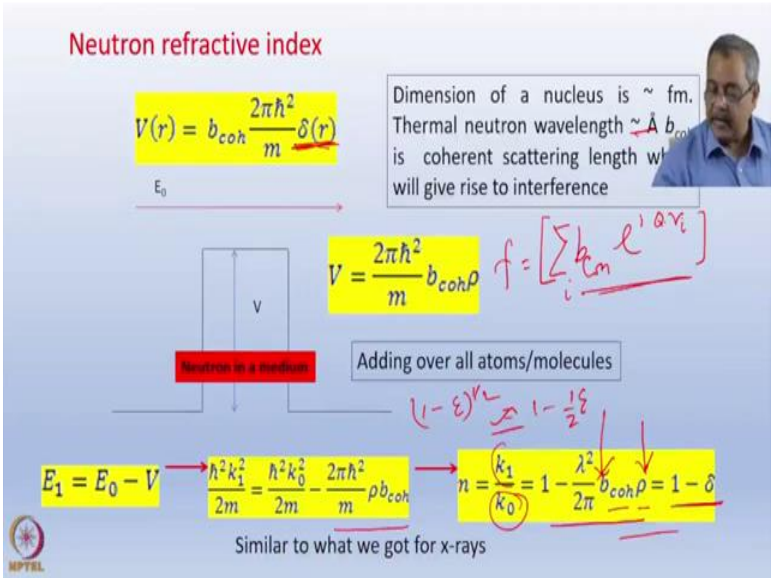
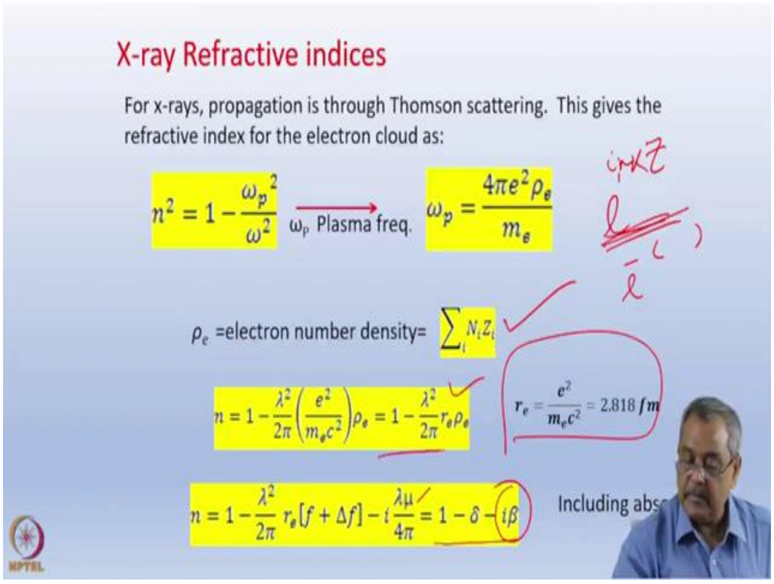







Neutron refractive index
V(r) = b_coh 2πħ²/m δ(r)
Dimension of a nucleus is ~ fm. Thermal neutron wavelength ~ Å b_coh is coherent scattering length which will give rise to interference
E0
V = 2πħ²/m b_coh ρ
V
Neutron in a medium
Adding over all atoms/molecules
E1 = E0 − V
ħ²k1²/2m = ħ²k0²/2m − 2πħ²/m ρb_coh
n = k1/k0 = 1 − λ²/2π b_coh ρ = 1 − δ
Similar to what we got for x-rays


X-ray Refractive indices
For x-rays, propagation is through Thomson scattering. This gives the refractive index for the electron cloud as:
n² = 1 − ωp²/ω²
ωp Plasma freq.
ωp = 4πe²ρe/me
ρe =electron number density= Σi NiZi
n = 1 − λ²/2π (e²/mec²) ρe = 1 − λ²/2π re ρe
re = e²/mec² = 2.818 fm
n = 1 − λ²/2π re[f + Δf] − i λμ/4π = 1 − δ − iβ
Including abs

Now, I can draw the similarity between x-rays and neutron. Apparent Energy (*E*) inside the medium is equal to energy in the vacuum (E_o) minus the potential (*V*). For a free neutron it is given by,

$$\frac{\hbar^2 k'^2}{2m} = \frac{\hbar^2 k_0^2}{2m} - \frac{2\pi\hbar^2}{m} b\rho$$

Here, I have replaced b_{coh} by b. This can be written as,

$$k'^2 = k_0^2 \ - 4\pi b\rho$$

$$\frac{k'}{k_0} = \left(1 \ - \frac{4\pi b\rho}{k_0^2}\right)^{1/2}$$

Since 2nd term is <<1, so it can be simplified as,

$$\frac{k'}{k_0} \cong 1 - \frac{2\pi b\rho}{k_0^2}$$

Putting $k_0 = 2\pi/\lambda$,

$$\frac{k'}{k_0} \cong 1 - \frac{\lambda^2 b\rho}{2\pi}$$

Now, refractive index is defined as, $n = {}^{k_1}/_{k_0} = {}^{k'}/_{k_0}$, so,

$$n \cong 1 - \frac{\lambda^2 b\rho}{2\pi}$$

This expression is equivalent to, $n = 1 - \delta$. This expression is similar to what I got for the x-rays $\left(n \cong 1 - \frac{\lambda^2 r_e \rho_e}{2\pi}\right)$ with $r_e\rho_e$ replaced by $b\rho$. In x-rays it was density of electrons, while here it is scattering length density multiplied by coherent scattering length. That gives me the total scattering length per unit volume and *n* is given by that. So, very similar to what we got for x-rays that

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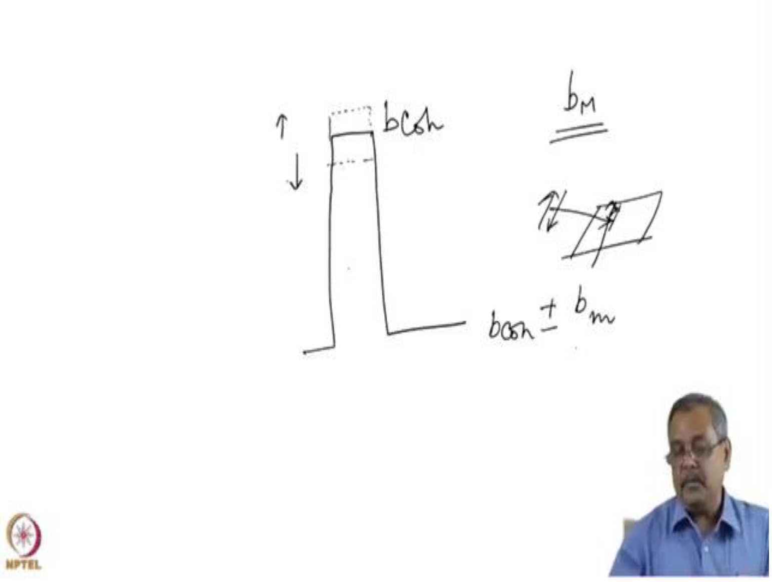

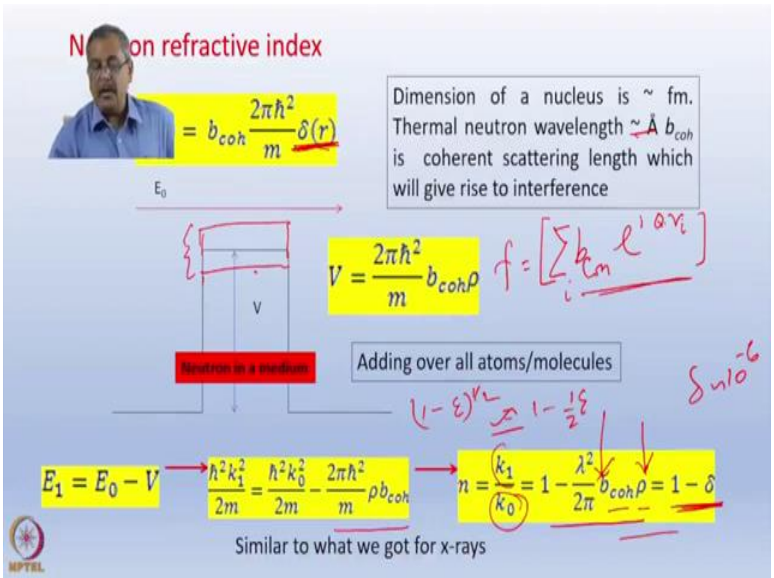

So far in this expression, I have not brought in any magnetism and the coherent scattering length (b_{coh}) till now has nuclear origin. However, if we consider interaction of polarized neutrons with a magnetic material an additional scattering length term, namely b_m, comes into picture which is magnetic is nature, as a coherent scattering length. In such a case, the scattering length term in potential is replaced by $b_{coh} \pm b_m$ and the sign here depends on the relative direction of the magnetization of neutron and that of the thin film medium. So, either the potential becomes slightly higher for the up neutrons or it is slightly lower for down neutrons. Now, what does that mean?

Let me talk in terms of a critical angle then it will become clear what does it mean? Earlier, I talked about nuclear potential $\delta(r)$ resulting in $V(r)$ which gives me a refractive index, $n = 1 - \frac{\lambda^2 b\rho}{2\pi} = 1 - \delta$ and as I told you this δ is small, of the order of 10^{-6}.

Now, I find that this potential height changes, if I have a magnetic scattering length in the medium. If I reflect polarized neutrons from this medium, then neutrons polarized parallel and anti-parallel to the sample magnetization will see slightly different potentials (for magnetized sample). This is very, very important for magnetic neutron reflectometry or polarized neutron reflectometry. I will come to it and will use this result again and again.

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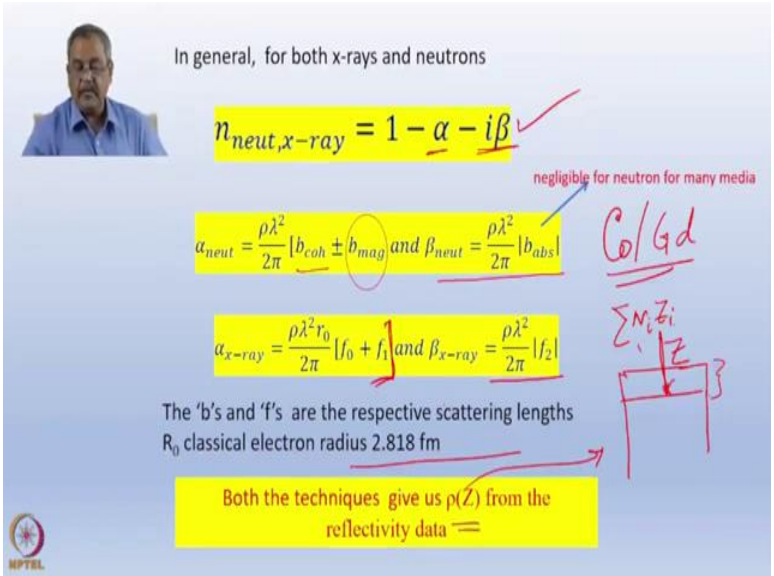

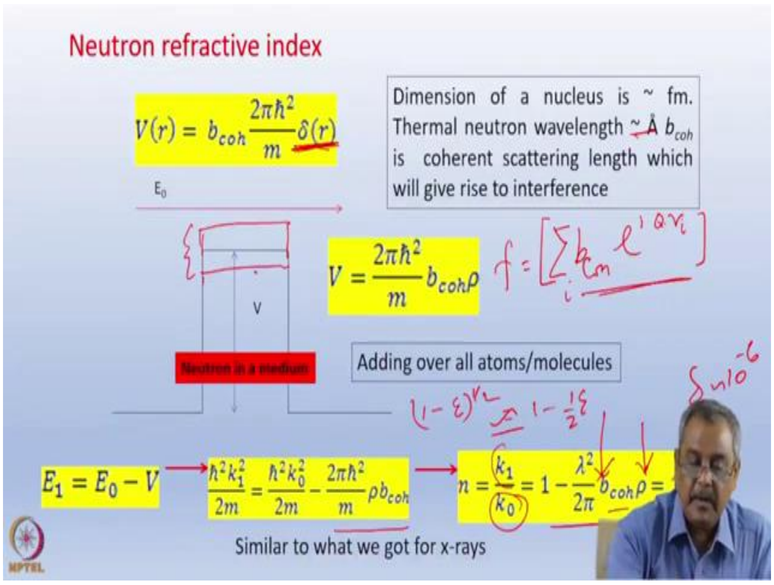

Now, let me just sum up that whether it is neutrons or x-rays, the refractive index is given by $n_{neut,x-rays} = 1 - \alpha - i\beta$ where the 2nd term is coming from e-density or scattering length density (for neutrons) and the 3rd term is coming from absorption. This is what I have written here,

$$\alpha_{neut} = \frac{\rho\lambda^2}{2\pi}[b_{coh} \pm b_{mag}] \text{ and } \beta_{neut} = \frac{\rho\lambda^2}{2\pi}|b_{abs}|$$

For neutrons often absorption term is negligible, except when you are talking about neutron absorbing medium like cobalt, gadolinium which are strong neutron absorbers and neutron reflection from them. Otherwise, this is often negligible for a very large number of materials. In case of x-rays, we have a very similar expression,

$$\alpha_{x-ray} = \frac{\rho\lambda^2 r_0}{2\pi}[f_0 + f_1] \text{ and } \beta_{x-ray} = \frac{\rho\lambda^2}{2\pi}|f_2|$$

Here f_0 comes from $\sum_i N_i Z$, but the fact is that there are some deviations from this number as you go up in Z values. Interpretation of β_{x-ray} is exactly the same as β_{neut}, which is given by the absorption coefficient of x-rays and that is large for almost all the medium for x-rays and negligible for neutrons.

The *b's* and *f'*s are the respective scattering length for x-rays and neutrons. And r_0 here, as I used it earlier, is classical electron radius. Both of these techniques, can give us density of a thin film as a function of *z* (depth), if we consider *z*-axis is the one which is going inside the medium starting from vacuum along normal to the film surface. As we go in the film the reflected intensity of x-rays and neutrons gives us the ρ(z) values from reflectivity data. How? I will tell you in the next module.