Theory of Group for Physics Applications
Prof. Urjit A. Yajnik
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 46
SU(3) and Lie's Classification - II

Further application which becomes much more important of SU 2 is that electron and neutrino also form an SU 2.

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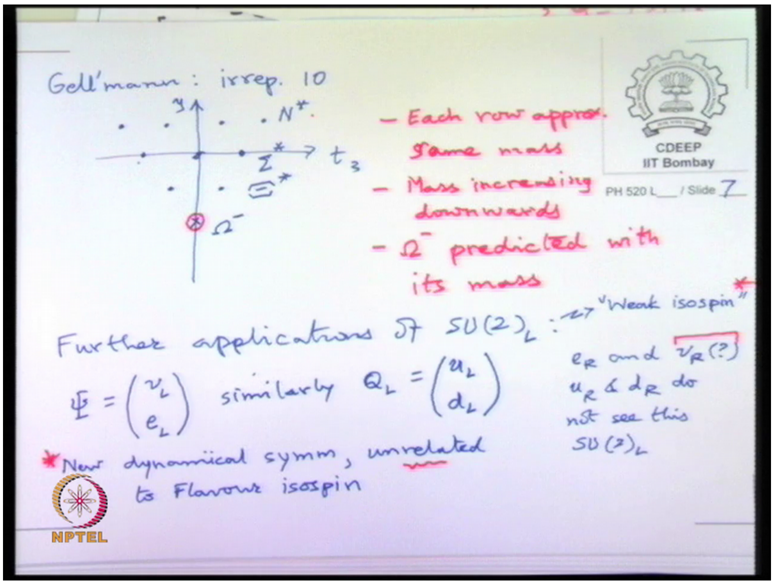

But this is left handed neutrino left handed electron, this form a SU 2 representation. So, there actually exists fundamentally as a metric, which can rotate electron into a neutrino now you will say that is very strange because the different charges and also because they have very different masses.

The answer is that the charge difference is not a surprise, because we have generators that exchange that increase and reduce charge tau plus tau minus of Pauli flips pin up and down. So, there are charge changing generators in this like here delta q would be equal to delta 3 plus delta y. So, if you are only delta 3 and only delta y then change charge can change. But the mass difference is huge, you know there is n u b and there is 0 essentially. Whereas these things have same approximately same mass that is what required in mention what is called spontaneous symmetry breaking and a long story. So, and similarly Q L equal to upward left handed downward left handed. So, the weak forces at the strange I may as well write SU 2 L.

The weak forces at the strange property that the afflicted only left chiral parts of the particles and not the right chiral once it is effected in a different way. So, there is this is the e e R and u R and d R. So, do not see this SU 2 L. And the right hand neutrino spread in exist as far as most people were concern for a very long time now it does exist. So, we may as well as add, but we do not know if it is exists ok, but if it exist then almost certainly exist, but we do not have a conclusive evidence.

On this all of those any way do not see the s weak SU 2, the weak iso spin SU 2. So, this is called weak isospin by a some abuse of trying to continue the nomenclature isospin, but it has nothing to do with the original isospin. This is completely distinct from the flavor isospin we call it weak isospin, but it is actually is not the flavor isospin. So, there were various SU 2s and there was some confusion for quite some time, but eventually it was sorted out.

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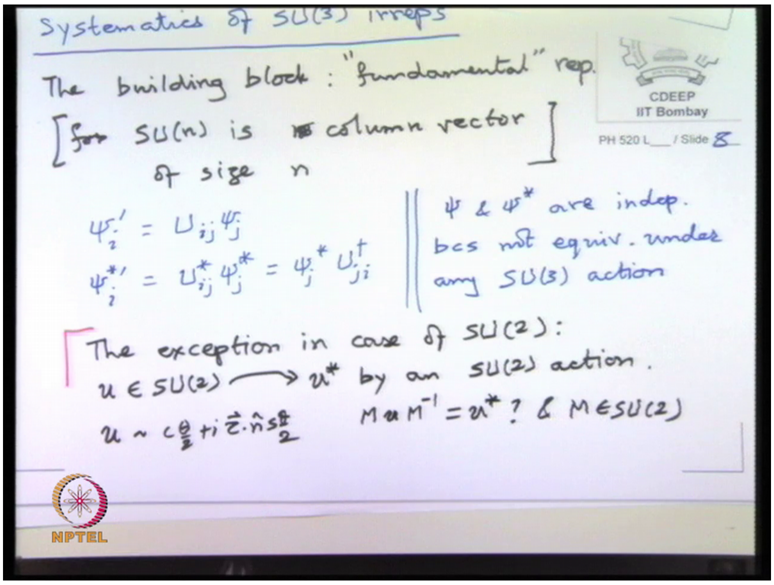

So, now, we come back to representations of SU 3 and method for finding the higher dimensional representations, main building block is the fundamental representations which for SU n, it is n vector n the column vector of size n.

And we have to state this column business importantly, because row and column have very different meaning now. So, column vector of size n, this is the fundamental representation of SU n, we call it fundamental its obvious its natural because I have n by n matrices. So, they act on this we call it the fundamental representation now. So, if I have.

So, just to write the notation, we say psi lower psi i prime equal to U i j s i j. I should have return lower case U because that is what we have been using, but somehow I can any way my notes contains this right now. So, I will stick to this. It turns out that psi star is an independent representation. So, psi star i prime would then will be equal to U i j star psi j star or which we could write as equal to psi j star and then U dagger j i.

The complex conjugate representation is an independent one the conjugate because the process of complex conjugation is not within SU 3. There is no SU 3 operation and you know SU 3 matrix can converted 3 column vector into its complex conjugate.

So, psi and psi star are independent not equivalent under any SU 2 operation or SU 3 operation you cannot use any SU 3 metrics to convert a psi in to a psi star. So, at this point just to take a put an important to carrier at which is important to a lot of physics is that, SU 2 a is an exception the smallest group SU 2 as by accident an internal cemetery that can convert its 2 dimensional representation into its complex conjugate.

So, all though this kind of is a break to the main thing we want to do, because its important enough may we as well write it. We can convert a u belonging to SU 2 can be made into u star by an SU 2 action. And the proof is very simple its because u is made of you know you remembered its cos by 2 plus i. So, it involves the tau matrices. So, the question boils down to whether a metrics exits which takes M u M inverse to u star and M belong to SU 2 that is the question of equivalence and the answer is that this is exactly.

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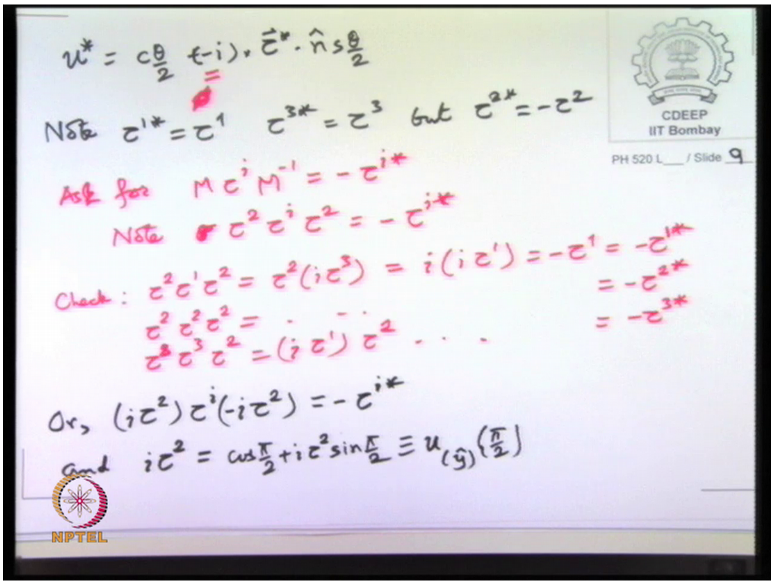

So, if you want u star you need to star everything here cos theta by 2 minus i times tau star dot n cap sin theta by 2. But now remember what are tau stars. So, tau 1 star equal to tau 1, tau 3 star equal to tau 3, but tau 2 star equal to minus tau 2. So, what can you propose that will render the signs of the tau 1 and tau 2 opposite, because there is the minus sign here and the point is that this can also be written by observing that if I have see if I have write it here.

The sigma 2 matrix does the job actually, that is the answer. So, what we may ask is whether there is a M that takes this to minus tau star. So, because of this minus sign we need to ask for M tau i M inverse equal to minus tau i star. So, this arrows is kind of confusing let me remove it, but does this implies that we need to ask for this to convert this a use u into u star there should be a matrix m which converts the tau i into minus tau i stars. And the answer is that this is done by M equal to sigma 2 that is same as tau 2, I am sorry tau 2 tau i tau 2 inverse, but tau 2 inverse is tau 2 actually is what we are looking for.

We know tau 1, tau 2 is equal to i tau 3, but tau 2, tau 3 is equal to i tau 1. So, is equal to minus tau 1, but tau 1 star is equal to tau 1. So, that is same as minus tau 1 star, tau 2, tau 2, tau 2. So, you can check this tau 2, tau 3 this is trivial because tau this is back to tau 2 and tau 2 is minus of tau 2 star, this tau 2 star is minus of tau 2 because is a imaginary matrix. So, that works out and similarly this works out by all the same things tau 2 tau 3 is i tau 1 and. So, tau 2 matrix works, but tau 2 is not an SU 2 matrix, but i tau 2 is ok. So, the inverse of i tau 2 is minus i tau 2 because tau 2 square is 1 and i tau i times minus i is 1.

So, this is equal to minus tau i star and i tau 2 is actually an SU 2 matrix because it is equal to cos pi by 2 plus i tau 2 times sin pi by 2. So, there exists an angle theta and rotation axis direction 2 so, that in our earlier rotation. So, u y cap pi by 2 is exactly the matrix i tau 2. So, there exists an SU 2 matrix which converts SU 2 matrix in to the one star. So, for a SU 2 the matrix the representation and its star are identical. So, this closes our side comment on SU 2. So, this, but for SU higher SU n s no such thing is available.

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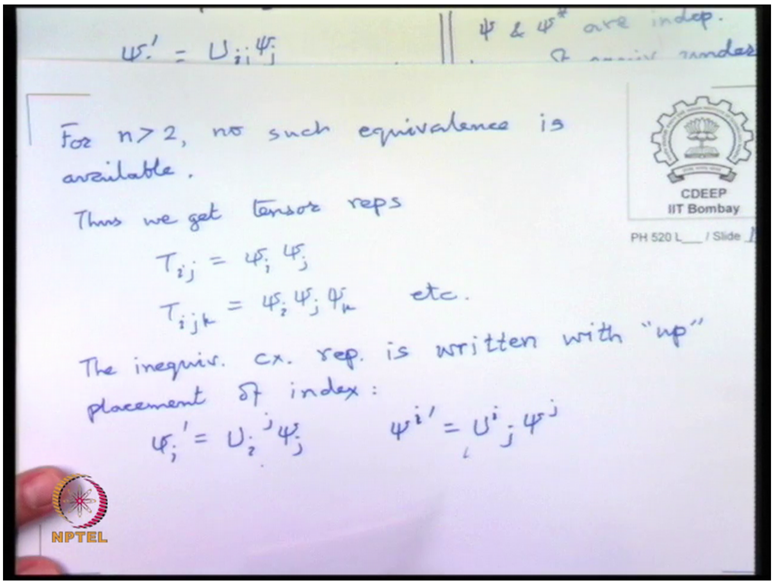

So, back to the SU 3; so we had written this notation psi, psi i prime equal to U i j psi and psi i star prime are they transformed by u adjoint if you treat them as row vectors. So, what we do now is to construct representations.

No on and left multiplication and which one no. So, on the group elements themselves you have to apply a similarity transformation, and on to the fundamental s. So, fundamental you will have to apply i tau 2 on to the fundamental. So, if you want we can complete that bit there that as a consequence psi SU 2 conjugate. So, that is a good point, psi equal to which what do you like upon u and d are neutron and proton u d. So, if you had psi u d, then it goes to i tau 2 times psi star u d i tau 2 times u d this yes.

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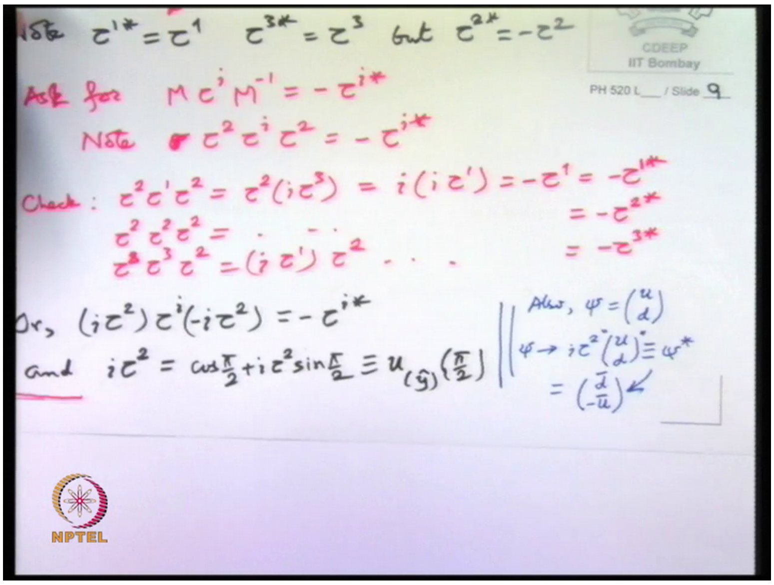

So, it becomes equal to minus d. So, I remembered no. So, d minus u and this you have to identify with psi star, the star representation is equal equivalent to this. So, it flips u ds; but therefore, there is a bar required on these this. So, they change that it the in particle physics you will have change that charge, you have to write the d bar and u basically d bar and u bar. So, for the time being let me put inverted commas like this. So, you act on it from the left, but you that is identified which use psi star, but psi star has u bar and d bar in it.

So, this is what happens to the fundamental by left actions, but you do not get anything new out of it that is the statement similarly. So, now, we go on to the higher dimensional representations and what I the 2 things that we want to say is that, from the building block of fundamental, we construct tensor representations. So, we what we do is, the in equivalent complex representation is written with different placement of the index.

So, we write psi i prime equal to; and now I have to be very careful U i j psi j, but psi upper i prime equal to U i j. So, it is transpose actually psi j. So, this is U and this notation actually makes it the. So, this is the notation adapted not only is it in the on the 2 by 2 matrix, not only is it important whether the index is up or down, but where it exactly it is located. So, if you want I mean this is the way to write it, because there is no easy row and column representation any more in terms of matrix, but this is the notation adapted and remember that the U i j written like this is essentially the U star. So, this equation is actually converted to this, by lifting the index up and the u dagger j is we write like this and this is the notation adapted.

Of course the inner product is. So, that is you to invariant inner product, I will not able to finish much this time I think, but inner product is psi i phi i.

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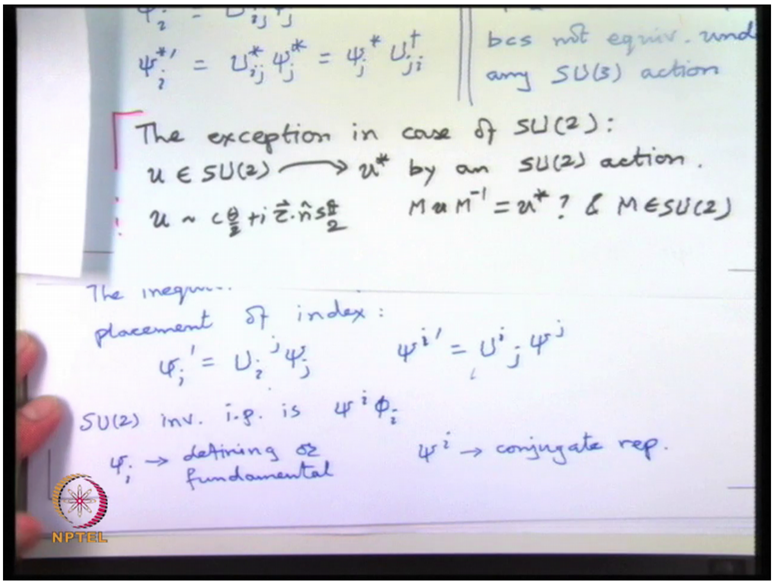

One index of one index down makes this an invariant. So, we call psi i the defining or fundamental and psi upper i to be the conjugate; these indices can be raise than lowered using a. So, there are 3 we will be not able to finish everything today, but we will continue next time.

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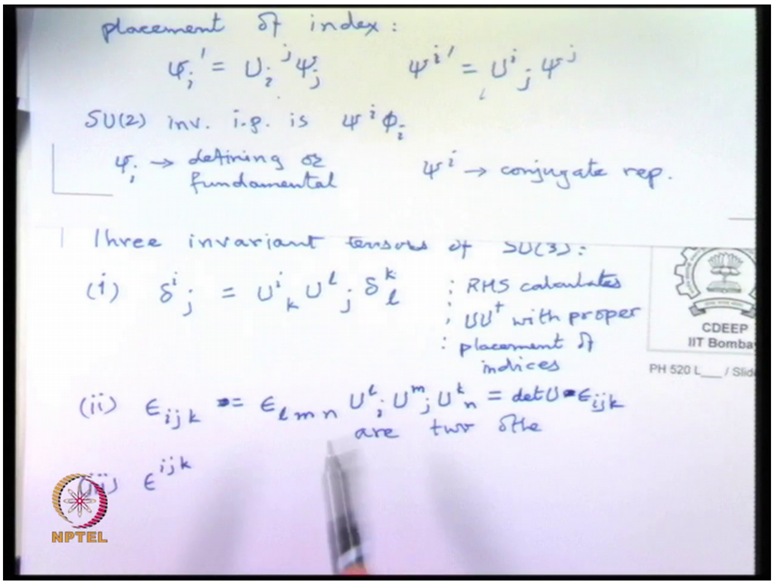

But one more thing is that there are 3 invariant tensors of SU 3, one is what we already saw the inner product. This is nothing, but delta i j, this is left invariant by SU 2 operations because you applied the u here and u there. So, let us write down how it works in this notation U i k and then U L j delta k l.

So if you sum this with this and this with this. So, what was the notation? So, we had this notation, that the low for the conjugate representation, it is in lower index that couples to the upper index of the conjugate index so, here u u U i k, the k couples to this k and this l I think this is fine. So, this is the convention we are going to follow ok. So, not the conjugate once, but they use themselves. From the use we can see U i k U l j, but if I multiplied by delta k l that, because u is unitary this actually U U dagger ok. So, r h s calculates with proper placement in dieses I just takes this and therefore, it basically gives 1 because u u dagger is equal to 1. The other 2 are epsilon i j k and epsilon i j k.

Because the up and down are not equivalent and you know what this is, its the (Refer Time: 27:27) tensors. So, patterns, so oh, but we have to check that it is invariant right. So, that is because it calculates determinant of u, have I written it somewhere. So, j k what alphabet you like l m n equal to det u times epsilon i j k. So, this is actually a theorem you can check that the epsilon tensor of the correct denasality which is same as size of these matrices. So, epsilon times doing this, basically just calculates the determinant of the matrix U this is a generic result.

But in our case determinant U is equal to 1. So, it becomes same as epsilon i j k.

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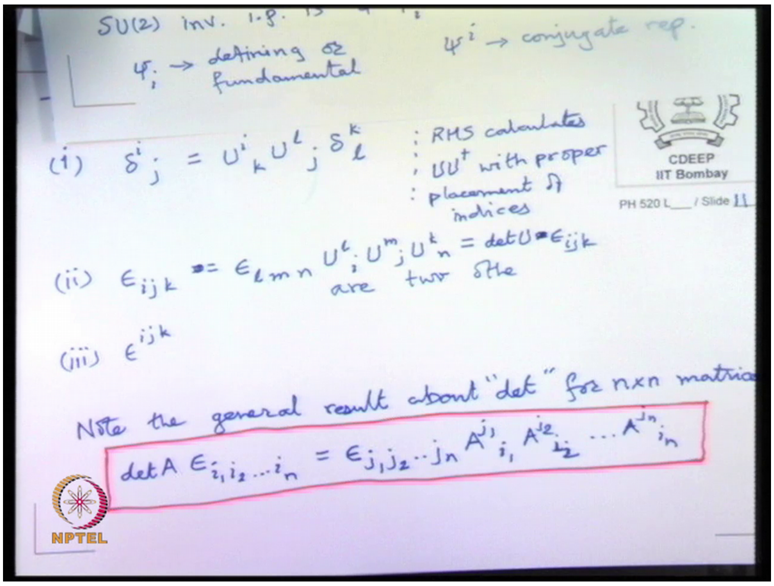

So, note the general result in n dimensions. So, sorry we have to write it like this i 2. I can justify it in 2 minutes why this is correct because you know how to calculate determinant right you are to take you have to select any one element and then keep selecting other elements, which are neither in the same row not same column. So, that is what the epsilon imposes on you if one side choose this row, I have to next in some column which ever you like, but then I have to take a row which is not the previous one, that is empty symmetric and some other column and so on.

If I do this choosing any column I like, but the point is I have to keep choosing a different column. So, so here is the statement I can keep choosing any columns I like if I choose any 2 of the column is the same I will get 0. So, on the left hand side what it says is, you better choose all the column also distinct and if you did that thing then you will get determinant of a that is what this statement is saying. And finally, you have to just once make sure of the correct sequence it is 1 2 3 and that is plus 1, but if it is 1 3 2 and then it is minus 1 and so on so, that all works.

So, this is the general formula of matrix algebra, and actually this is the way of that what is the symbolic way of defining determinant. We are always taught determinant as a operation like which has described, but this is the statement of what the determinant is ok. So, that applied here will give you that the epsilon tensor is invariant because epsilon tensor.

So, given this tensor, if I wanted to transform it what do I have to do? I have to transform each of its index the application of a U. But process of transforming is nothing, but because the tensor itself epsilon tensor, it produces det u times corresponding other epsilon, but det u is one because its SU n. So, the epsilon tensor is an invariant tensor and similarly the upper epsilon is invariant. The point is that using these you can convert one representation in of the other in to another ok.

So, we will stop here with this now. And next time we will finish young tableaux and the general scheme of what SU n groups are.

Thank you.