Special Theory Of Relativity
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Lecture - 6
Length Contraction and Time Dilation

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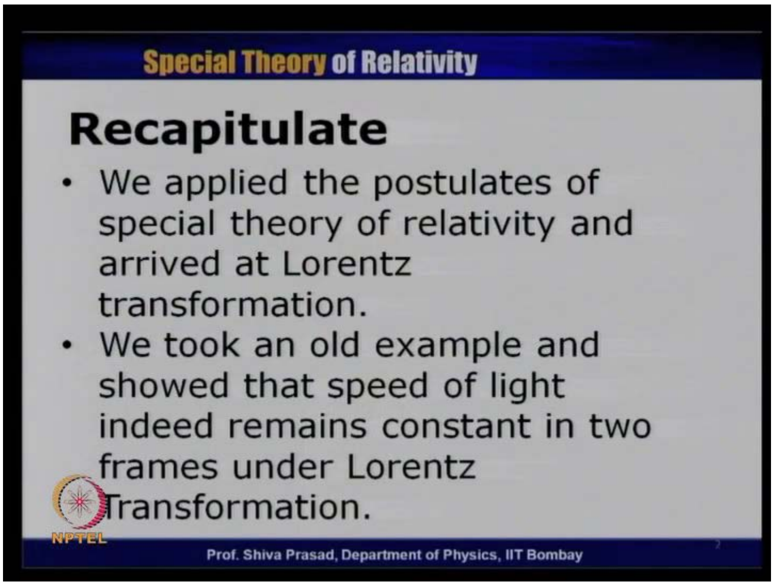

In our last lecture, we had applied the postulates of special theory of relativity and eventually arrived at Lorentz transformation. We do not say that we derived Lorentz transformation. We just say that, we arrived at it. We presented a series of arguments on how Lorentz transformation could be arrived at. A real test that Lorentz transformation is believed on is on experiment. If we find an experimental support, then only we believe a Lorentz transformation. After that, we took an example, which was an old example of a light being emitted from the origin.

We had earlier found out that, under Galilean transformation, the speed of the light does not remain constant, if we change the frame of reference, which was obvious because the classical mechanics was not designed for that. Then we apply Lorentz transformation on the same example and said that if we apply Lorentz transformation, then the velocity of light turns out to be same in the other frame also, which is what Lorentz transformation is based on or the second postulate of special theory of relativity is based on. So, that is the way we proceeded in our last lecture.

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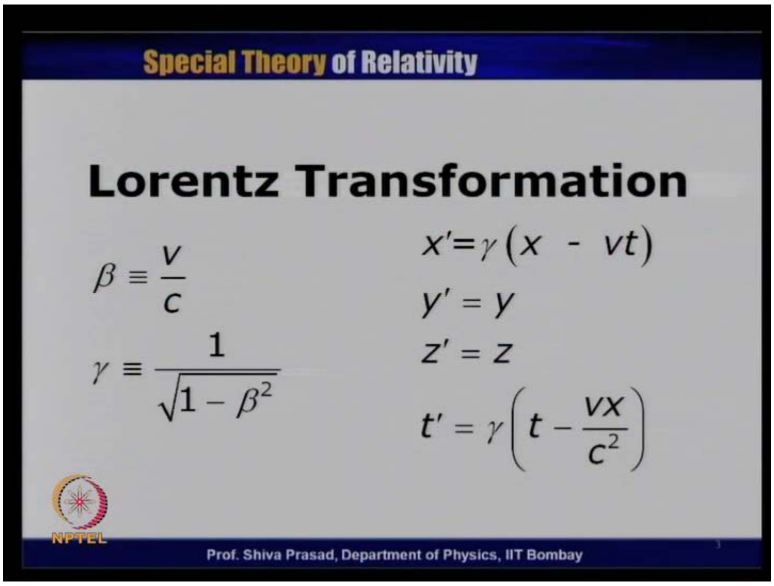

Just to re-capitulate capitulate, let us write or let us discuss the Lorentz transformation again. These are the equations governing Lorentz transformation. These are the equations. We introduced two symbols; one is beta, which is V divided by C. Just to remind, V is the relative velocity between the frames and C of course is the speed of the light.

Then, we defined another parameter called gamma, which was 1 upon under root 1 minus beta square. If we use these two parameters, the Lorentz transformation equations become like this. For X co-ordinate, it is x prime is equal to gamma multiplied by X minus V t. Y prime remains equal to Y and Z prime remains equal to Z and t prime becomes gamma multiplied by t minus V X divided by C square. We had earlier said that in the non relativistic limit, meaning when V is much smaller than C, this beta is negligible in comparison to 1 and gamma is essentially close to 1. Then X prime just becomes X minus V t, which is the standard Galilean transformation. These two equations are common in a Galilean transformation

Here, if V is very small in comparison to C, this particular term is neglected or can be neglected. This gamma will tend to 1, then t prime will be tending equal to t, which is again the standard classical Galilean transformation. So, these equations effectively become important, only when V is close to C, where this factor beta square cannot be neglected in comparison to 1.

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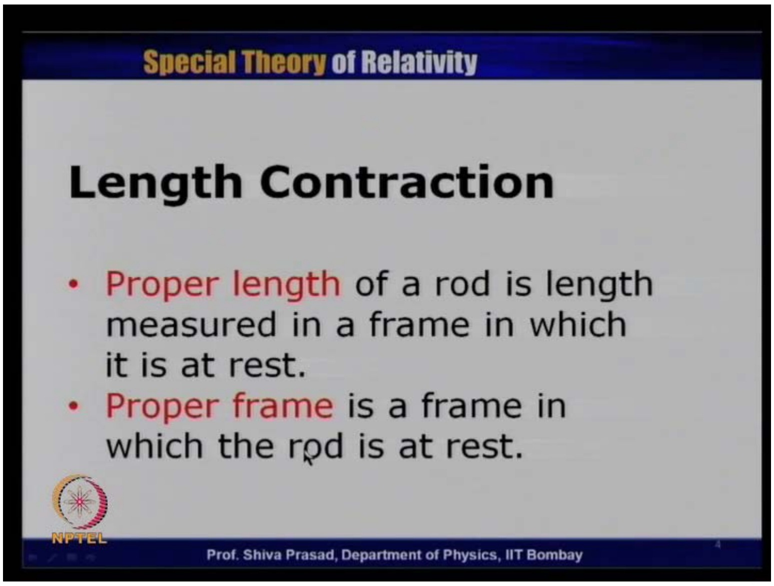

Today, we will try to discuss some of the example, some other examples of Lorentz transformation. Specifically, two very very important and popular results arrived out of Lorentz transformation. One of them is called length contraction. The length contraction, as the name suggests, means that the length of an object may not be constant, if an observer in a different frame observes the length of the same object. In fact, as you will be seeing, there is a contraction or length becoming smaller is associated with this phenomenon, which is called length contraction. But before we really derive it, let us understand a few terms, because important to derive or to work with length contraction formula correctly. It is important that we are familiar with these things.

The first term is the proper length. Proper length as the name suggests or as we have defined of a particular rod is length measured in a frame in which it is at rest. So, let us imagine that we have to measure the length of this particular rod. This is a rod and the length of it you have to measure. Now, this particular rod is stationary in my own frame of reference. So, I do not see any motion of this particular rod because as I said, this is stationary in my frame of reference. So, whatever is the length that I would measure for this particular rod would be called proper length. If this particular rod was moving in my frame of reference, then the length that I will measure of this particular rod would not be call proper length.

So, proper length has a specific meaning that it must be measured in a frame of reference, in which this particular rod is at rest. That frame of reference, in which it is rest, is in which it is at rest, that particular frame of reference is called proper frame of reference. That is what we have written in this particular transparency. Proper length of a rod is length measured in a frame, in which it is at rest and proper frame is a frame in which this rod is at rest. Now, let us assume that this particular rod is along the x direction. Remember this particular x direction is the direction of the relative velocity between the frames. So, let us assume that this particular rod is along inclined or it is put along the x direction or it is put exactly on the x axis to be more precise.

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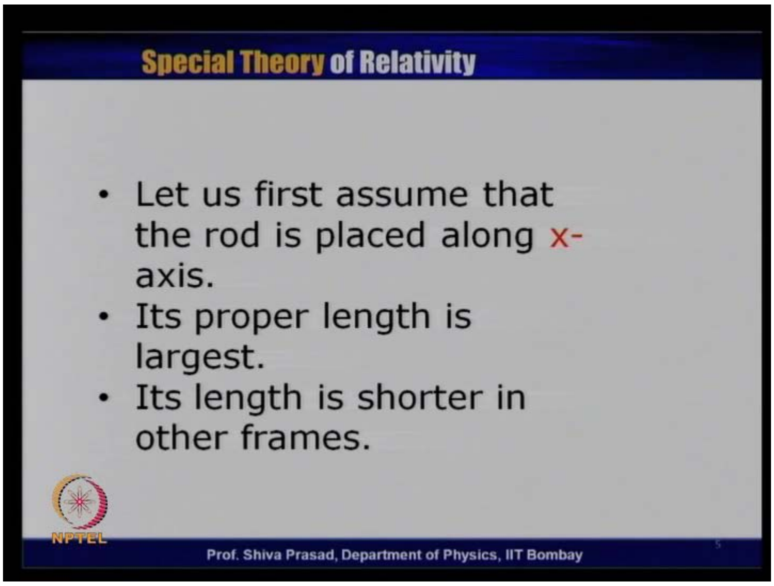

So, this is what I have written. Let us assume that the rod is placed along the x axis. Then we will show just now, that its proper length is the largest and if we go to any other frame, in which there is a relative motion along the X direction, then its length will turn out to be shorter. That is why it is called length contraction.

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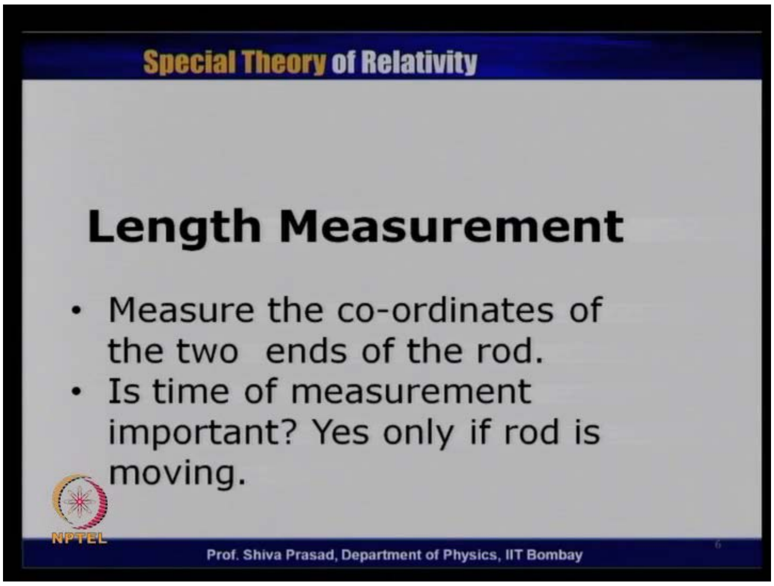

Now, let us suppose that, we have to measure the length of this particular rod. How do I measure? The question is very simple. I measure the co-ordinates of the two ends of the rod. If I measure the co-ordinates of the two ends of the rod, actually by subtracting these co-ordinates, I can always find out what is the length of this particular rod. So, this rod length is to be measured, I can measure what is the co-ordinate here, I can measure what is the co-ordinate here. Then take the difference of these two and this will give me the length of the rod. But the question which I am posing is that, is time important. Let me explain what I mean to say. Suppose, I measure this particular length, this particular end or co-ordinate of this particular end of the rod, let us say at t is equal to 0. Now, this end, this particular co-ordinate, can I measure at any other time is time? Is time important for its measurement? See, the answer to the question is that, no, it is not important, as long as this particular rod is at rest.

So, I can measure this co-ordinate now, go and have a cup of tea, come back and then measure the other co-ordinate. Still the difference of these two co-ordinates will give me the length of the rod, because this rod is fixed in my frame of reference. It is not moving. So, this rod is not going to change its co-ordinate as a function of time. So, it is immaterial when I decide to measure the co-ordinate of this particular end and when I decide to measure the co-ordinate of this particular end. But suppose this particular rod was moving in my frame of reference, then the same statement is not correct. Because if I make measurement of this particular end now and make the measurement of the co-ordinate of this end little later, during this particular time this rod has been moving. So, this rod is moving. I make a measurement now and by the time I go the other end make the measurement, by that time the rod as been displaced from its position. So, the co-ordinate has changed

Therefore, if you still decide to take the difference of the two co-ordinates, that will not give me the correct length of the rod. This is what I have said. Measure the co-ordinates of the two ends of the rod. Is time important for the measurement? Yes, only if the rod is moving. Then it is important that the two ends of the rod must be measured at exactly at the same time.

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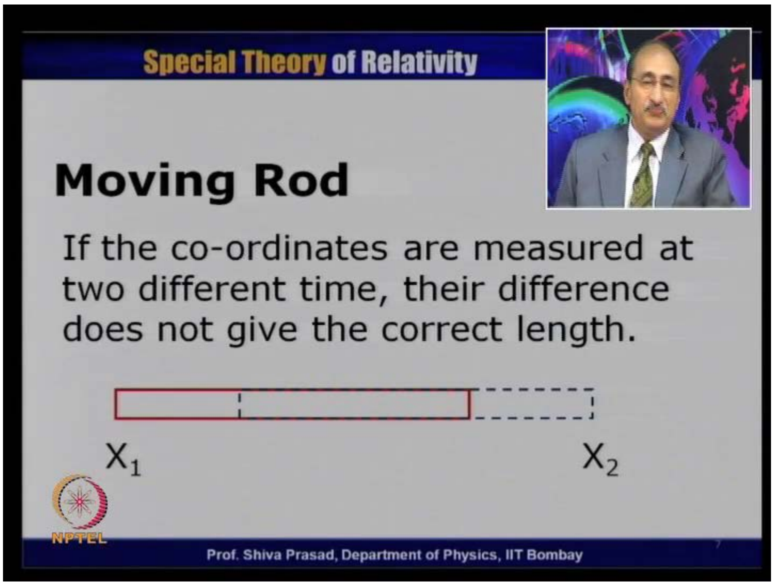

This situation, I have pictures here in the next transparency. I have depicted a moving rod. This red is the position of the rod at a given time. Let us call time t is equal to 0. Now, let us suppose the time t is equal to 0. I made the measurement of co-ordinate of this particular end and find it out that is equal to X 1. Now, I make some delay. I do not immediately make the measurement of this particular end. But after some time, I decide to make the measurement of the other end of the rod. Then by this particular time, this rod is moved ahead. So I, at a later time, what I would be measuring is probably these co-ordinates which is X 2. You can clearly see that X 2 minus X 1 is not equal to the length of the rod. It is a different question.

From these two measurements also, I can find out the length of the rod, provided I take care of this displacement, that the rod has taken or the displacement that the rod as undergone during the difference of the time, during which I have to measure. That is a different question. That also I will give an example little later. But at this moment, what I want to do is, I want to measure the length of the rod by just measuring the co-ordinate differences. For that, if the rod is moving in my frame of reference, it is important that the difference the two co-ordinates must be measured in at the same time. Only under that condition, the difference of the co-ordinates will give me the correct length of the rod. So, this is what I have written. If the co-ordinates are measured at two different times, their difference does not give the correct length. That is what I have said.

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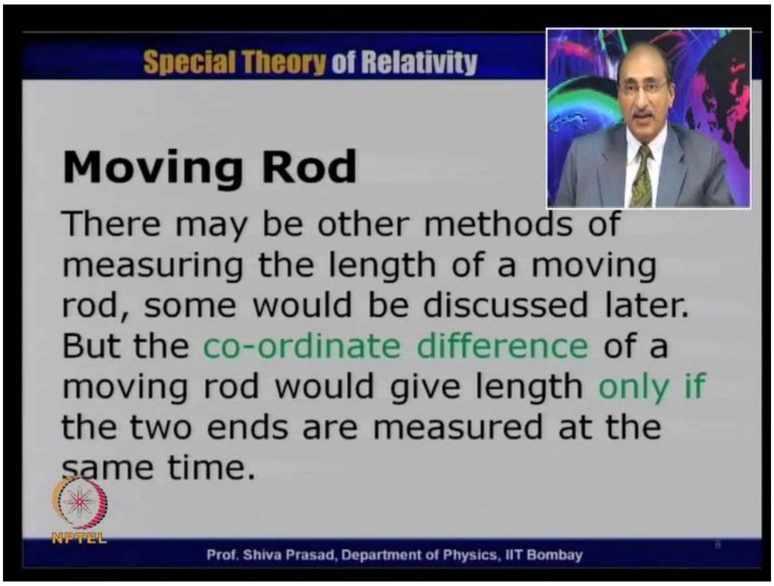

There may be other methods of measuring the length of a moving rod. Some would be discussed later. As I have mentioned, if I can calculate how much is the rod, how much the rod has moved during the time difference that I have taken or the time I have taken in making a measurement of one end to the other end. Of course, I can find out the length of the rod. That I will do little later. But if the co-ordinate difference of a moving rod has to give me length, then for a moving rod, the two ends must be measured at the same time. That is what I have written. But the co-ordinate difference of a moving rod would give correct length, only if the two ends are measured at the same time.

This is an important point to note, because this is where I find a very large number of students make a mistake. They do not bother about time. They just decide and they just say just x 2 minus x 1 is equal to length, which may not to be correct. Because if the object is moving in the frame of reference, then x 2 minus x 1 need not give the correct length, until these two measurements were carried out exactly at the same time.

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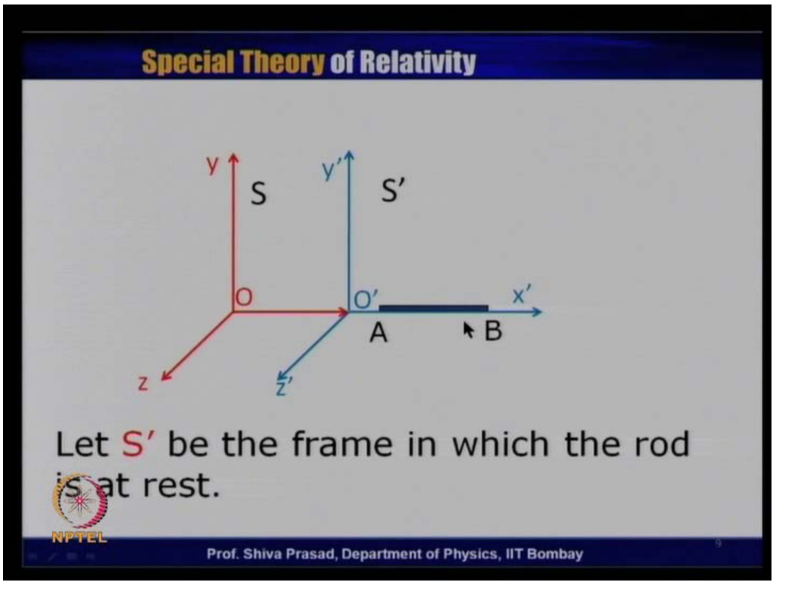

Now, let us come back to the problem. Let us assume that this particular rod is at rest in S prime frame of reference. We have already defined what are our S frame and S prime frame of reference. Lorentz transformation is applicable when we impose sudden conditions on S and S prime frame of references, which we have discussed earlier, that the S' should be parallel and the relative motion should be along x direction. So, that is anyway have to be valid. But what I am assuming in this particular case that it is the S prime frame of reference, in which this rod is stationary

It means, it is S prime frame which is the proper frame. So, this rod A B, which I am calling here, A B is at rest in S prime frame of reference. Obviously, it means that in the S frame of reference, this particular rod is moving. I can always imagine that I have put a rod, let us say in a train and the train is moving with respect to an observer on ground. So, the person sitting on the train feels that the rod is not moving, but a person sitting on the ground, and feels that the particular rod is moving. Remember, this x direction is the direction of the relative motion. This is in this direction that S prime is moving relative to

S. Now, we have earlier agreed that relativity things are simple, if we define events. Generally, it is always a good idea to define events. So, let us suppose we define events relating to the measurement of the length of this particular rod and these events, I am defining here, calling event number 1 and event number 2.

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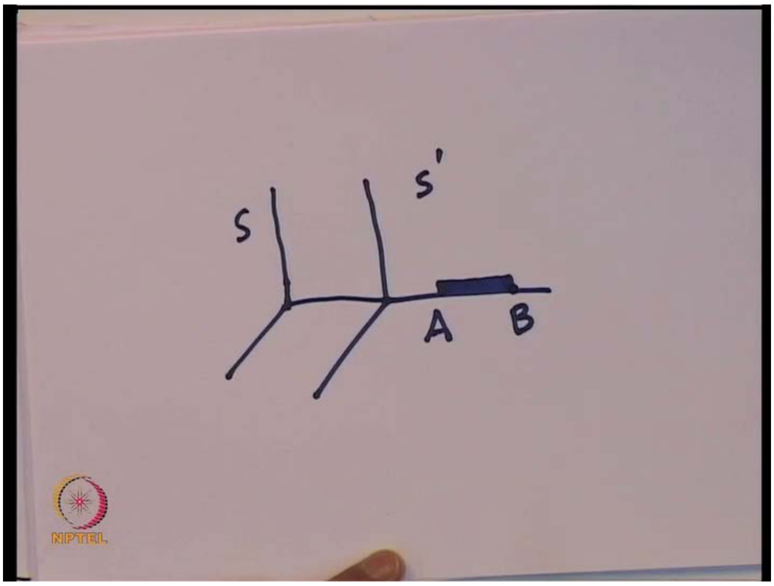

So, this is my event number 1. Let me just write to show the rod here. This was my S prime frame of reference and this rod was at rest here. This was my end A, this was my end B and this rod is stationary in S prime frame of reference, being also observed by another observer, which is an S frame. In this S frame, this rod is actually moving to the right, as the frame of reference, S frame is also moving to the right with respect to S.

So, my event number 1 is that, I make the measurement of the co-ordinate of this particular end of the rod. Let us assume that this particular measurement is being done by an observer in S frame of reference. So, he makes a measurement of this particular, the co-ordinate of this particular end of the rod. This is my event number 1. Then he goes here. Let me now say goes here, but he also makes a measurement of the end B of the rod. This I call as event 2. Now, as this particular rod A B is not stationary in S frame of reference, so in peaceful he cannot go from A to B and make the measurement. The measurement of the co-ordinate here and co-ordinate here have to be done at the same time, if the co-ordinate difference of these two events has to give me a correct length of the rod.

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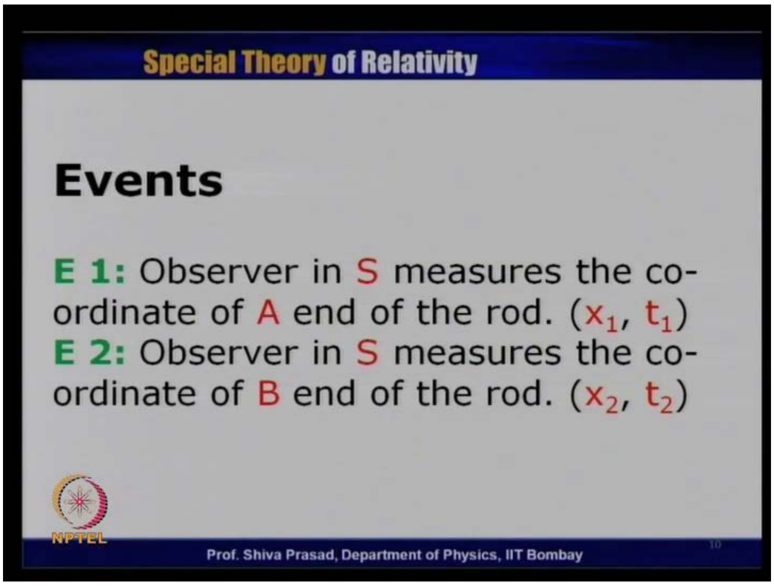

So, this is what I have written here. I have defined my events. My event number 1 is, observer in S measures the co-ordinate of A at the end of the rod. Let us suppose this rod be x 1 and t 1. Normally, we have a tendency of now including time also as a co-ordinate because as you will be seeing going ahead in the course, we realize that for many purposes, time x acts as the another co-ordinate. So, because this is event number 1, so I have put a subscript here 1. So, this event number 1 occurs at a co-ordinate of x 1, at a position co-ordinate of x 1 and at a time co-ordinate of t 1. Similarly, the observer in S measures the co-ordinate of the B end of the rod. Let us suppose, he does it and finds out the co-ordinate to be x 2 and the measurement is done at time t 2. So, the co-ordinate of this particular event is x 2 t 2, while the co-ordinate of the first event is x 1 t 1.

As we have earlier realized and discussed, this x 2 minus x 1 will give me correct length of the rod, only if t 2 is equal to t 1, because it is in x frame that the rod is actually moving.

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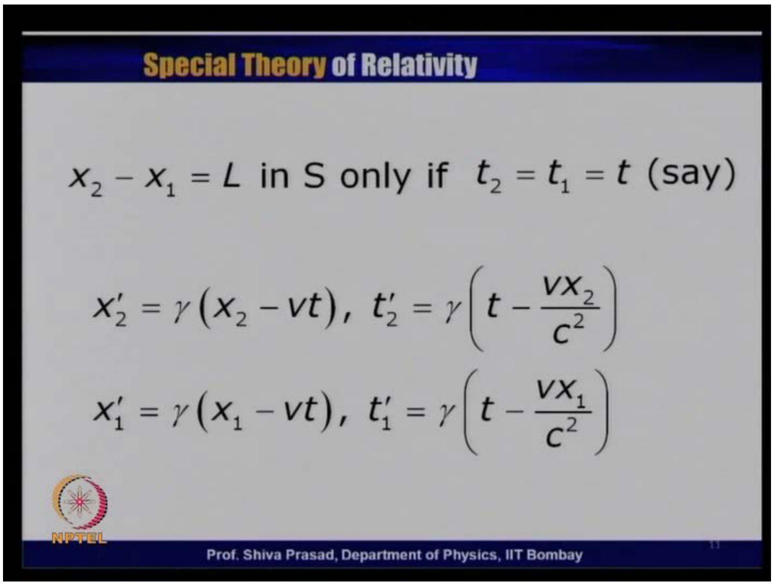

This is what I have written here. x 2 minus x 1 is equal to l in S, only if t 2 is equal to t 1, which I have said. Let us assume that is equal to t rather writing same symbol. So, as I said, let us assume that the t 2 is equal to t 1 is equal to t. Then these two co-ordinates, I used Lorentz transformation to find out the information in S prime frame of reference.

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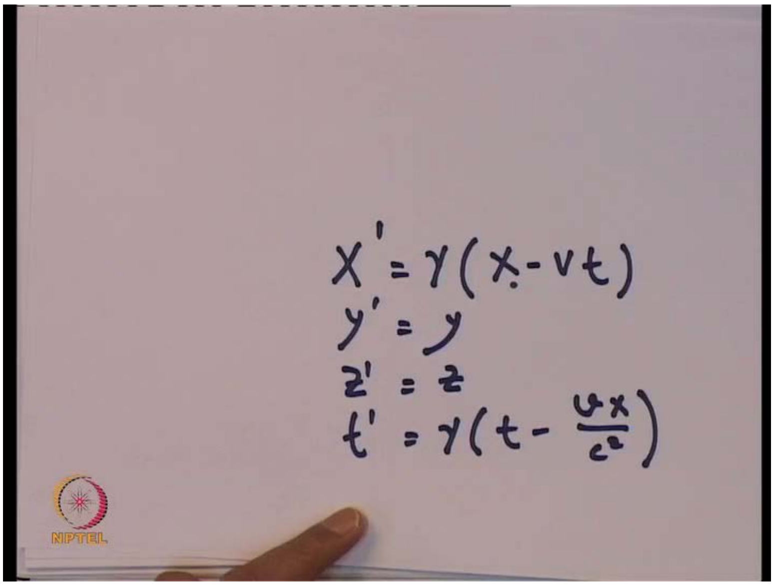

Let me write the Lorentz transformation once more here. Just to remind you, it was x prime is equal to gamma x minus v t. y prime is equal to y. z prime is equal to z. t prime is equal to gamma t minus v x divided by c square. This is what was my Lorentz transformation. Now, let us apply the Lorentz transformation on the two events. If I apply, let us say first event number 2, it means the value of x was x 2 and time t was t 2. I will get the co-ordinate of that particular event in S prime frame of reference. So, I will get x 2 prime, the co-ordinate of the second event in S prime frame of reference.

Similarly, if I substitute here x 2, substitute here t 2, I will get t 2 prime, which is the time at which the second event occurs according to an observer S prime frame of reference. This is precisely what I have written here. If you go back to the transparency, it is x 2 prime is equal to gamma x 2 minus v t, because t 2, I have taken to be equal to t. This is what I have written earlier. So, this I have just put it equal to t. Then t 2 prime will be given by gamma t, which is just t 2 is equal to t minus v x 2 divided by c square. Exactly the same thing. I put the co-ordinate position and time here. For the first event, put position and time for the first event, I get these two equations. You can very clearly see that, as far as S prime observer is concerned, t 2 prime is not equal to t 1 prime. See this t is same as t, but this x 2 and x 1 there has to be difference. If the length has to have some, if the rod has to have some length, then x 2 cannot obviously be equal to x 1. Therefore, this t 2 prime is not equal to t 1 prime.

So, what it means that, according to the observer in x prime frame of reference, these two events occurred at different time. It should not be surprising to us because remember earlier, we have discussed the simultaneity relative. I am I shall be discussing and I shall be harping a number of times saying that, simultaneity's relative. It is in S frame that the two events occurred on the same time, and hence, they where simultaneous. According to S prime frame of reference, these two events did not occur at the same time. Therefore, they are not simultaneous.

So, according to him, the two events occurred at a different time. Would it mean that X 2 prime will not agree that X 2 prime minus X 1 prime is equal to length? No, that is not correct because in x prime frame of reference, this rod is stationary. So, it is not important whether the two ends should be measured at the same time. What is important is that, we just make measurement at any given time.

So, even if the measurement of one end of the co-ordinate in S prime frame of reference was done in a different time from the other end, the rod being at rest at this particular frame of reference, the time, the position difference will still give me the correct length.

So, what I want to argue out that x 2 prime minus x 1 prime is equal to the correct length of the rod in x prime frame of reference irrespective of the fact that t 2 prime is not same as the t 1 prime.

While in S frame, we could not have done a similar argument. I have to force the time to be same in S frame of reference. Only in that case x 2 minus x 1 would have given me the correct length. Now, if I want to find out a relationship between the length measured in the S and S prime, all have to do is to take the differences. So, I know that x 2 minus x 1 x 2 minus x 1 was the correct length in S frame of reference because these two times are same. x 2 prime minus x 1 prime will anyway be correct length in S prime frame of reference, even though the times are different. So, I will say that x 2 prime minus x 1 prime is L prime. The length of the rod in S prime frame of reference and call x 2 minus x 1 as the length of the rod in S frame of reference.

So, if I subtract these two, we will just realize that this particular term v t here would cancel and time being identical, so you will just get x 2 prime minus x 1 prime is equal to gamma multiplied by x 2 minus x 1.

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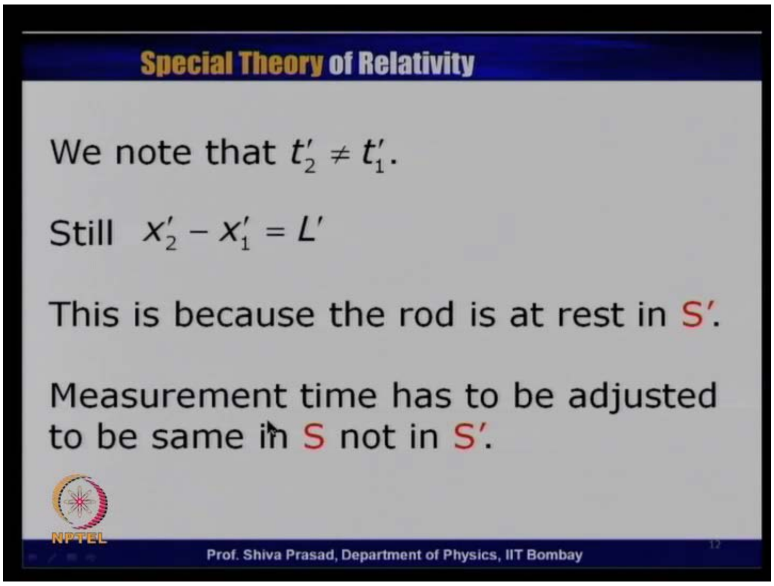

This is what I have written in the next transparency. We note that t 2 prime is not equal to t 1 prime. Still x 2 prime minus x 1 prime is equal to L prime. This is because the rod is at rest in S prime. Measurement time has to be adjusted to be same in S and not necessary in S prime.

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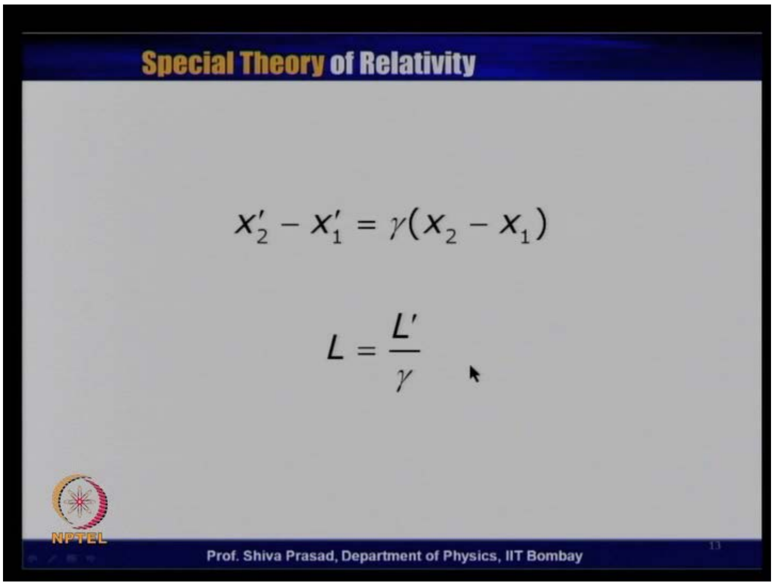

I have taken the differences. I get x 2 prime minus x 1 prime is equal to gamma x 2 minus x 1. As we discuss, this is the length in S prime frame of reference, this is the length in S frame of reference, I call this is L prime and I call this is L. So, this is L prime is equal to, this L equal to L prime divided by gamma, which is what I have written in this particular equation. As we as seen earlier that, gamma is to be always greater than 1, because you remember, gamma was equal to 1 upon under root 1 minus v square by c square.

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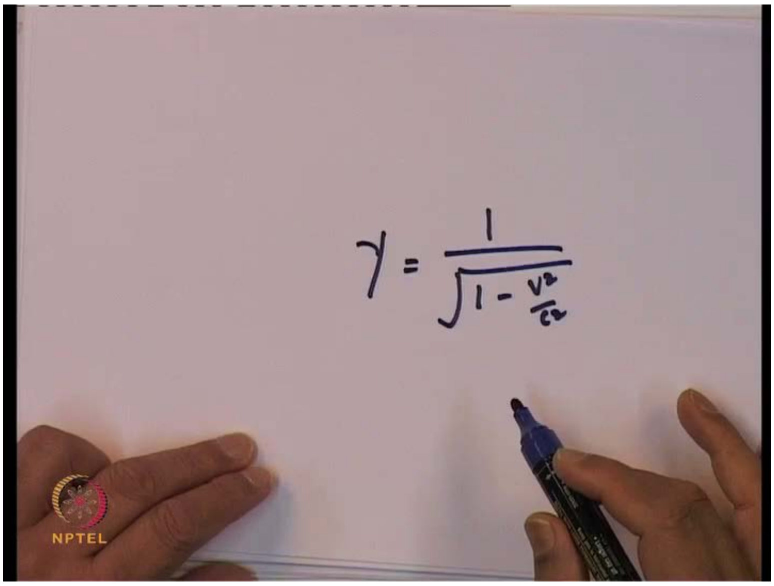

So, you are always subtracting some quantity from 1. So therefore, this particular factor is always going to be less than 1 and 1 divided by something less than 1, always make a quantity greater than 1. So, gamma would always be greater than 1. Therefore, this L would always be smaller than L prime.

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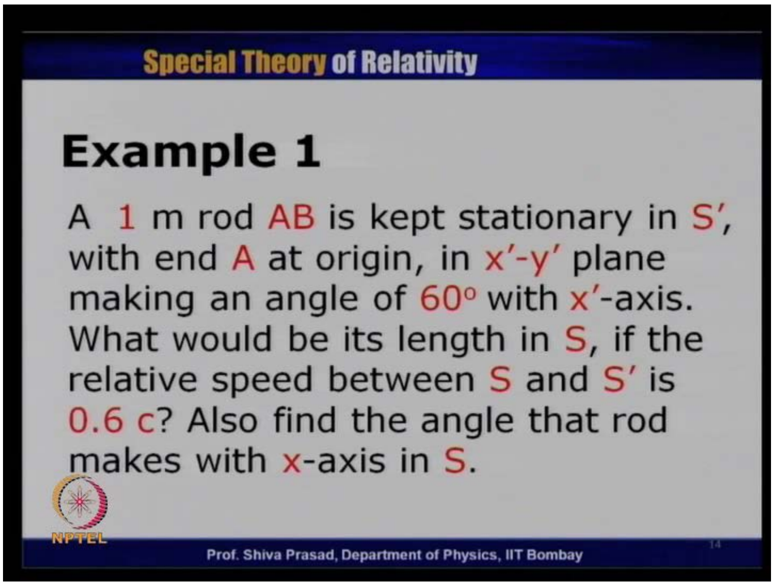

So, we say, length is contracted. It is called length contractions. Let us take an example. Slightly different example, in which, I do not make the rod along the x direction. But let it have a slightly different orientation. So, in this particular example, I have assumed that this particular rod is line actually the x prime y prime plane, I still assume that it is x prime frame of reference, which is the proper frame of reference for this particular rod.

So, this rod is actually stationary in an S prime frame of reference. But only difference I say that, this rod is no longer along this particular direction, but making some angle. You know somewhere here like this. So, my rod is like that A B and this is S prime frame of reference. So, I assume that the end A of this particular rod is at the origin here. I assume that length of the rod is 1 meter. So, effectively in S prime frame of reference, I know the x co-ordinate of A which is 0 0, x co-ordinate of B which is I can find out easily, if I know the length and if I know the angle. What I have to do now is to find out the length of this particular rod in S frame. That is what is the question.

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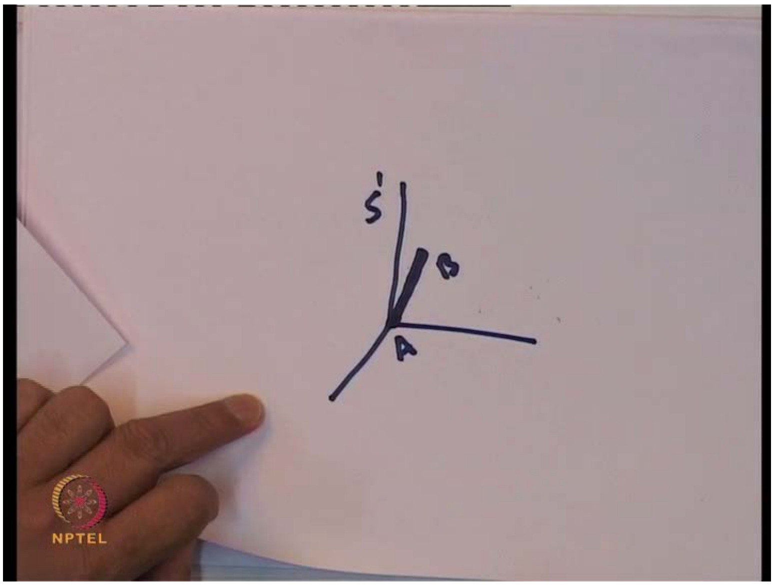

So, let may read the statement of the question, statement of the example. There is the 1 meter rod A B, the one which I have said. The length has been given to be 1 meter rod and is kept stationary in S prime frame of reference, which is the proper frame of reference, with end A at origin, which I just now showed in the picture, in x prime y prime plane making an angle of 60 degrees. So, I know theta value which is 60 degrees with x prime frame, with prime axis. What would be its length in S, if the relative speed between S and S prime is 0.6 c? So, I have given some numbers. So, you can calculate. The answer, a numeric answer. Also, find the angle that the rod makes with x axis in S. So, according to S observer, at what angle this particular rod is inclined with respect to x axis.

So, this is essentially the question. The same picture, which I had drawn on this particular piece of paper, and we have S prime frame of reference. This particular rod is at rest with A being at the origin. This end B being here. This angle being given 60 degrees and this length is being to be given 1 meter.

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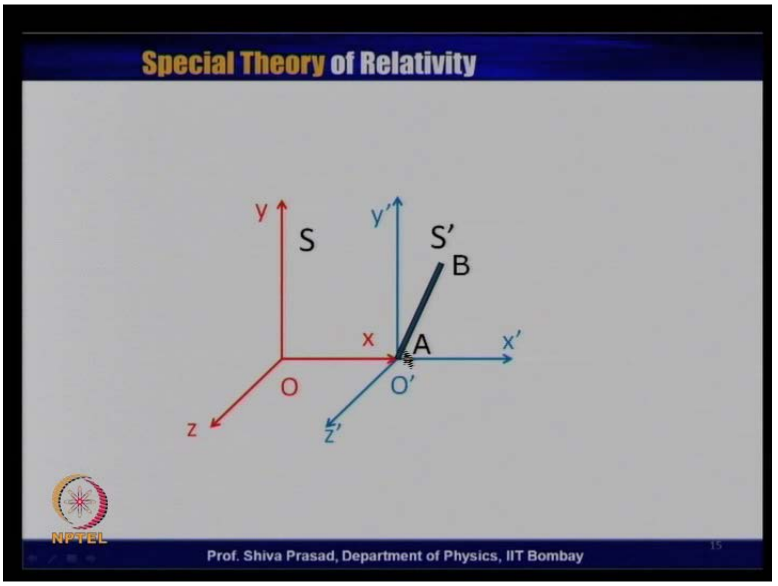

Now, like before, I will define the events and will define the events to show that these two events occurred at the same time in S frame of reference. Because it is the S frame that this particular rod is moving. Also, because here y co-ordinate is also involved; see in earlier example, we did not bother about the y co-ordinate, because y was 0 in S frame. It had to be 0 in S prime frame of reference. But here, because this particular other end of the rod will have a different value of y than the first end of the rod. Begins will have a different y than the a end. Therefore, is better talk about the y co-ordinate also. So, we will define our events.

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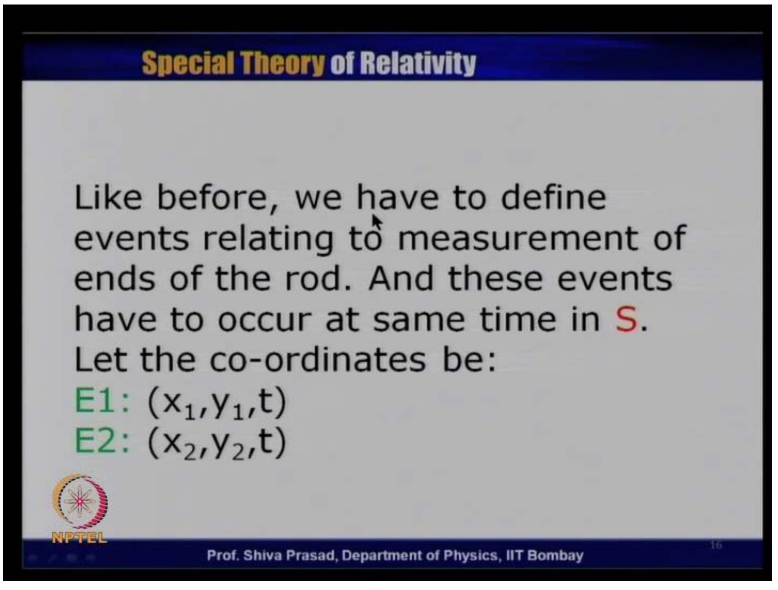

So, I said like before, we have to define events relating to measurement of the ends of the rod and these events have to occur at the same time in S. Let the co-ordinates of these events be again, like before, event is x 1 y 1 t. So, this co-ordinate occurs, this particular, sorry, event occurs at co-ordinates x 1 y 1 and t, according to an observer in S frame. Similarly, event number 2, I have put subscript 2, happens at x 2 y 2 and at time t. These two times I have to put same to ensure that actually the measurement of these co-ordinates will give me the correct length of this particular rod. So, in principle, as we will be discussing, if I take the difference of this and take the difference of this and square and add, take the under root, I will get the length of the rod in S frame. Again, we have to carry out the Lorentz transformation to find the corresponding co-ordinates of these two events in S prime frame of reference. The only difference in this particular case is that I already know the co-ordinates, so I can write this equation. It makes not such a great difference, but just ensure that we understand the situation well.

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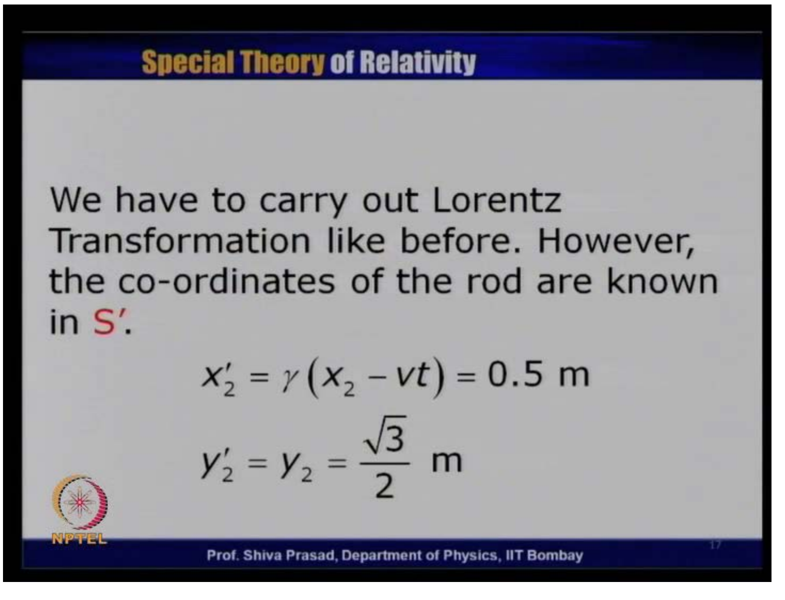

Now, we applied for the second event the Lorentz transformation, which is x 2 prime is equal to gamma x 2 minus v t. Just like in the earlier example of length contraction.

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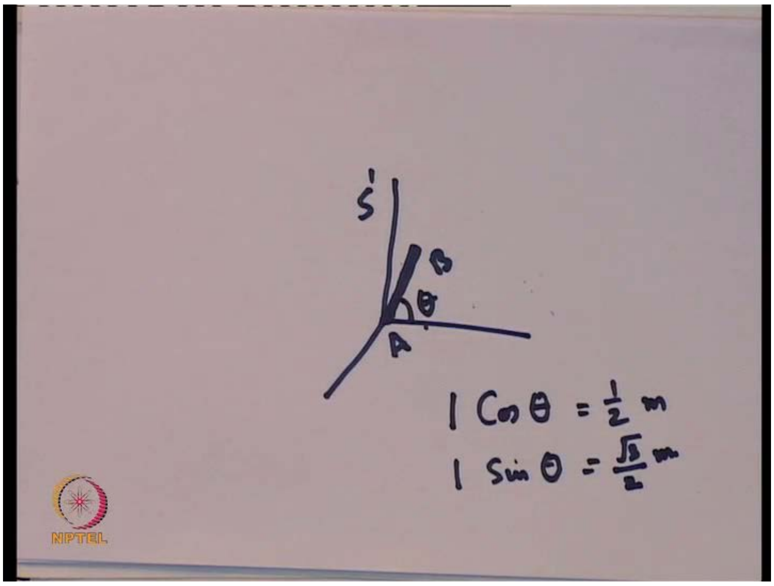

Only thing I say that, if this particular length was 1 meter, this angle is theta, then the x co-ordinate of this particular rod will be 1 cos theta, because 1 is the length of the rod. The y co-ordinates of this particular rod will be equal to 1 sin theta. So, this is the x co-ordinate, this is the y co-ordinate, this will be 1 cos theta, this will be 1 sin theta, because theta has been given equal to 60 degrees. So, 1 into cos 60, which is half and this will be half meter and 1 sin theta will be equal to root 3 by 2 meters. So, this particular co-ordinate, the y co-ordinate would be equal to root 3 by 2 meters and while this x co-ordinate will be equal to half meter.

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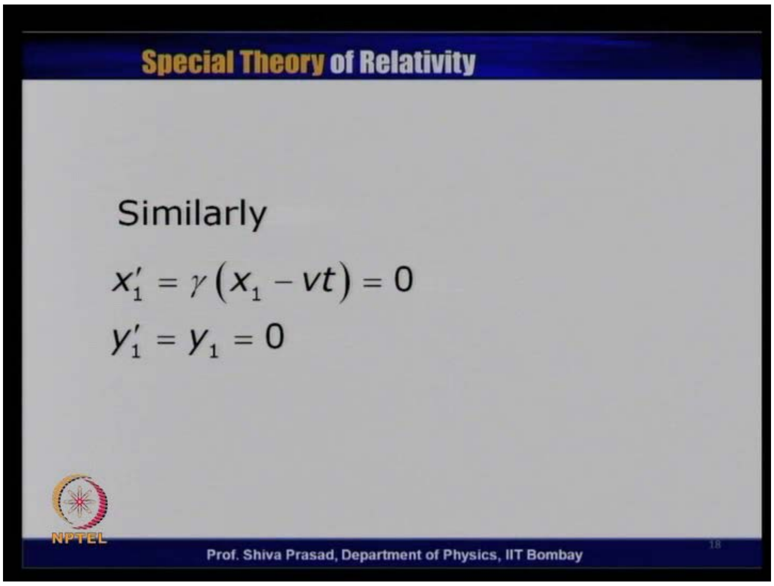

So, this is what I have written in this particular transparency, that x 2 prime is equal to gamma x 2 minus v t, which I know is equal to half meter and y 2 prime is equal to y 2, which I know is under root 3 by 2 meters. Similarly, I can take the Lorentz transformation of the x co-ordinate, which gives me x 1 prime is equal to gamma x 1 minus v t. This co-ordinate is equal to 0. y 1 prime is equal to y 1 is equal to 0, because I know in S prime frame of reference and A is at origin. So, x 1 prime and x 2 prime, I am sorry, x 1 prime and y 1 prime both must be equal to 0.

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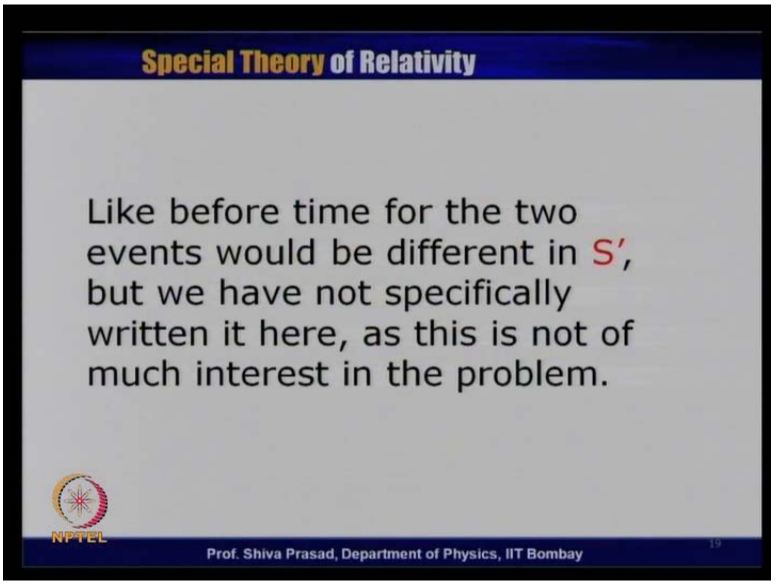

Like before, we take these two events to be different in S prime. The time will be different in S prime. But have not specifically written in this particular case, the equations involving the time difference. Refer the time transformation because as far as he problem is concerned it is asking is only for the length. It is not asking for time. So therefore, I am not written the equations relating to time transformation. I have just written the x transformation because for the problem, that is not important. Only thing I have to ensure, that the time of the two events is same in S. That, I have to ensure by putting the 2 times to be same. That is all.

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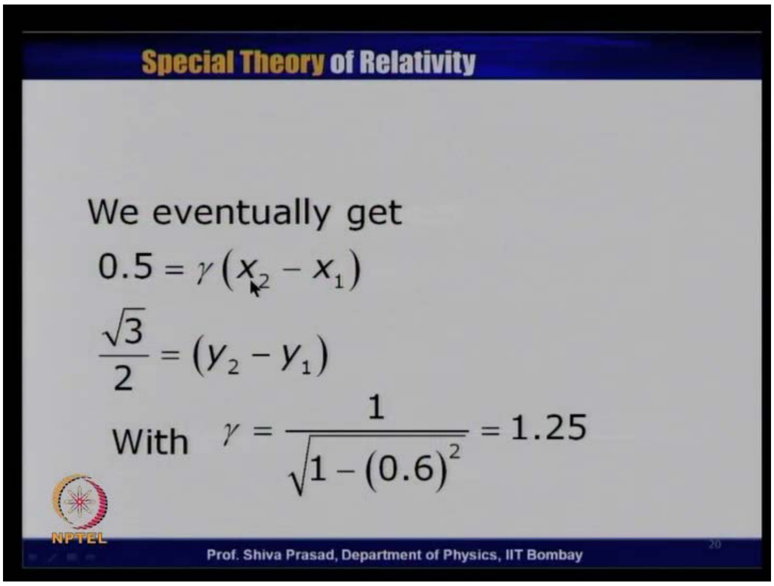

Like before, I take the difference of these two, just like what we did in the earlier case. I will find gamma x 2 minus x 1 to be equal to 0.5 because remember one was 0.5 and another was 0. So, this difference is just 0.5. Similarly, y 2 minus y 1 was equal to root 3 by 2 because one was root 3 by 2 and another was 0. So, this, this. Of course, for velocity equal to relative velocity equal to 0.6 c, I get gamma is equal to 1 divided by under root 1 minus v square by c square, which is 0.6 square. This comes 0.361 minus 0.36 under root is 0.8. So, this is equal to 1.25. So, gamma is equal to 1.25. This 0.6 gives you a very clean number for gamma.

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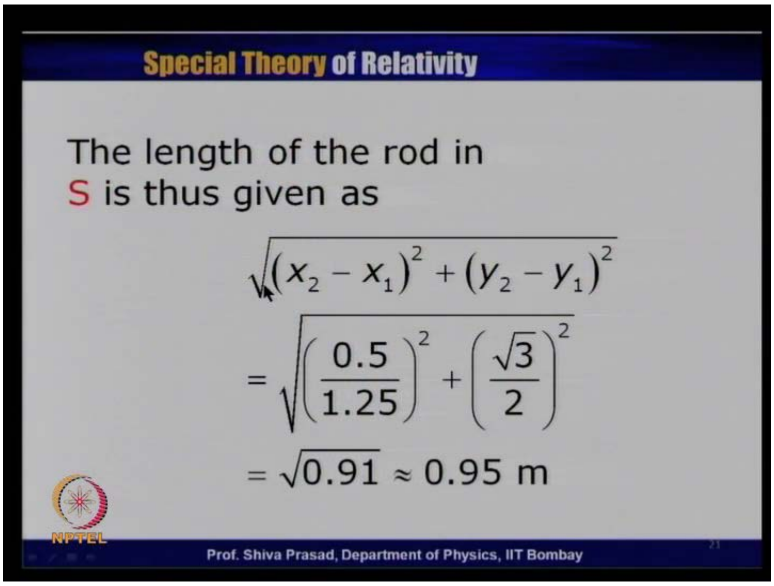

So, in many of the problems to make the mathematical simple, making the algebra simple, we choose the gamma value, which is something 0.6 0.8. These gives clean values of gammas. So, I substitute these values of gamma here and find the length. As I have said earlier, the length is given by x 2 minus x 1 plus y 2 minus y 1 square. Look back at this particular rod. You find out the co-ordinate of this particular rod. Even if they are not 0, so what I have to take the difference of the x co-ordinate and the difference of the y co-ordinate, this length plus this length, this length square plus this length square will give me this length square. If I take the under root, I will get the length of the rod. So, this is what I have written here. The length of the rod in S is thus given by x 2 minus x 1 whole square plus y 2 minus y 1 whole square.

We have just now seen from the previous transparency here, that x 2 minus x 1 is equal to 0.5 divided by gamma and gamma is equal to 1.25. So, I have written here is 0.5 divided by 1.25 square plus, here there was no gamma, it is just root 3 by 2 because y prime is equal to always y. If I calculate this number, we get under root 0.91, which is approximately equal to 0.95 meter. Hence, the length of the rod in S frame of reference would turn out to be approximately equal to 0.95 meter and not 1 meter. I have to calculate the angle and angle again, if I have to calculate this angle, I have to calculate this distance, and this distance, divide by 2 and this will give me tan theta. So, this is y 2 minus y 1, this is x 2 minus x 1 and divide the two and that will be equal to tan theta.

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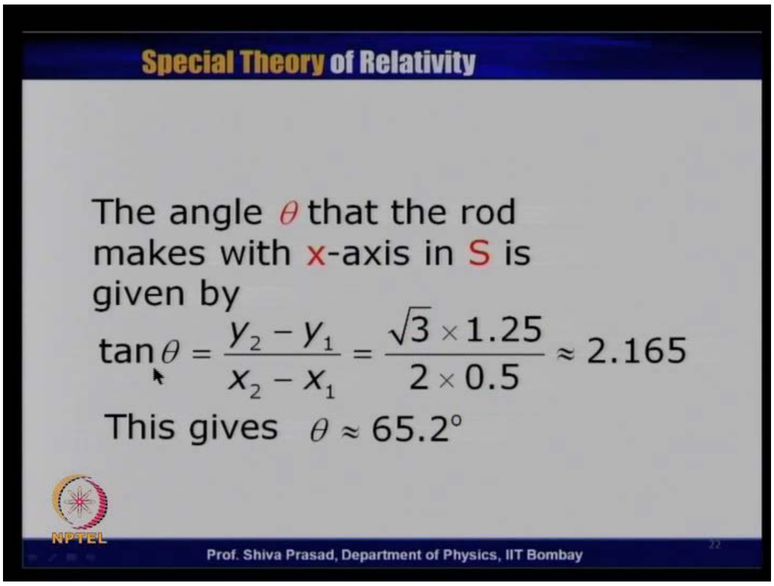

So, this is what I have written here in the transparency, that tan theta is equal to y 2 minus y 1 divided by x 2 minus x 1. y 2 minus y 1 was root 3 by 2. x 2 minus x 1 was 0.5 divided by 1.25. So, 1.25 comes into the numerator. If you calculate this number, approximately turns out to be 2.165, which gives theta approximately equal to 65.2 degrees.

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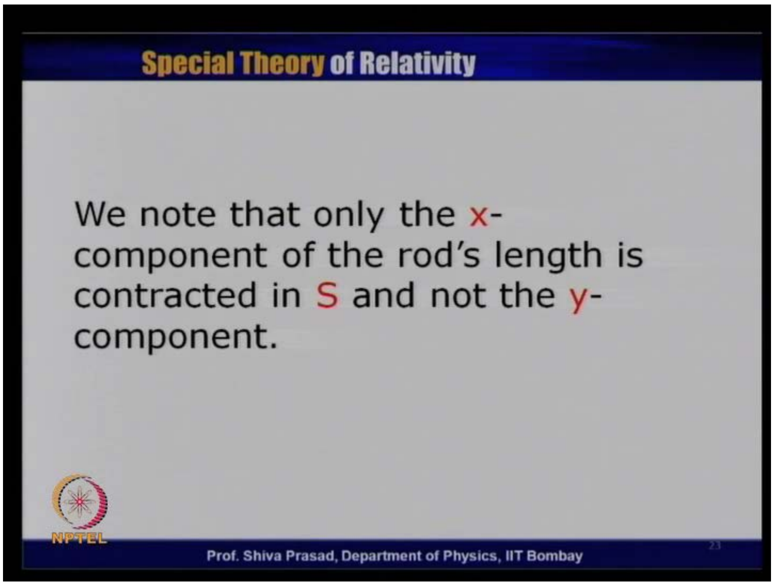

So, according to an observer in S, this rod is inclined not in 60 degrees, but 65.2 degrees. I hope you would have appreciated one particular aspect from this particular derivation, that the gamma factor has arrived only in the x component and not along the y component. Therefore, actually it is only the x component which get contracted and not the y component. So, this is the point which I have mentioned here specifically, that we note that only the x component of the rod length is contracted and not the y component.

Similarly, not even the z component because y prime tends out to be y and z prime is tends out to be equal to z. Therefore, there is no gamma factor involved. Gamma factor involved is only in the x component and that is the factor, which is the relative velocity direction along which the contraction what really occur.

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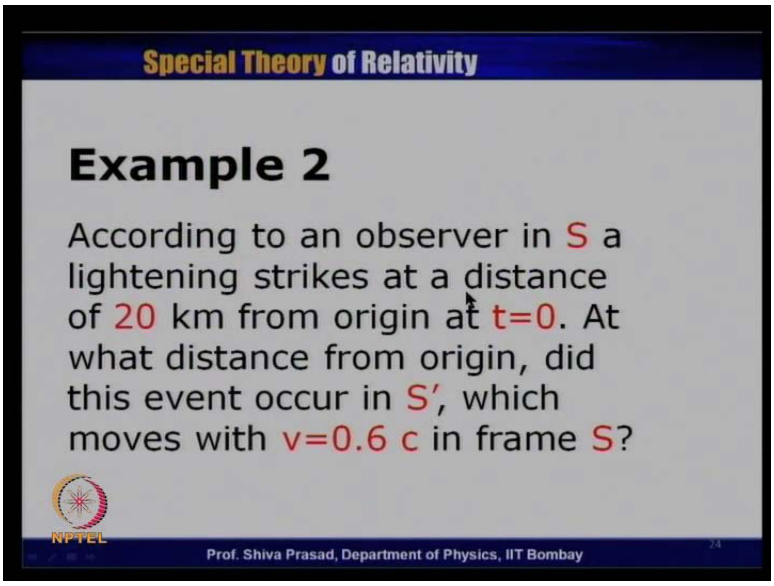

Now, let us taken another example. There is an observer in S and he finds that a lightening strikes at a distance of 20 kilometers from origin at t is equal to 0. So, for example, I am sitting here in this particular place and 20 kilometers away from me, lightening strikes. It could be any other event, but I am sort of taking as a lightening. So, at 20 kilometers away, some lightening strikes and that was time t is equal to 0 as far as I am concerned.

So, I am sitting at my origin and I find out that at time t is equal to 0, lightening had struck at a distance of 20 kilometers away. Remember, at this time, t is equal to 0. According to me, the origin of S prime was also passing by my side. So, the observer in S would feel that the origin of the S prime passing by his side and this event occurring at t is equal to 20 kilometers happens exactly at the same time, which is t is equal to 0.

Now, the question is, at what distance from origin did this event of lightening occur as far as observer in S prime frame of references is concerned? This S prime observer is moving relative to me with a speed of 0.6 c. So, let us read the statement again. According to an observer in S, a lightening strike at a distance of 20 kilometers from origin at t is equal to 0. At what distance from origin did this event occur in S prime, which moves with a speed of 0.6 c in frame S? Let me first to give a wrong answer for this particular example. My experience tell that a very large number of students have a tendency of giving this wrong answer. Then we will see what is the correct answer and why the wrong answer is not really correct.

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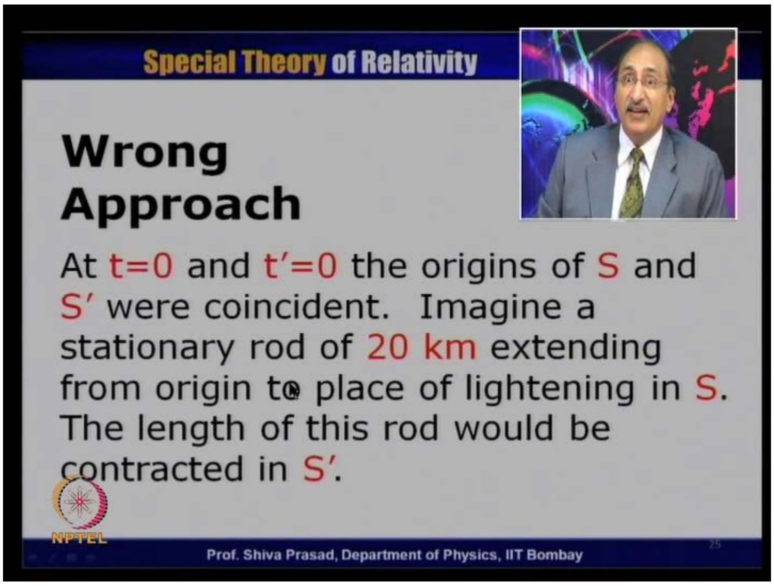

So, standard wrong approach to this particular problem is essentially applying blindly the formula of length contraction. So, I may start thinking that t is equal to 0 and t prime is equal to 0, the origins of S and S prime were coincident. Let us imagine a stationary rod in S frame. That in S frame there is a stationary rod, which is 20 kilometers long; very long, which is extending from the origin to the place of lightening. I can always imagine that there is a huge long rod, which is going right from origin to 20 kilometers away and this is stationary in S frame. Now, I expect that the length of this particular rod would be contracted in S prime because this S, this length of the rod is proper in S frame.

So, probably I can apply the length contraction formula. Let me first just read before I go to the next transparency. At t is equal to 0 and t prime is equal to 0, the origins of S and S prime were coincident. Imagine a stationary rod of 20 kilometers extending from origin to the place of lightening in S. The length of this rod would be contracted in S prime.

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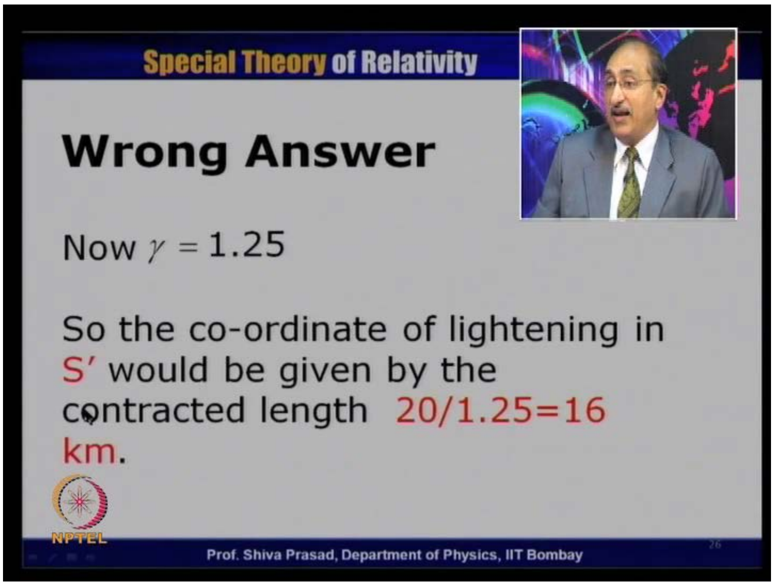

So, I just now said that gamma is equal to 1.25 for v is equal to 0.6 c. So, gamma is 1.25. So, this 20 should be divided by 1.25 because this is the contracted length and that answer is 16 kilometers. Therefore, this particular lightening must have struck in S prime frame at a distance of 16 kilometers from origin. Let me read. So, the co-ordinates of lightening in S prime would be given by the contracted length 20 divided by 1.25, which is 16 kilometers. I am again repeating, this is the wrong answer I have written. Only wrong answer. Let us try to find out the first correct answer. Correct answer. Whenever there is the confusion, apply the Lorentz transformation directly. You will not making any mistake. Lorentz transformation. I know the co-ordinate of this particular event, which is at S is equal to 20 kilometers, and t is equal to 0. I can always find out what is S prime and what is t prime by applying Lorentz transformation.

So, this is the correct answer. The co-ordinate of event in S is x is equal to 20 kilometers and t is equal to 0. Hence, using Lorentz transformation, the x co-ordinate in S prime will be given by, x prime is equal to gamma x minus v t. x is 20 kilometers and t is 0. So, this particular quantity becomes 0. 1.25 multiplied by 20, 25 kilometers. In fact, this length appears not to be contracted, but extended.

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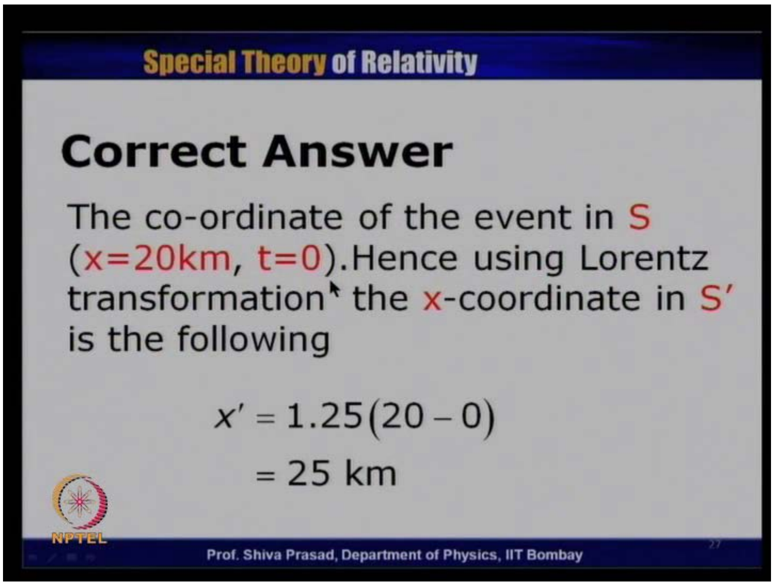

So, according to the correct answer or according to the answer, which have you obtained from Lorentz transformation, this particular event occurred at a distance of 25 kilometers from the origin. Not at 16 kilometers. Not at 20 kilometers, but at a distance of 25 kilometers. The length has not contracted. Does not it look funny? Let us try to find out what is the reason and why we are getting different answer. Let us try to understand and let us try to, we will think that why this earlier result was wrong and why this particular result is correct. Why this discrepancy? Look at time.

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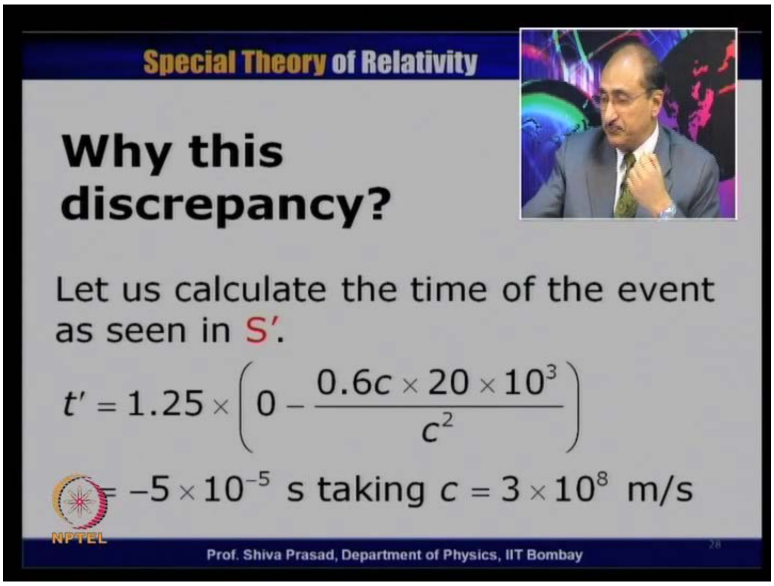

Let us first do the time transformation. t prime is equal to gamma t, which is 0 minus v which is 0.6 c, relative velocity between the frame. x co-ordinate of the event, which is 20 kilometers, 20 into 10 to the power 3 meters divided by c square. If I take c is equal to 3 into 10 to the power 8 meter per second, I find t prime is equal to minus 5 into 10 to the power minus 5 seconds. What is this negative? There is a minus sign here. So, this thing becomes minus. What does that minus sign means? It means, this event occurred before 0 time.

Remember, according to S prime of observer also, at t prime is equal to 0, its origin was coincided with the origin of S. This event t prime, this particular event of lightening struck according to S prime, at t prime is equal to minus 5 into 10 power minus 5 seconds. It means it appeared before. It means, the origin of S and S prime were not coincided metal that particular time, when the event occurred. The two origins become coincided at a later time at 5 into 10 to the power of minus 5 seconds after this particular event occurred in S prime frame of reference. This is somewhat shocking, but this is what will be the perception of S prime frame of reference. Observer sitting on S frame of reference, that this particular event occurred before the origins of the two frames were coincided. This is what I have written.

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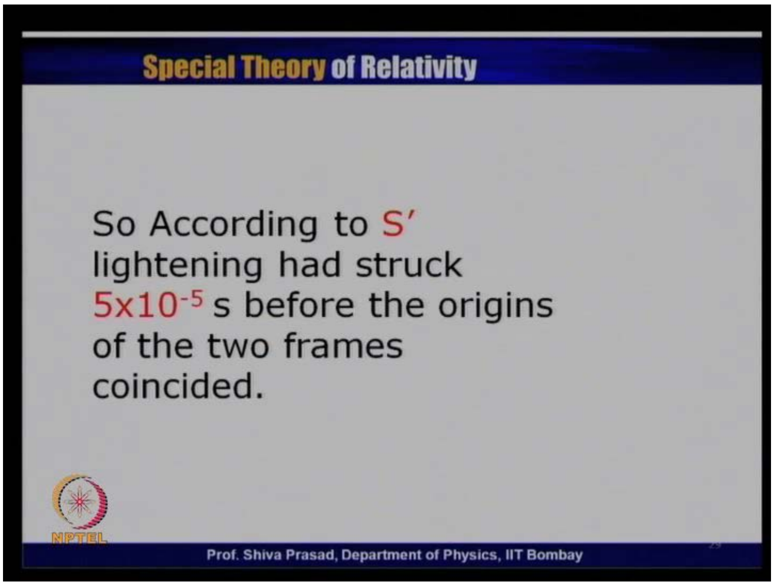

So, according to S prime, lightening has struck 5 into 10 power minus 5 seconds before the origins of the two frames coincided.

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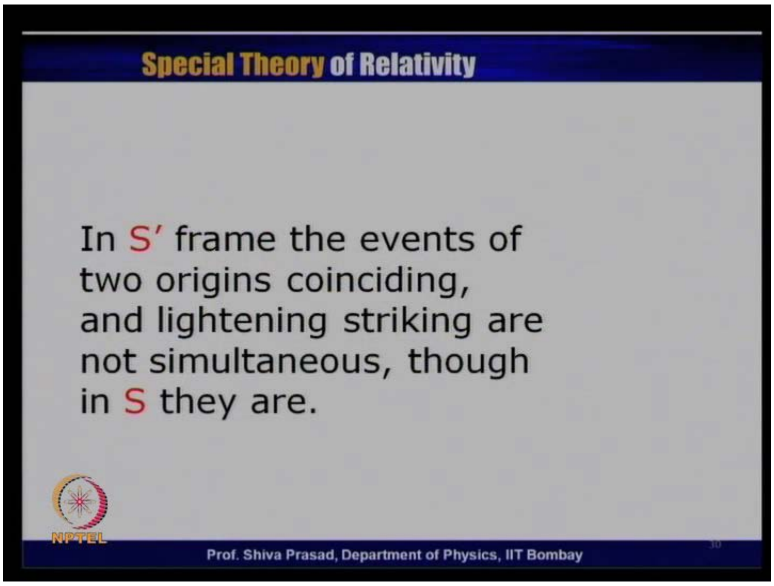

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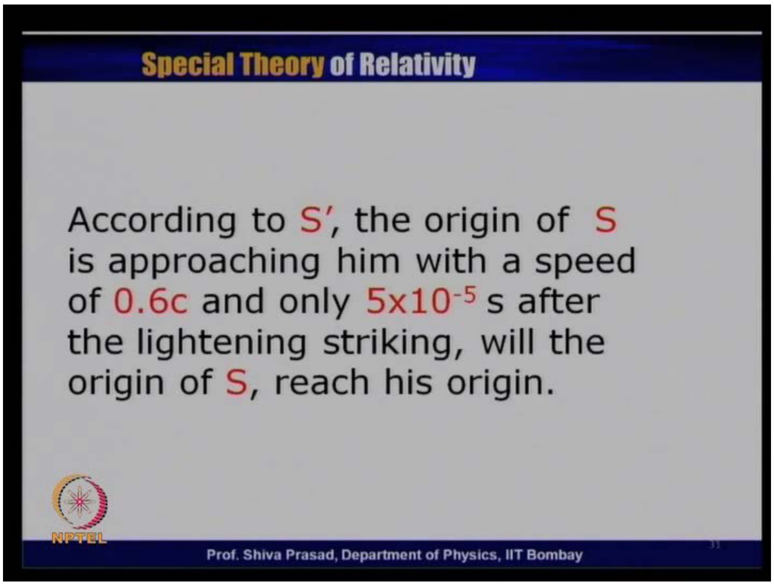

In S prime, the events of the two origins coinciding; if you call this as an another event, then the two origins being coincident, this event of the two origins coincident and the lightening striking are not simultaneous, though in S they are. According to S observer, lightening striking and the origin of S prime passing, both occur at time t is equal to 0; same time. Therefore, the events were simultaneous. According t prime, first the event of lightening struck before the second event. That is, the coincident of the origin occurs later. They were not simultaneous. Simultaneity's relative, this is what we are discussing by giving many examples, that this is something which is very different from the classical ideas to understand that simultaneity is relative in special theory of relativity. According to S prime, the origin of S is approaching him with a speed of 0.6 c.

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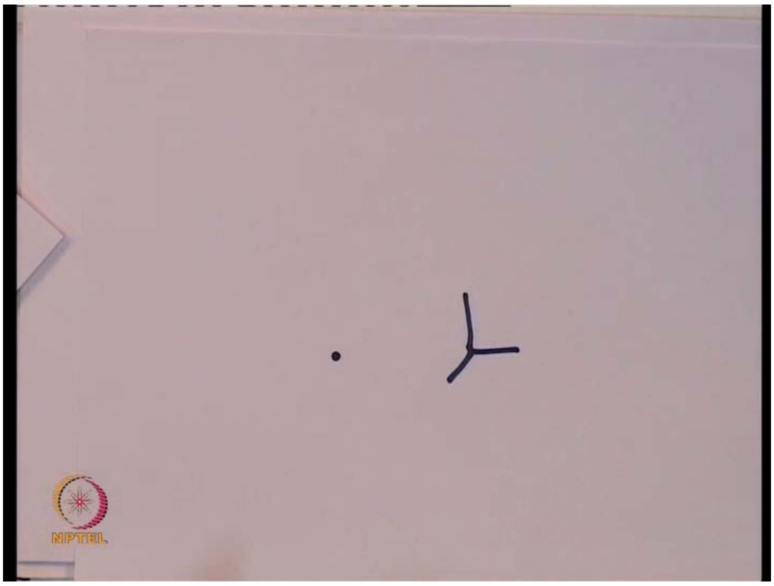

If you are looking with respect to an observer an S prime frame of reference, this particular origin S was actually moving towards him. Only after 5 into 10 to the power minus seconds, will the origin S reach his origin. So, this origin is always coming towards him and it will take 5 into 10 to the power minus 5 seconds before this origin will really come and meet him while lightening has struck. Earlier a lightening was struck here, when the origins where not coincident. This is what I have written. According to S prime, the origin of S is approaching him with a speed of 0.6 c and only 5 into 10 to the power minus 5 seconds after the lightening striking, will the origin of S reach his origin.

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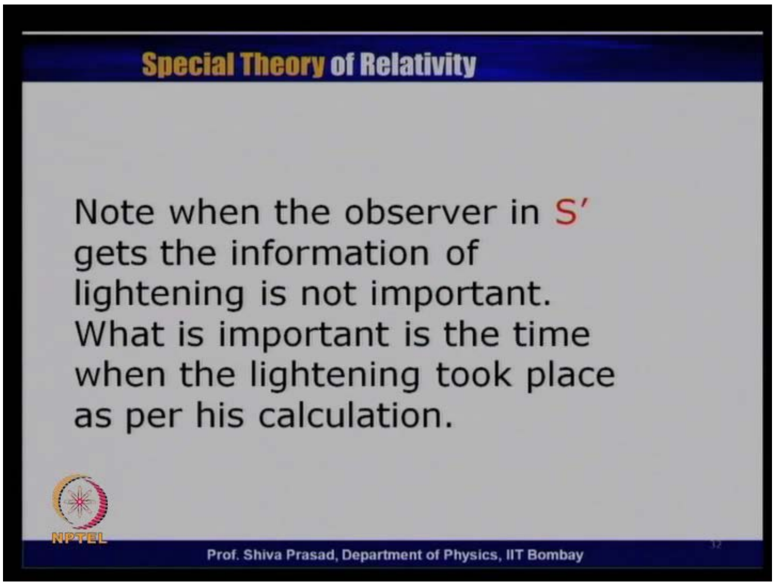

I wanted to make one particular point, which sometimes, again I find that students are somewhat confused. See, it is not important when the observer in S prime gets the information of lightening because the lightening, the information will take certain time to reach. What is important is the time of the event. Suppose, one of our friends is departing from the railway station at a particular given time and I come to know that his train departed exactly at 10 o clock. This information may not reach me exactly at 10 o clock. The person who has gone to see him off may come here, may take 1 hour to reach my place and may give me information that the train had departed actually 10 o clock. It does not mean that the event occurred at the time I got the information. Even, it had occurred earlier.

So, what is important is the time of the event. I know that the train is departed at 10 o clock, even though I am have got the information late. So similarly, an observer in S prime, may have got this information later because it always take a finite amount of time for the information to reach him. But all this time, if we try to calculate and take into account, he can find out at what time the event actually takes place.

So, for example, from the railway station, if somebody sends me a light signal saying that train is departed, I know how much time the light would have taken to reach me here and I can calculate at what time this particular event would have occurred. So, when I am saying the time of event, it is important to realize that this is the time the event actually occurred. So, this is what I have written. Note. When the observer in S prime gets information of lightening is not important. What is important is the time when the lightening took place as per his calculation.

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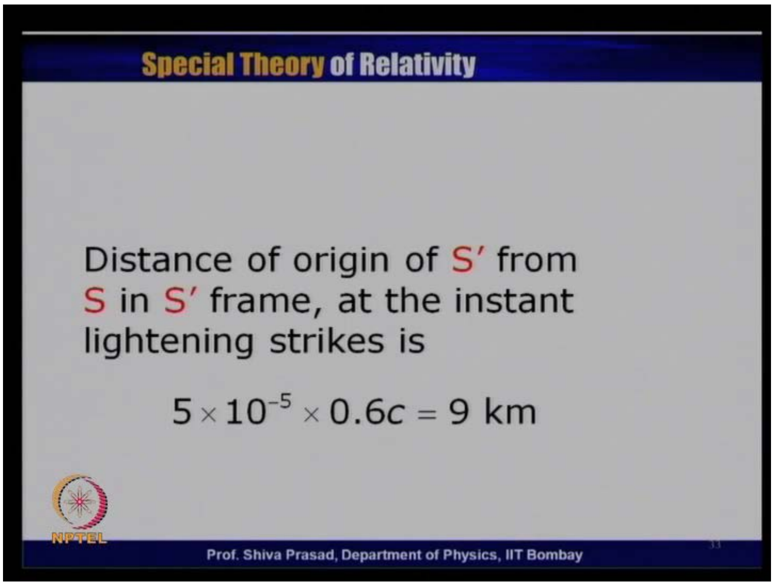

Now, as we have said here, that after 5 into 10 to the power minus 5 seconds, the origin of the two frames will reach and at that time, according to S prime will be time t prime will be equal to 0. I can find out what is the distance here. How do I find the distance? I know that after 5 into 10 to the power minus 5 seconds, this origin will reach him; will reach here and I know what is the speed, which is 0.6 c. So, I can calculate what will be this distance at the time when lightening struck. So, I can find out what is the distance of the origin of S from the origin of the S prime at the instant lightening struck according S prime of observer.

Remember, all these things I am talking now with respect to S prime observer. We should never confuse the frame of reference. When I am talking of one frame of reference, let us assume that we are sitting in that frame of reference and try to get all the information in that frame only. So, I calculate this particular time, 5 into 10 to the power minus 5 seconds multiplied by the speed with which the origin is approaching me. This distance is 9 kilometers.

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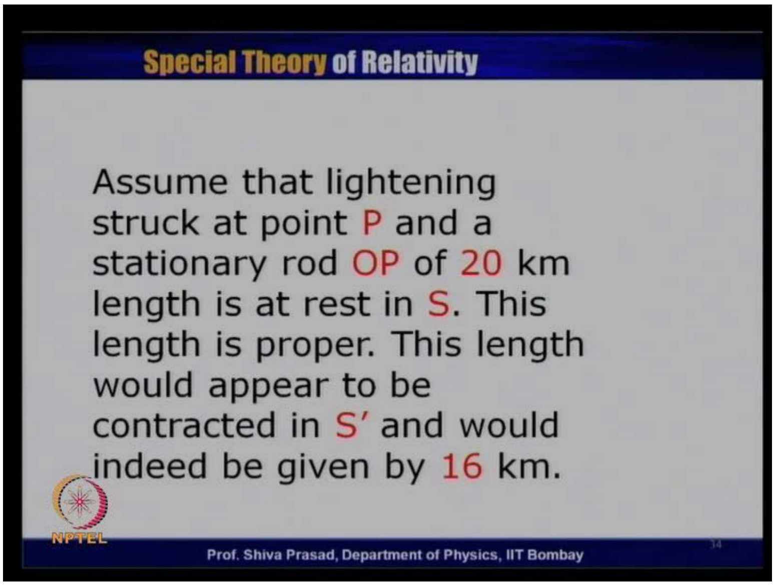

So, let us now recall our ideas, and readjust our things. Assume that lightening struck at point P and a stationary rod OP of 20 kilometer length is at rest in S. As we said earlier, this length is proper. This length would indeed appear to be contracted in S prime and would indeed be given by 16 kilometers. About that particular aspect there is no problem. The problem is only that, this particular length contraction, whether my co-ordinate gives me the correct length or not. This is what I have written here.

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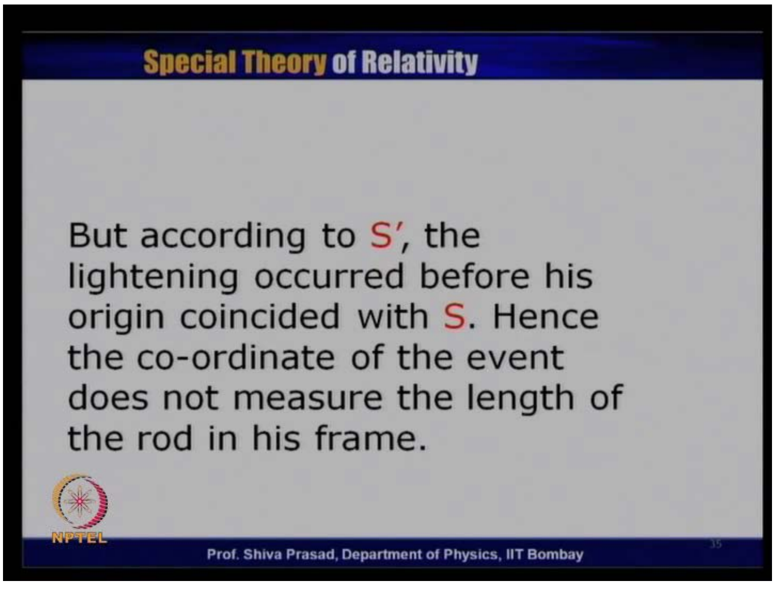

But according to S prime, the lightening occurred before his origin coincided with S. Hence, the co-ordinate of the event does not measure the length of the rod in his frame. While, what I had calculated by length contraction is the length, but the question that has been asked is, what is the co-ordinate of that particular event and that co-ordinate is not equal to the length of the rod.

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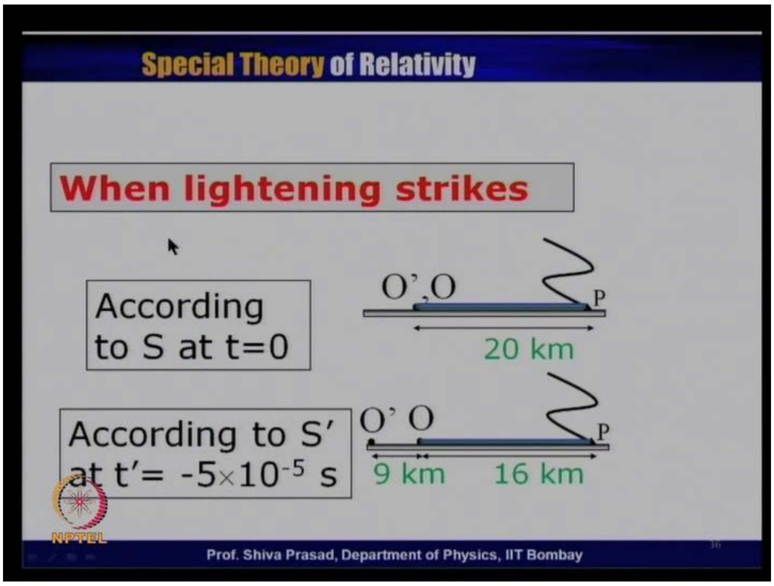

So, I have given the situation in the picture. When the lightening strikes, assume that lightening strikes, and at that instant, the picture, according to S, the picture that S observer will draw, he will draw that the origin of O prime and O were constituted at that time and at a distance of 20 kilometers, this lightening struck. This is the way S is going to picturise the event. But according to S prime, the picture will be somewhat different. According to S prime, he will feel that his 9 kilometers will be away from the origin. This event actually occurred at 25 kilometers. But then this rod OP, according to him will be 25 minus 9 kilometers, which is 16 kilometers, which is indeed the contracted length that we had observed. So, that is what is the difference of perception. According to this observer S prime, he was 9 kilometers away from the origin when the event occurred.

So, the co-ordinate of this particular event is actually 25 kilometers, what we have obtained from Lorentz transformation. P 9 kilometers away from here and 16 kilometers is actually length of the rod OP, which is indeed the contracted length.

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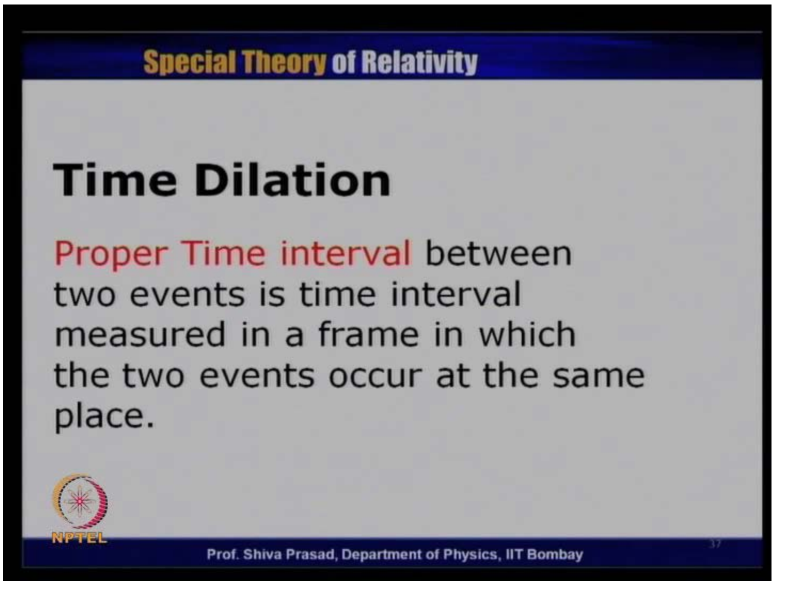

Before I end this, let us discuss one more important aspect of this particular Lorentz transformation, which is also very popular, which is called time dilation. For discussing time dilation, we have to define a proper time interval, just like we have defined a proper length. Proper time interval we can define between any two events. There could be any two events; event 1, event 2. Now, we have defined many types of events. We will also be defining in the course of lecture many other events. If these two events occur at the same position in a frame of reference, then that time interval, which is measured in that frame of reference, is called a proper time interval.

So, let us assume that two events appear to be occurring at exactly the same position. So, I am sitting here. Something happens here. Then again I sit at the same point. Something happens here exactly at the same point. So, if I take the time difference between these two events, that is what is called proper time interval and I measure the time interval between these two events. What I am trying to say is that, if anybody else who is moving relative to me, measures the time interval between these two events, he will find that this time interval is larger. That is why it is called time dilation. Like, the eye specialist dilates the eye. The ball becomes bigger.

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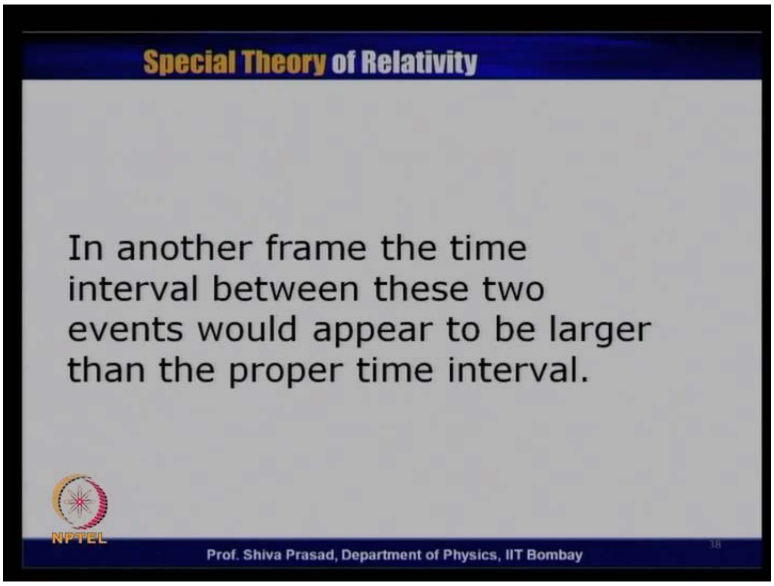

So similarly, this particular time interval becomes dilated. It becomes larger. That is why it is called time dilation. In any other frame, the time interval between these two events would appear to be larger than the proper time interval. That is why this is called time dilation.

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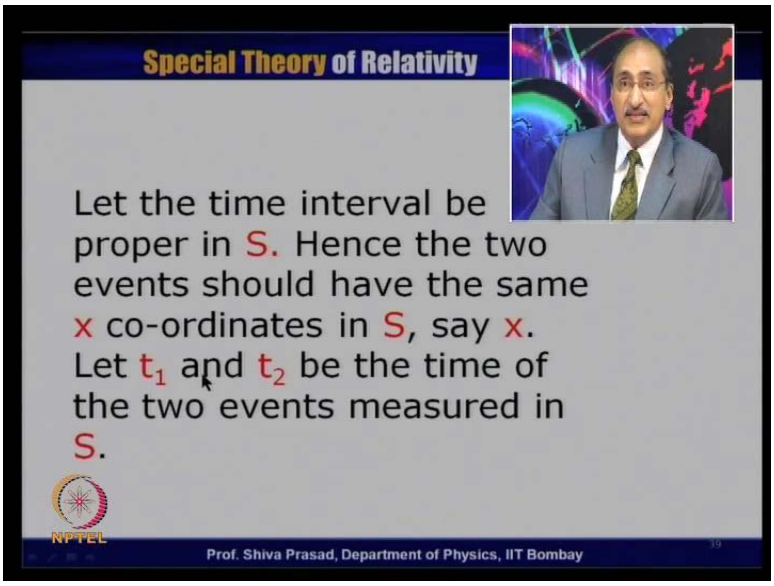

Now, let us assume that it is S frame. We already discussed what is S frame and S prime frame. That is, S frame in which the time interval is proper. It means these two events must have occurred at the same position in S frame. If they have occurred at the same position, it means that their co-ordinates, x co-ordinates would be same. So, it means these two events, event 1 and event 2 occurred at the same value of x. Let us suppose the time is measured by an observed in S is t 1 and t 2 for these two events. So, t 1 minus t 2 or t 2 minus t 1, whatever you want to call and the difference of time interval is the proper time interval as measured in S. Because in S, the two events occurred at the same value of x or same position.

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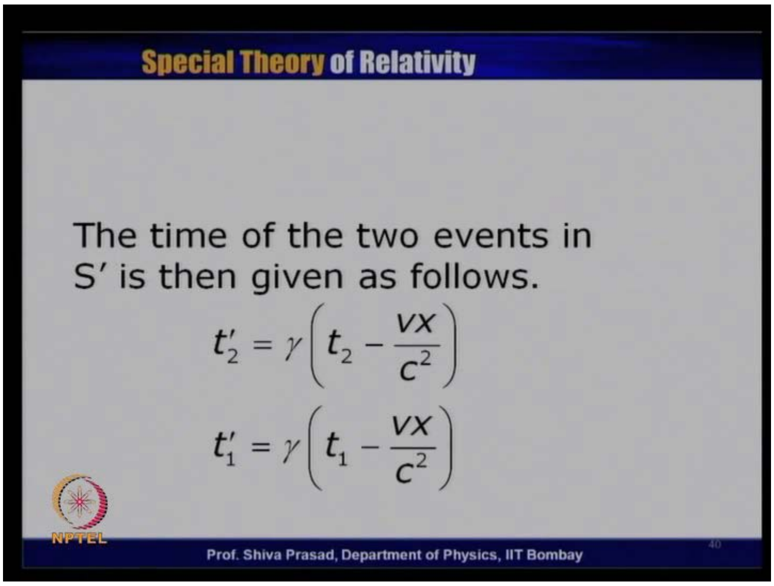

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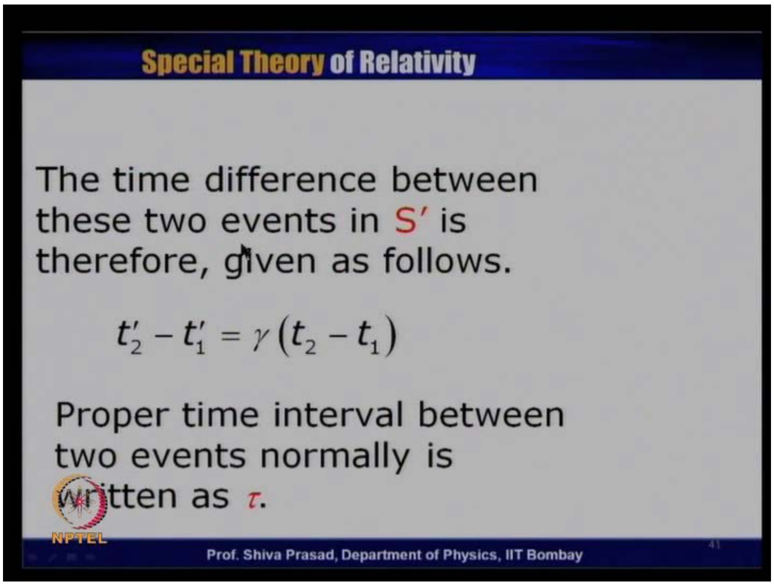

Now, I apply Lorentz transformation. If I apply Lorentz transformation, exactly like before, I get t 2 prime is equal to gamma t 2 minus v x upon c square. This x, I have taken to be same. So, this x, I have not written x 2. For the first event, I have written t 1 prime is equal to gamma t 1 minus v x by c square and this x being same. I take the difference of these two. Again, these two will cancel out. I will get t 2 prime minus t 1 prime is equal to gamma t 2 minus t 1.

This is what I have written here. The time difference between these two events in S prime is therefore, given as t 2 prime minus t 1 prime, which is equal to gamma t 2 minus t 1. This t 2 minus t 1 was the proper time interval, because this was measured in a frame of reference, in which the two events occurred at the same position. So, this is proper time interval. Gamma being greater than 1 and t 2 prime minus t 1 prime will be larger than the t 2 minus t 1 and therefore, time interval will be dilated. Many times, this proper time interval between the two events is written as tau. This is one of the symbol, which is very well commonly used. There are one or two small comments, which I would like to make.

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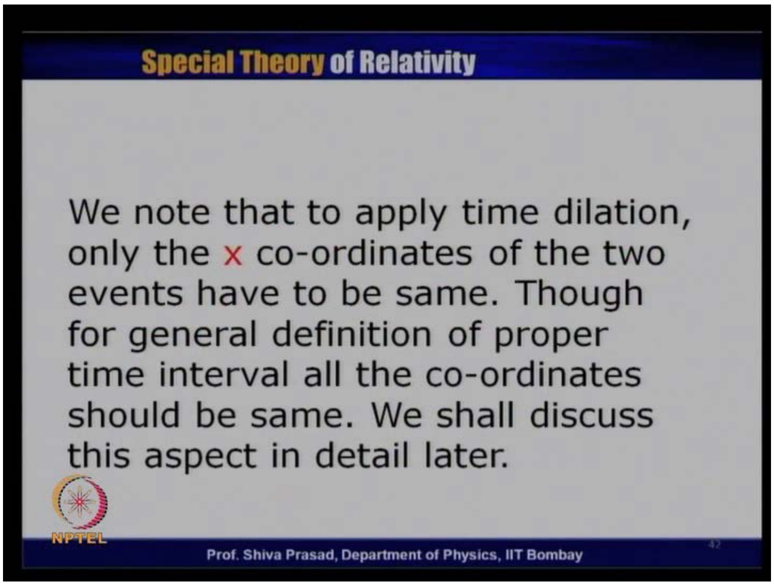

First, that you will note that to apply time dilation, only the x co-ordinates of the two events had been used. Strictly speaking, if the y co-ordinate of the two events were different, one event occurred here and another event occurred at some other height. One event occurred here and another event occurring at some other height, but at the same position. Still time dilation formula is applicable. To strictly speak that the time interval is not proper, we will define exactly a correct definition of proper time interval a little later. But as far as the application of time dilation formula is concerned, only the x co-ordinates of the two events need be same.

So, that is what I have written. We note that to apply time dilation, only the x co-ordinates of the two events have to be same. Though the general definition of proper time interval, all the co-ordinates have to be same. As I said, we shall discuss this aspect in detail later.

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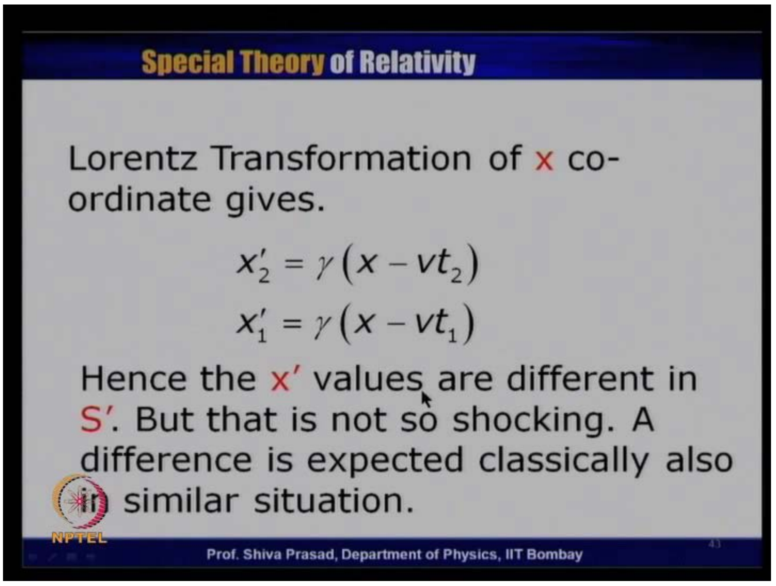

The second point, which I want to mention is that, if I apply the Lorentz transformation for X co-ordinate, according to S prime observer, these two events would indeed appear to be occurring in different positions. Like, in the length contraction, according to an observer in S prime, the time difference of the two events was not same. It was different. Here, the X will be different. But this is not shocking because even classically also that what is what we see. If an event for example occurs here, it will classically we do not have to go to relativity.

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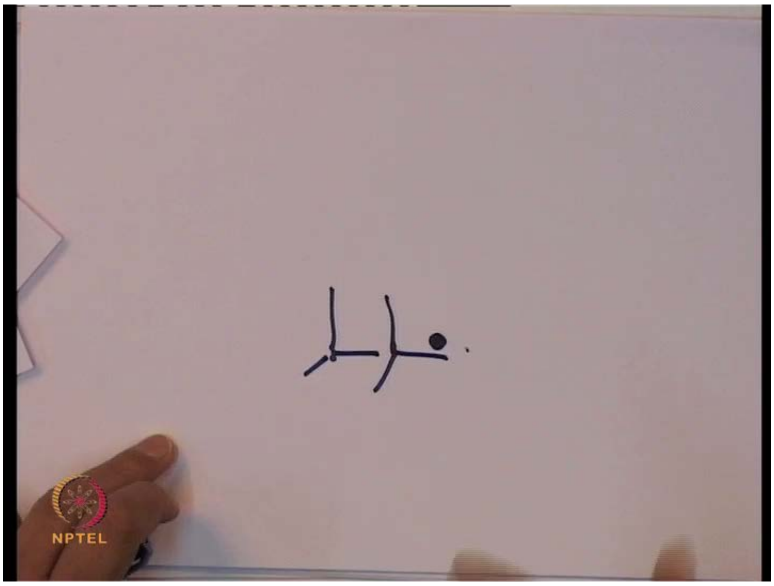

If an event occurs, let us say a mole here, somewhere here and one frame was here. Another frame, let us say here, also supposed to be here. They observed that these two events happen exactly at the same value of x. But the second event when it occurs, the second observer has moved to the right. Therefore, the event, the x co-ordinates has change. Therefore, the x co-ordinates being different is not at all that shocking. This is also expected. Classically, it is a time difference being different in different frame, that appears to be much more shocking.

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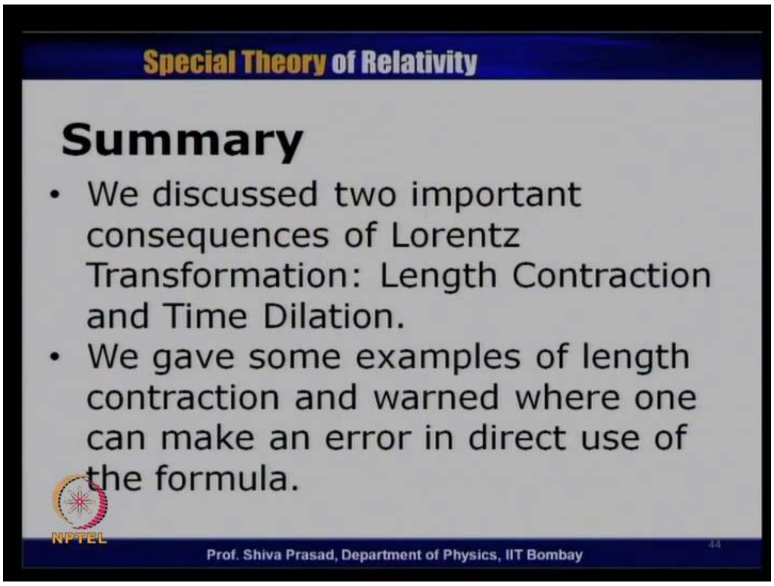

So, that is where I will end my lecture, just by giving a small summery. We discussed two important consequences of Lorentz transformation; namely, the length contraction and time dilation. Then we gave some examples of length contraction and warned where one can make an error in direct use of the formula.

Thank you.