Marine Hydrodynamics
Prof. Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 38
An Introduction to Boundary Layer Theory

Welcome you to this lecture on Marine Hydrodynamics, under the NPTEL program. The last couple of lectures we have seen how (( )) equation is used to several analyze various few problems. In particular our emphasis in the last few lectures was when the flow is unidirectional in the beginning we started with a clipping motion at the Reynolds number is very small. And then we talked about unidirectional flow, where the conductive inertia term was neglected and in the process, and the flow was only in the actual direction one of the actual direction.

And in the process we have seen that large number of it there is a class of problem is can be analyzed for solution, and solutions are obtained explicitly and but this class of problem is very few. There are very few such problems, where we can get explicit solution, of course we have looked into three types of problem; one is acceleratory solution, one is transient solution, one is a steady state solution.

However, all this unidirectional flow problems are the fully developed flow problems are limited, when the Reynolds particularly when the Reynolds number is greater than the clipping flow motion, but still they are small. There is a another class of problem, but I talked about in the last class also that that is called where the Reynolds number is moderate in nature.

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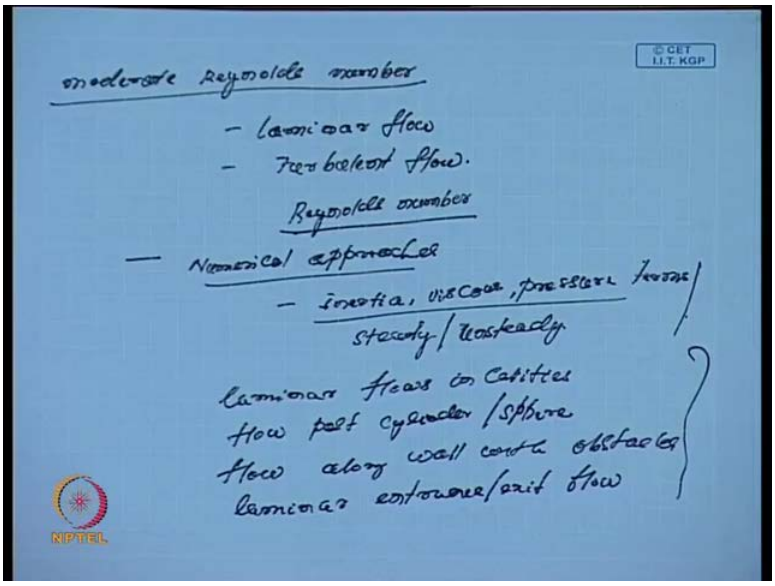

So, basically there is a class of problem where the Reynolds number is a moderate, moderate Reynolds number and for this again and I have already discussed that we have two types of flow is the laminar flow, the other is the turbulent flow. We have also dependent particularly the one of the factor which depends whether the flow is a laminar or turbulent is the Reynolds number. So, for moderate Reynolds number the the moderate Reynolds number there is a class of problem for which rarely any closed form solution is available and we always go for approximate solution.

However, last class of problem in problems mechanics are in real life problems are handled by this various numerical approaches, associated with, and these are as your last class of problem they are basically higher. In fact, as the inertia term and the viscous term inertia viscous and pressure terms plays a significant you cannot ignore none of this terms and both the study unknown study and here both the we have study here and case of unsteady.

So, here we cannot neglect all these, because even if they are there moderate their role is very important flow in pipes and there are many flows, flow in exit, flow in pipes and particularly to near the nozzle and where all these things are very important. But, in particularly if I look into laminar flows in cavities, flow past are cylinder, sphere, flow with the wall which are how longs are wall with obstacles Laminar's entrance or exit flow, more important in this case of this kind of entrance and exit flow (( )) get, where there is a the get’s account the control of flow. And then these are class of problem where the Reynolds number is very moderate, but it cannot survive a wide range of problems in physics and engineering. Then we have Reynolds number, when the Reynolds number becomes high all have seen that for high Reynolds number I talk about for which the boundary layers flows.

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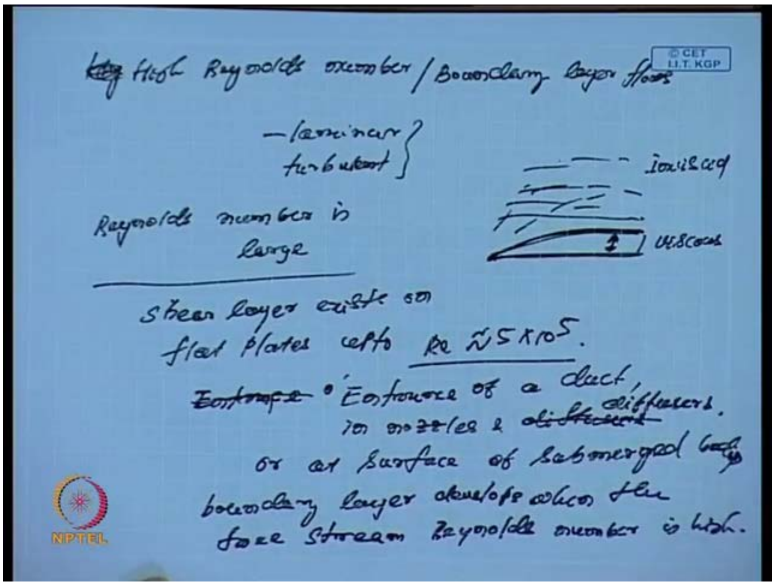

So, what happened just we have, we knew that there are two types of laminar flow, so one is one is laminar or one is turbulent, but what happened there is a class of problem. Where, just when we have a boundary, just along those any boundary is there or a wall is there just note the wall, the flow remain viscous the near the wall the flow remain viscous and beyond this wall boundary if you look at the flow, any flow here. So, the flow, becomes inversed here, but in this region flow becomes viscous and that this there is a layer what we call the when which this happens.

And this layer we call the this distance we call the boundary layer thickness and then beyond that and this happens when the Reynolds number is large is large, particularly if you look at the flow in a flow past a plate. We can see for example, the shear layer shear layer exists and on flat plate up to Reynolds number is 5 into 10 to the power 5. And are similar such situation occurs in a the entrance, when you look at the entrance or exit of a duct entrance sorry if you look at the entrance of a duct in nozzles and diffusers or at surface of submerged bodies surface of submerged body.

The boundary layer develops in the free stream Reynolds number is high is high, so this are this is another example here, we can see that it is a large number, even if you for a flat plate. And entrance of duct nozzles on diffuser this is in diffuser sorry I call it diffusers or at surfaces of submerge bodies boundary layer develops. And apart from this we have seen that there are a large numbers of problems in which it is observed that the viscous pate are felt, just in the immediate neighborhood as I at the term immediate neighborhood of the solid boundary.

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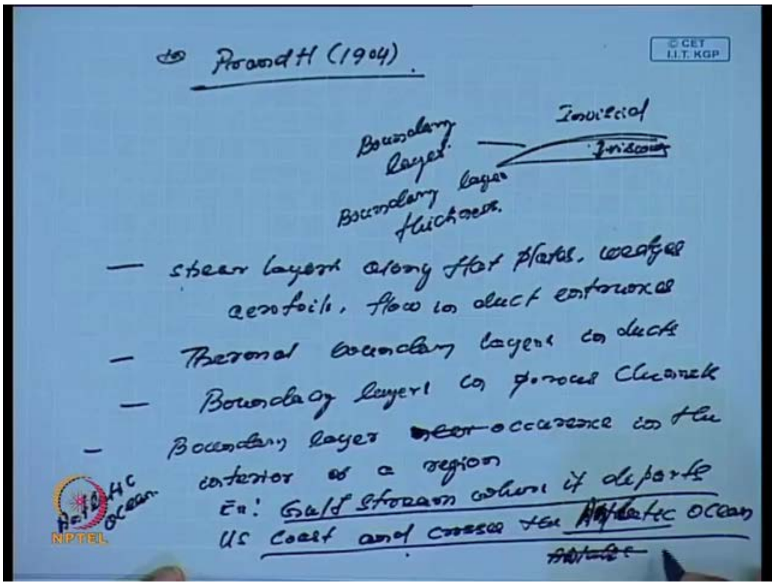

This is the solid boundary and this is just in the immediate and this this distance is very small, in fact in 1904 nineteen hundred zero four in prandtl 1904 on a paper on fluid mechanics, he talks about that the Navier stoke equations holds in a very thin layer near the surface of a boundary. And then in which inside which the flow is considered a viscous beyond, which a closing invertical this is inside (( )). And here the in fact a here the both the and here basically in this region the viscous fact is always comparable with that of inertia effect, and then thin layer near the wall of this solid boundaries.

This thin layer near the wall of the solid boundary, we as called the boundary layer and this distance is called the boundary layer thickness, so it is not in additional I will say that is not always that you have to have a open boundary or for the boundary layer to occur. There is a that can be this boundary layer can be also occur in the in a moving fluid region at an interface, near a wall or not only near a wall at an interface of 2 plate this can occur.

For example, for example, I will give you one typical example that how the at the interface about to that, so basically I will just first write few examples of the occurrence of boundary layer. Again that say shear layer, where the occurrence of boundary layer occur shear layer along flat plate, wedges, aerofoils, flow in duct entrances as I mentioned duct entrances and then we have thermal boundary layer thermal boundary layers in ducts, boundary layers in porous channel.

Then we have also as I say that the interface of a two fluid houses here one of the example of a fluid region, it is not only on the surface. For example, this example is very important that if you look at the gulf stream the gulf stream where it departs the USA the u s a coast and crosses the Atlantic ocean antlatic ocean crosses the antlaticocean. There we see the thin layer of a boundary layer occurrence of a boundary layer is a Atlantic atlantic sorry atlantic ocean.

So, we also see the presence of a boundary layer and then with this then; that means, a here in this case the Reynolds number is although the Reynolds number is large, but there is no turbulent motion, but there is the occurrence of a boundary layer, but again if the motion becomes more beyond this certain in in the high Reynolds number region R e for certain values of the Reynolds number this boundary layer formation takes place and after it reaches certain optimum value.

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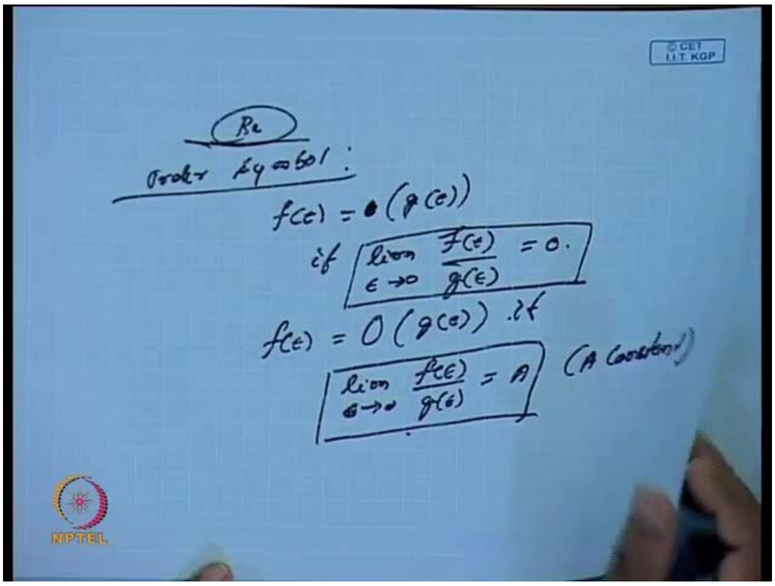

Then it in the flow becomes further with an increase in a Reynolds number the flow, becomes turbulent. Now, if we are going to the some discuss some of the characteristics of the boundary layer, I introduced two concept what is called the order of symbol, this has a two meaning the order of symbol. Suppose, I say f of epsilon is equal to small of small o of g of epsilon, if I call this then this is small o, this is order if a epsilon of order of g of epsilon. Then in that case it happens if limit epsilon tends to 0, f of epsilon by g of epsilon is equal to 0, this is small o and on the other hand I say f of epsilon is equal to order of g of epsilon.

If limit epsilon tends to 0, f of epsilon by g of epsilon is equal to is bounded is equal to some a constant value, so this is the basic difference between a small o and capital O. In fact, while going to discuss the theory of a boundary layer equation that derivation of the boundary layer equation, we will come across this small o and the capital O very frequently. So, this is very important in this concept, and otherwise also in mathematics is a very important con notation small o and capital O one as two.

In fact while analyzing various flow problems always we compare that it is of x is f order y or f x is of order g x; that means, if it is small o. Then f x by g x if is 0, as x tends to 0 and if it is capital O f x by g x is equal to bounded quantity as a x tends to 0, this is what order symbol, now I will use this because we know that now I will come to what about the boundary layer thickness just basic definition.

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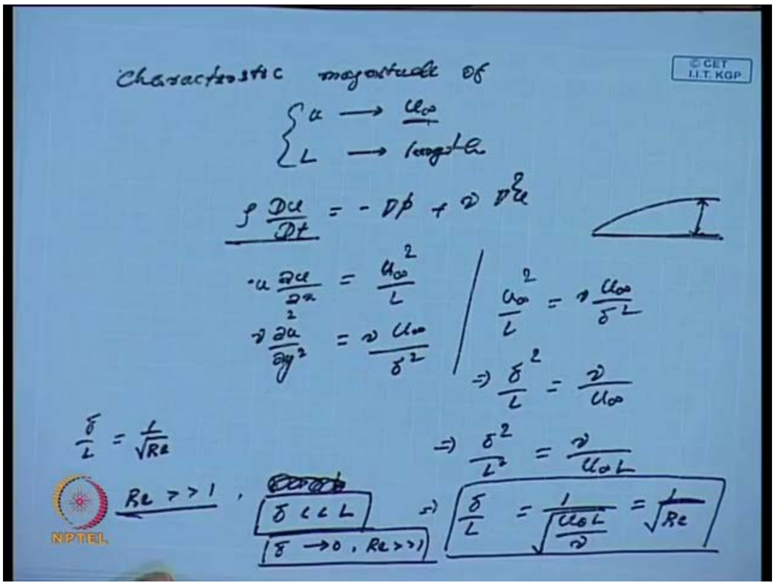

Let us consider a characteristic length some values characteristic magnitude of horizontal component of velocity u h you infinity, like you have seen this u infinity is kind of a suppose you have a viscous fluid. And as I have said that beyond the boundary layer the flow is initiative and maybe a three dimension flow of stream, some kind of characteristic length.

And then I will discuss the last distance maybe the u is equal to infinity, that is the large distance, so this is called a characteristic magnitude of the velocity component. Similarly, let L be the characteristic magnitude of length, and that depending on that nature of the problem this L and u infinity you have defined then, so what happen over this characteristic length u changes to u infinity. So, that what exactly you mean, and then if I say that if you look at the equation of motion we have rho d u by d t is equal to minus gravity plus mu square u.

If I look at the component wise there is a convictive from here the first convicted to m will be or sometimes I call it the convictive term will be. Before, going to the convictive term I will just look at one thing what will happen here u del, one of the convicted term is u del by x, convictive in your center what will happen. If I put it in the form of order that, basically the inertia term then u is of the order of character change magnitude is u infinity, so there should be basically gives me u infinity square by x it has a characteristic length here.

So, u del u by del x is u infinity by L u infinity square by L, on the other hand if I look at one of the term of this is a component of our that is the diffusion term or the viscous term. If you look at the viscous term one of the component will be nu del u by nu del square u by del y square, and what is and what is the corresponding characteristic length that will be nu. This is u infinity that square u by del y square and by delta square, why I said delta square, because I consider delta as the characteristic length y and x is the characteristic length, L is the characteristic length along x.

So, then in that case what will happen this these terms are comparable u del u by del x, because one of the term is u del u by del x and the other term here is, so they are of same characteristic length. If they have to have the same characteristic length; that means, I will have u infinity square by L will be nu to infinity by delta square. And if that is happened then what will happen by delta square by L that is nu by delta square I brought delta square by L, so nu by u infinity.

So, which gives me delta square by L square is equal to nu by u infinity, L which gives me delta by L is equal to I can now just write u naught L by nu root over and this is like nu del is nothing, but the Reynolds number. So, I can call this one by R e root over, so that gives me del by L, where I consider del is a typical as I say that is this is a typical distance along the y axis this is the characteristic, so del by L is one by R e.

So, from here we can see if del by L is one by R e root over, now if I say R e is large if R e becomes very large this quantity will be very small so; that means, delta will be very large compared to L, sorry R e is large delta is then this quantity. Then if R e is large then this quantity will tend to 0; that means, delta is very small compared to L so; that means, this boundary layer thickness is very small number for high Reynolds number. That is what and again this delta will tend to 0, for a when R e becomes small or greater than one basically, so that is what we have got from here.

Now, with this understanding I will go, I will derive further boundary layer equation says, so we have understood that for large Reynolds number the boundary layer thickness tends to 0. And again if I look at the length scale along the x axis is L and delta is the length cell, along the you axis then delta is much less then L when r is write r is class. And again this relation holds good delta by another thing here also, I will come a little later before coming to this before going coming to that little, so let us look at what happened the simplest boundary layer equation that is called the in two dimension, let us derive this and that will clarify many of the questions in two dimension.

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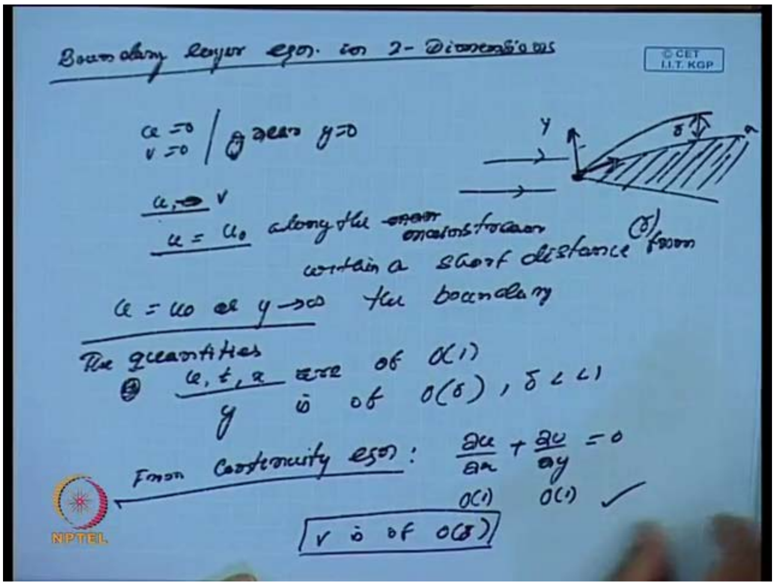

Let a consider a body boundary, there is a credential wave flowing and then what will happen as I say that I assume that u is equal to 0, v is equal to 0 at near y is equal to near y is equal to 0. And the x axis is along the wall boundary, so if say this x axis is along the boundary of the wall this becomes my y axis is the x axis, and I say that here it is 0, so there is a fluid flowing here, so what will happen then this distance I call delta.

So, what is happening this is a which the boundary of wall there is a fluid is flowing the velocity at this tip is 0,zero this is a rigid boundary, so the on the wall the velocity will be 0, but between this each distance between this region. It is a the flow is a called viscous beyond, which the flow is we call the flow as indecent and then and we say that u is parallel to the wall, the velocity component u basically it is along the x axis. So, u is the x component of the velocity and v is the y component of the velocity. So, what will happen near the and again when u becomes because I say u is 0, v is 0 near you is equal to 0.

And the u from the velocity component u from 0, it will be raised to u naught as along the main stream, and I say along the main stream; that means, within the within a short distance; that means, within a short distance from the boundary. That short distance that short distance I call delta and this delta I assume that this is the build, which u is equal to u naught, so u is u naught sometimes also we call this u, becomes we u is u naught as y tends to infinity.

So, this is what I will say we can also say that u is equal to u naught when y is greater than delta, so that will come. now I assume few things, suppose I assume based on this, I assume component of velocity u. Then the is the time variable x x variable, these are the quantities rather I will say the quantities u t x r of order one and i will say, y is of order delta that delta is much less than 1.

If this is the case then what will happen then what will happen to from continuity equation, if you look at if you look at the continuity equation we have del u by del x plus del v by del y, because it is a is equal to 0. If that is 0, and u and x both are of order 1, for 3 1 x are order order of 1, because this quantity is of order one, this quantity also has to be order one, and if I say y is of order delta And if I say y is of order delta. Since, because u t x are order one, so this is of order one, and where as the del v by del y also has to be of order one.

However, if I say y is of order delta, so that gives me v is of order delta, so that is one of the important conclusion we have drawn from this, which why I say that I assume the that means along the x axis with a small changes. The changes if x u t x there is a small change, there is not much changes will be affected on the other hand it will tend to if there are very small quantity, but where as if I say there is a change order of delta means other y delta will I will say that y will tend to 0 here.

Whereas, your u t x tends to 0, if I go by the definition of order, so v is of a order of delta, as u is a order delta by continuity equation, we have one v is a order delta. Now, with this understanding, now let us look into what will happen to del v by del t.

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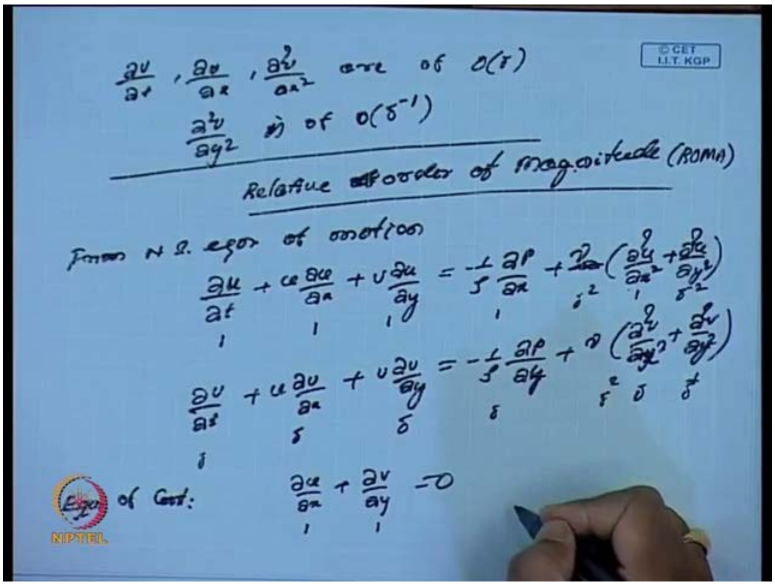

So, in the process we have del v by del t del v by del x del square by del x square because x is of order t t x, they are of order one, so these quantities are of order delta, on the other hand what will happen to del square v by del y square, because y is of order delta v is of order delta. So, this is of is of order delta to the power minus 1 because y is of order delta your y square will be of order delta square here, it is v of order and this is a this way of defining order is called relative order of magnitude relative order of magnitude, and sometimes you call it as a roma.

In fact, this is very important to derive the problem like this equation in two dimensional into dimensions, so now, if you look at the now let us look at this compare the terms of the rest of equation rather the equation of motion in two dimension based on this order of magnitude, if u do that now from Navier stokes equation motion in two dimension. You have del u and del t, I am just explaining the Navier stokes equation plus u del u by del x plus v del u by del y, this is equal to minus 1 by rho del p by del x plus nu y, rather I will say nu del square u by del x square plus del square u by del y square.

And again we have to look at del v by del t plus u del v by del x plus v del v by del y, so now, if you look at this let us look at the order of each term, so v is of order (( )) u is of order one, t is of order one, so this is of order one, here also is of order one, here this quantity is of order because v is there y is there. So, it is again of order one this is of order delta, this is of order one, this is delta and again I assume that this is again of order delta and then nu, nu is of order delta square, because we have just earlier.

When we have compared the just we will have seen this just comparison between nu, I will come to that, so nu is of order delta square, I will prove that if it is required del square u by del x is again order one. And this is of order one where as this is of order delta square this is of order one, so what will happen if I look at that term here first this is a if I multiply this del square u by del x square is order is your product delta square, where as the nu del square u by del y square is of order it is minus 2, is of minus 2.

So, in fact, all these terms are of order one except the term nu del square u by del x square which is of order delta square, so this term is a very small term if I look at the other terms. So, what will happen we can neglect this term, because delta square will be very small, so these term can be neglected. If I neglect that that will give me, then in the same way, so these term is of order delta square, now if I look at this term again here what is happening this is of order delta, this is of order delta, again this is of order delta.

Then sorry this is del p by del y, and again this is a of order delta again delta square this is nu is of order delta square, del square v by del y square this is of order delta and this is of order del square p by del x square and this is and this is 1 by delta. So, if you multiply this this is of order delta delta q and this is of order delta, so and then we look at del u by del x the equation of continuity, you have del u by del x plus del u by del y is equal to 0, because we have this is of order 1 and this is of order 1. So, if you just look at I just take the consider the terms which are of order one and the terms and only consider the terns which are order one.

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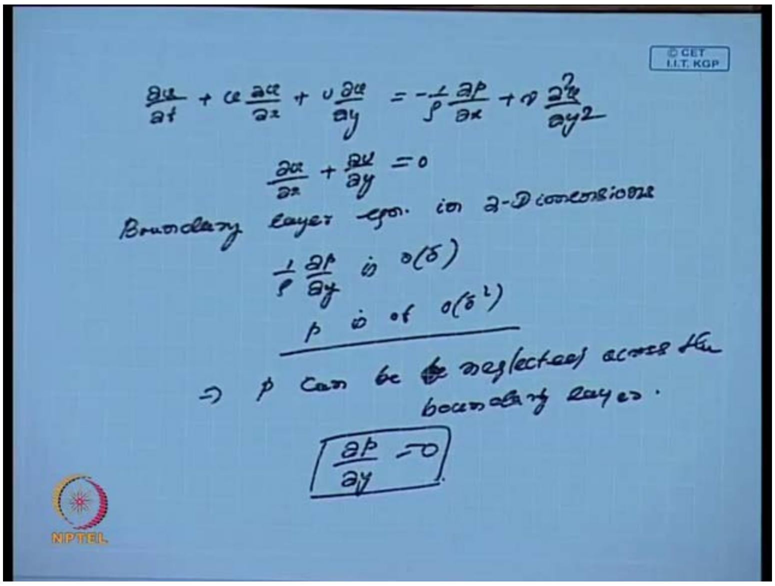

Then what it will give me that will give me, the x component of the velocity del u by del t plus u del u by del x plus v del u by del y is equal to minus 1by rho del p by del x plus nu del square u by del y square, this becomes and then you have del u by del x plus del v by del y 0. So, this becomes the boundary layer equation in one dimension, in two dimension and this is what was derived wave pattern and there in the similar way we can we also seen that one by rho del p by because here what will happen one by rho del p by rho del y, because this term we are not taking.

This will be of order is of order delta and if it is of order delta; that means, p it can be easily seen p is of order delta square so; that means, p is equal to p is 0 plus plus and may be neglected and, so p is 0 along the vertical axis. So, what will happen, so it implies p is of order, so you what we say, so p can neglected across the boundary layer, we suggest that p can be neglected across the boundary layer, so that in because it is of this, so in the process i can call it to del p by del y is 0.

So, that is on the other hand so; that means, p is a constant, so which del p by del y means p can be a function of x only and which normally, you can see that p is considered as a constant normal to the boundary layer; that means, along the x axis.

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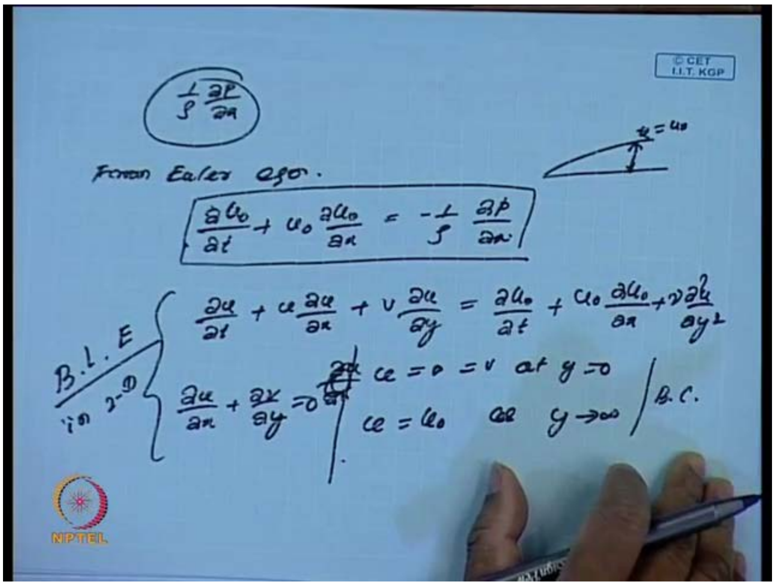

So, if that is the case then what will happen to one by rho del p by del x, because we know that if we have the boundary layer beyond this line u is equal to u naught, and if u is equal to u naught beyond the boundary layer. Then from earlier equation and here the flow is free stream flow, so from earlier equation we can get what will be 1 by rho del p by del x from earlier equation, it will be del u del u naught by del t plus u naught del u naught by del x.

That will give you minus 1 by rho del p by del x and here the gravity term is included inside it, so that is what my del p by del x will give. If I substitute for this equation in the boundary layer equation; that means, I will get what i will get i will get del u by del t plus u del u by del x plus v del u by del y is equal to del u naught by del t plus u naught del u naught by del x then plus nu del square u by del y square.

And we have del u by del x sorry u is equal to 0 equal to v, but y is equal to 0, and u is equal to u naught as y tends to infinity, so this becomes the and here we have already I have already told that it will satisfy this equation u will satisfy this equation and also the continuity equation, that is del u by del x plus del p by del y is equal to 0. So, these are the these two are the boundary layer equations into the whereas, these are the boundary condition, so this two equation has to be served subjected to this two boundary condition.

And now this is called the boundary layer equation in two dimension, now I will analyze few things based on the boundary layer equation, so I will again go back just certain observation we will make from this equation this is from this boundary layer equation. If I if i look at this equation, this boundary layer equation we have seen just I will look at that unidirectional flow, in case of unidirectional flow we have taken we have a pressure gradient term, but what had happened there we had taken the flow convictive inertia terms are neglected, here neglected.

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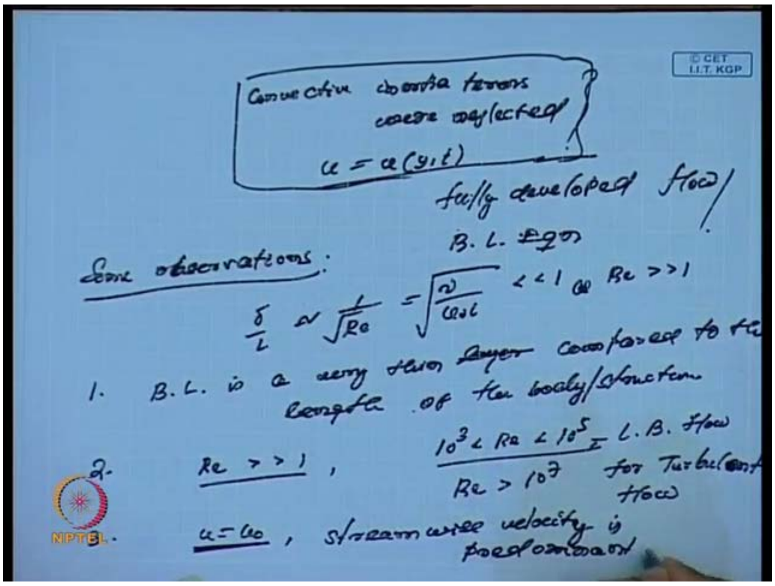

And we had called our flow as u is equal to u y t, often it appears that as if, but there the fluid was viscous throughout, here what is happening we have the fluid is viscous within a boundary layer, but near the near the wall boundary, but beyond the boundary layer the fluid is invasion. So, this is the basic difference between the laminar flow one of the basic difference with the laminar flow and the boundary layer flow boundary layer equation, that is fully developed flow in and then the what I say the boundary layer equation.

Now, I will just some others let us look at some of the observation some observation, these observations are very, very important we have seen that del by del by L is rates like one by R e and that is nothing, but nu by u naught L. And this is most less than 1, because R e is much greater than 1, as R e much greater than one, now, so what we have observed that the boundary layer is a very thin layer boundary layer is a very thin layer and it is thin layer.

Basically, other I will say boundary layer is very thin compared to to the length of the submerged body to the length of the submerged body of the body wall body or the length of the body or whatever of the structure, because del by L is much less than 1. And again here two we have the Reynolds number is large r is much large for example, when you look at the laminar flow I have mentioned, when you look at the laminar flow past a plate. Then our Reynolds number is varies written to that t is less then R e is less than 10 to the 5 and only when R e is becomes this is for laminar boundary layer flow, on the other hand R e is greater than 10 to the power 7 for turbulent flow, when there is just a flat plate.

On the other hand, and here if you look at that the steam wave velocity here, because stream wave velocity that u is equal to u naught, we call this as the stream wave velocity. Just beyond the boundary layer we have this u is equal to u naught, so I can say that a stream wise velocity is dominating velocity is predominant in the boundary layer. Again we can see because this boundary layer is the fourth point I will come to that, you can easily see that if you look at v naught by u naught is of the same as v by u and this is of the nature v is of the order delta, v is of order length delta by L is most less than 1.

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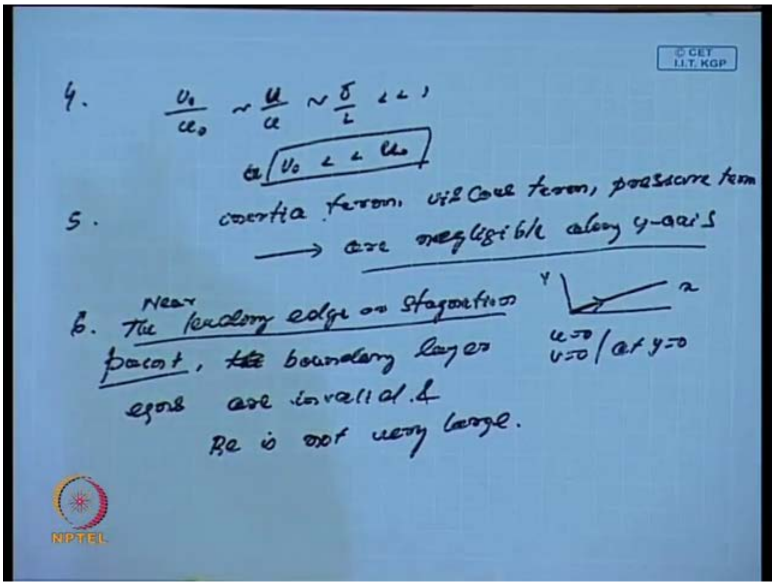

So, that means, what it says; that means, the vertical velocity component it is most smaller if you look at the, so that shows that v naught is most less than u naught, that is another observation. Then another fifth observation I will say if you look at the momentum equation in the y direction here most all the terms are neglected except all the terms almost neglected.

If you look at the inertia term look at the viscous term or the pressure term or the I will say the inertia term is viscous term and pressure term, all these things are negligible along y axis. And in fact; that means, even if the mass is basically the inertia term inertia force viscous force pressure force body force or the pressure force all these things are negligible along the y axis only it is predominant along the x axis.

So, these are some of the observation what we have, then we have another observation is that that easily we will see that because when we have seen these problem, we have taken u is equal to 0 and b is equal to 0 at y is equal to 0. And this is the flow is along the x axis this is y this is along the x axis, so you have seen that, so nearest stagnation point the leading is one of the very important point the leading is or stagnation point.

This leading is at the stagnation point where we have seen that in case of failure foil, there is a flow simulate each occurs and in this situation the leading as near the leading as rather I will say. The boundary layer equations are invalid are invalid here flow is a u is 0, v is 0, we initially describe this and this is the way, so if here the near the leading is or the stagnation point the boundary layer equations are invalid and the in this situation R e is not very large this is another point.

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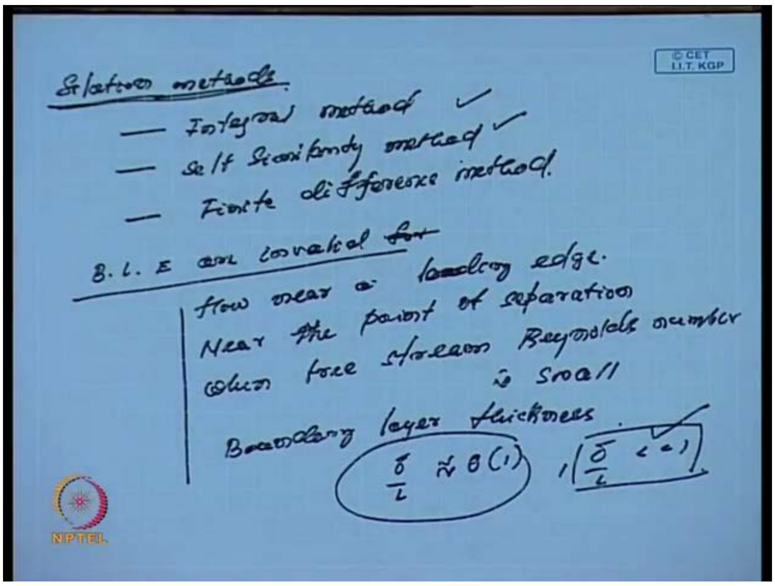

So, these are some of the observations on the boundary layer equation, now this understanding if I go into solution methods, if you look at the how we look into the solution method for the boundary layer equation because we have a earlier seen when we have looked into unidirectional flows. We have considered the flow is of transcend nature study structure either the motion is study or un study and then we have seen is to apply this flow is acceleratory or non acceleratory.

And one of the method used to simulate a transformation transformation method or we have used and within the case of we have also mentioned that is case when the Reynolds number is a more than that of the that of the unidirectional flow motion. Then we can use only approximate methods that is, but what will happen in case of boundary layer, on case of a boundary layer on the solution methods what we will use is.

There are three methods one is the integral methods, another method is called the self stimulatory method, and the third method is called the final difference method. In fact, I will come in detail to this these methods in brief I will discuss about this integral method and self stimulatory in the next lecture though certain examples. And another thing is that this boundary layer this method this rather I will say boundary layer equations are invalid for they are invalid to deal with the flow near a flow near on a a leading edge and also the near the separation point near the point of separation.

As we have seen that the shears force will be 0 shear will be 0, and then the tree stream number is too low when tree stream is small and again this equation will not well when the boundary layer thickness. That means, del by L is of order one because we have seen that when del by L is only of less than or equal much less 1. Than only bounded this equation will hold for the two dimensional boundary layer equation, but if del by L equation is for a one then boundary layer equation as derived will not hold good.

So, this is valid or this is discuss it, so all this cases we cannot apply the boundary layer equation to deal with a flow of this nature and with this understanding on the next class we will discuss how to solution methods and associated with boundary layer equation through various practical examples, now this we will stop here.

Thank you.