Marine Hydrodynamics

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Lecture - 29
Worked Examples on Wave Motion (Contd.)

Welcome you to the series of lectures on marine hydrodynamics. In the last couple of classes we have spent good amount of time on understanding various aspect of wave motion in a fluid, particularly in water. To be specifically in the last class, we have few examples to understand the wave propagation in specific cases, will continue that in today's lecture so that our understanding on the wave motion will be very clear. As I have told you in one of the lectures that when we look at waves in the ocean we across waves whose period is can be few seconds and it can be few hours, it can be in number of hours.

Like when you look at waves, like in case of a it is in few seconds, whereas in waves and further in the other waves which are generally day to day affairs, then it comes around 15, 10 to 15 second or 20 second . There are waves like tsunami, storms, where the wave period can go within minutes or in hours. When you go to tides, wave period again goes more than, it can be in hours. Whereas the waves can be in number of days, the details one can easily get it from various text book of water waves particularly the book where C C Mai, on the applied dynamics of ocean surface waves.

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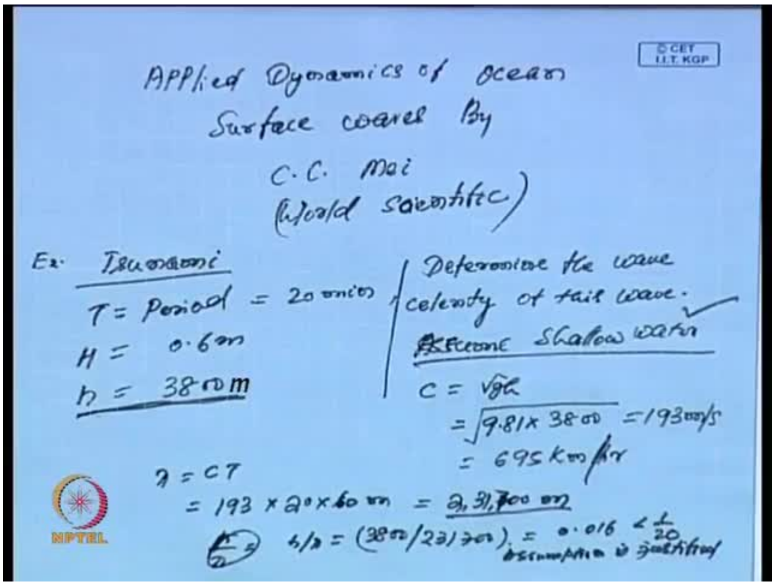

Lot of information can be obtained from this applied dynamics of ocean surface waves by C C Mai. So, today what will do let us work out couple of more examples, for better understanding about the wave propagation. So, let us have look at a wave in a tsunami, I am looking at waves in a tsunami. It has a period, the wave has a period 20 minutes and height period that is T, the wave height is H at 0.6 meter, at a point in the ocean were the water depth is H is 3800, so initially this was the data given. Now, this wave determine the celerity of wave determine the celerity of wave, determine the wave celerity of this wave.

Further, so the celerity will be C is equal to . Assume, if I assume if I assume if I assume shallow water assume that in shallow water, if I assume that then what will happen? C will be root g h and that is 9.81 into H is 3800. In fact 3800 this is almost double of the ocean depth and that will give me 193 meter per second. So, that is the speed at which the wave is propagating at the depth of water . Now, if I put in kilometer it is perhaps it will be approximately 695 kilometer per hour, so it can be considered at the speed of aircraft. Now, what is the corresponding lambda? Lambda is the C T, because I am considering once lambda is C T, that is C is 193 into 20 into 60, last 20 minutes 60 second.

So, that is the meter and that becomes 231000 meter, 700. So, look at this 231 kilometer at large, if you look at H by lambda, if you look at H by lambda H by lambda will be H is 3800 divided by lambda is 231700 and this becomes 0.016 which is less than 1 by 20, thus the justification of shallow water is justified, for assumption is justified, so the assumption of shallow water is justified. Now, with this understanding, now suppose what will happen to this wave?

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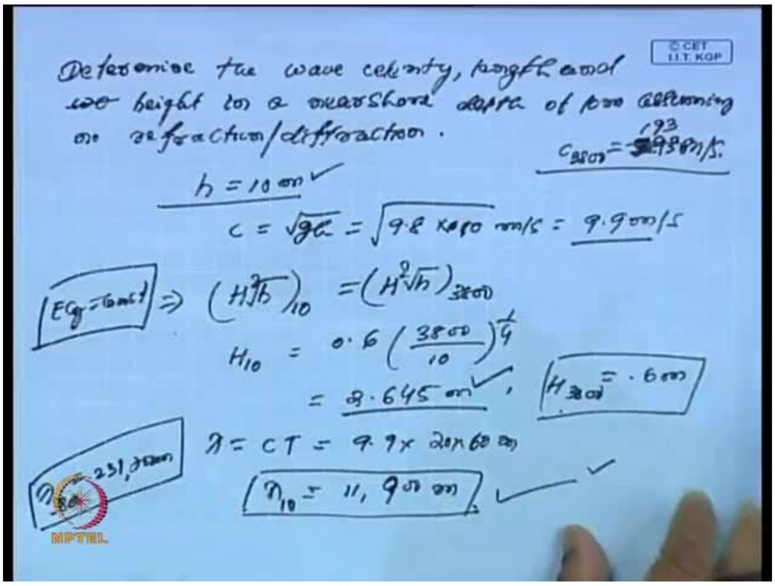

Now, I will go to the second part of this portion, second part is said determine wave celerity, length assuming the and height in a near in a over shore depth of 10 meter, assuming no refraction and diffraction. Where wave means if there is maximum distance same wave is propagating to the shore line, then what will happen to the vysolative wavelength and wave height, were in depth at the water depth. So, that means the nearby region H is 10, 10 meter if H is 10 meter, then what will happen? Because again if I look at the C root g h, g is 9.8 into 10, sorry into per second and that will give us nine 9.9 into per second, so that is the speed where in the near shore region.

On the other hand in the beginning C was 193 meter at C 3800, we have seen that this was 92, 93 meter, sorry 193 meter per second 193 meter per second, whereas now near the when the water depth, so that means near the shore line that depth speed as decreased, but the proportional speed has significantly reduced. Then what happens? What is going? If we apply energy conservation law, H square h, root h 10 is h square root h to the power to 3800, this is comes from the law of conservation of energy plus that I have repeatedly using, so I am not. So, basically it is coming from the equation E c g is equal to constant, in case of shallow water this becomes this.

Then if I, what will happen to my h 10? This will happen to 0.6, 0.6 into 3800 by 10 to the power 1 by 4 and that becomes 2.645 meter. So, we are interesting to observe the speed of propagation is reduced, but whereas the water depth as gone to 2.645. Whereas, the initial water depth at H is 3800, the water depth was 0.6 meter. So, that is what is more interesting to observe, the water depth as increased nearly by 2 meter and if you look at lambda, at this depth lambda is C T and that is the 9.9 into t is 20 into 60 meter and that gives me 11 kilometer 900 meter is my lambda, so this is what the wave.

So, what we have seen that, initial the wavelength when lambda was 3800, there the wavelength was 230 it was 231 kilometers, whereas this was this. On the other hand here, it as become lambda 10 has become this, so the wavelength has significantly reduced speed also significantly reduced water depth as increased that is what. This is one very interesting example to understand the wave propagation, when the wave propagate from the deep water to shallow water, particularly in case of a tsunami and all depends on the its becoming large, because it is quantity is becoming large because it is because of the wave period of 20 minute. Now, I will go to another example it is also another interesting, this example will give us about, let us look at the business lesson.

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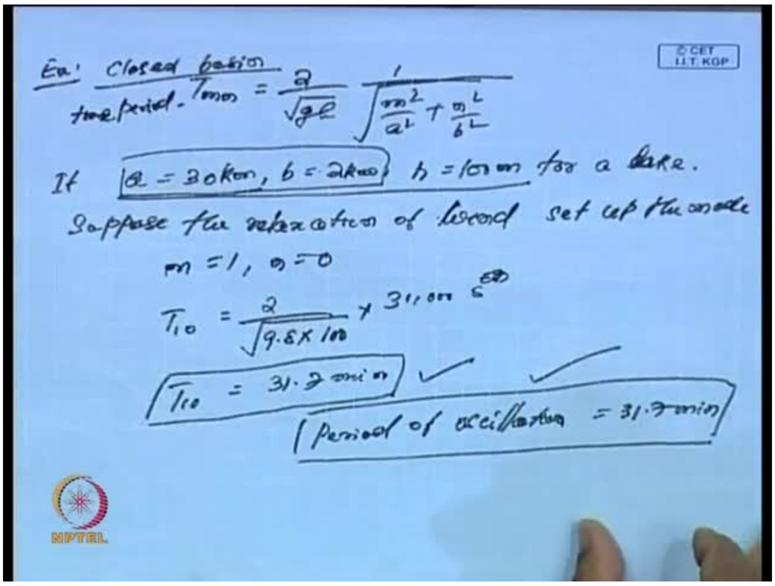

In the last class we have talked about that the time period associated to the rectangular lesson is 2 by root here, close to here 2 by root g h, if it is in shallow water depth, 1 by m square by a square plus n square b square, this is by period of this lesson. Now, what will happen if this is another example? Now closed basin, consider closed basin basin this is of time period time period. Now, if I consider my basin is of length 30 kilometer, it is length of 30 kilometer length, b is a width is a 2 kilometer and the water depth of the lake is 100 meters, this is the data given for a particular lake for a lake.

Then suppose the relaxation of wind set of a wind set of the mode, m is equal to 1, n is equal to 0. Basically there was a wind and then it calculated the harbor, the lake starts oscillating and it set of the mode m is equal to 1, n is equal to 0 and then what will happen in this case T 1 0? 2 by root g h, g is 9.8, h is 100 into 30 kilometer, m a by m, m is 1, n is 0, so this is a is 30 000 and that gives me, sorry it is second second and that gives me how much? This will give me exactly 31.7 minute. So, look at this, so this is period of oscillation of the harbor plus that is your T 1 here.

In fact this kind of oscillation as we have seen that this kind of oscillation in lake makes bring a lot of damage to the infrastructure that is available in the lake, whether it is the sea one, whether it is some of the existing facilities . 31.7 minute that harbor will go go the rather the lake will go oscillating and that will generate the wave, which will say infinitely create a lot of damage to the. So, this becomes the periodic solution. So, these two examples give us how the wave propagates in case of tsunami and in case of harbor oscillation are particularly oscillations of lake.

Here look at the length of lake it is very large lake, 30 kilometer length and 2 kilometer width, very large area, but it creates wave of this period. So, this two examples, that gives us a very good understanding about the wave period and the wave length and the wave length associated with a period of period of oscillation of basin particularly . In previous example it gives us the about the idea, how the wave length and speed of a wave that changes when a tsunami propagates. With these two simple examples, now I will go to another class of problem.

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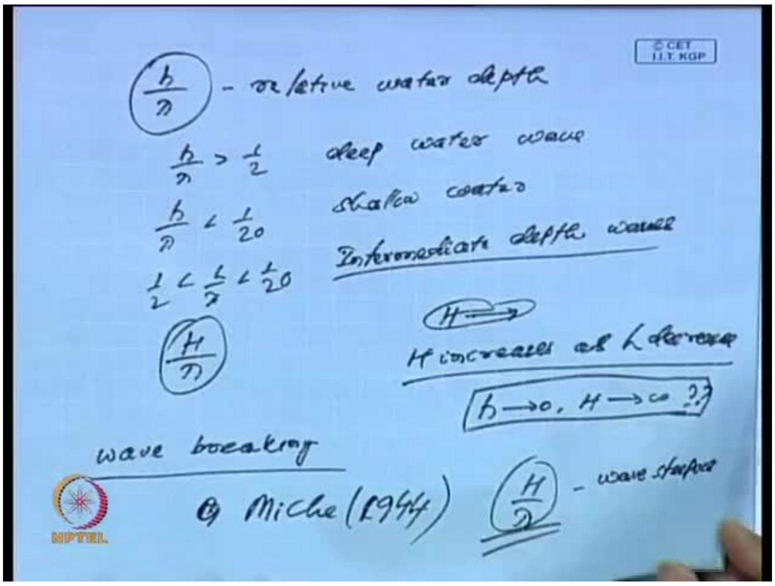

Basically what happen? We always say that when h square lambda will always talk about, this is called the relative depth. This is the relative water depth and when h by lambda when h square lambda is greater than half, say it is of deep water wave, when if say h by lambda is less than 1 by 20, say it is the case of shallow water. On the other hand, when 1 by 2 is less than h by lambda is less than 1 by 1 by 20, say it is the case of intermediate water depth, weighs in intermediate depth. depth. So, that is about h by lambda, what will happen to capital H by lambda? Further this affect the motion of propagation of the wave.

So, when it comes to this, in fact sometimes this h by lambda because wave height increases. We have seen in the previous example one of the example is in case of tsunami, that the wave height increases when the wave approaches towards the shore line, h increases as the depth decreases. The question comes can it a when h is tending to 0 whether h will tend to infinity? That is my question, but in real what will happen to this situation? So, what happened to the wave, when wave propagate from the deep water to the shallow water region? So, there must be a phenomenon of wave breaking and this wave breaking criteria it has developed in 19 Miche in 1944 in (( )) type wave breaking criteria and which all depends on the H by lambda and so that is called wave stiffness. The stiffness of the wave increases, this factor is called the wave stiffness.

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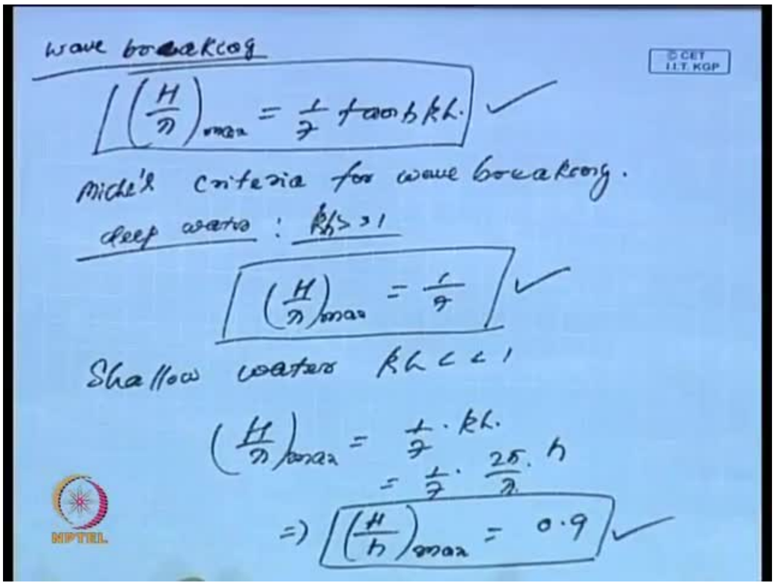

In fact Miche suggested, after lot of module test that this is for lambda ratio, give a limit for this and that limit in case of default is, that is H by lambda, maximum value will be 1 by 7 tan hyperbolic k h and that is the criteria of wave breaking, Miche's criteria. You can see that in case of default, in case of default we have k h is much 1 and in that case tan hyperbolic H by lambda, maximum it can obtain 1 by 7. On the other hand in the shallow water region, on the other hand in case of shallow water, where k h is much less than 1, then we have H by lambda maximum value will be obtained is 1 by 7 into k h. That k h is nothing but 2 pi by lambda, 1 by 7 into k h is 2 phi by lambda 2 phi by lambda into h. So, this is what which gives, which can be written as H by h, because lambda is here, lambda is here. So, this is similar, so this gives H by h maximum is equal to 0.9.

So, this is also particularly in case of shallow water this becomes the limit and in case of deep water this becomes limit, this is in case of water depth, so this is the criteria for, these are the criteria for wave breaking. So, in fact of this wave breaking criteria there are three different factors on which the wave breaking criteria will depend. So, there are, I will not to go in this lecture to that details, but particularly when wave how they break? Particularly when the wave propagates towards from the deep, from deep ocean to shallow ocean what happens?

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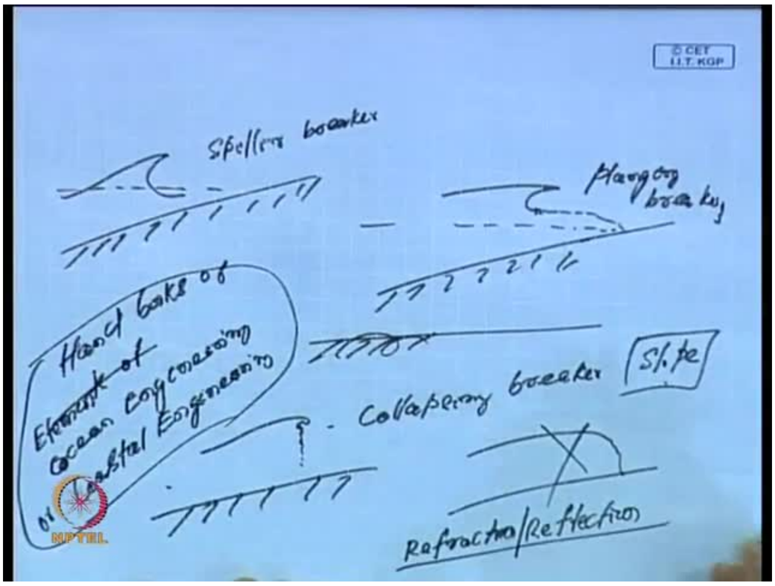

If I say that this is my water depth and what will happen? Wave propagates, sometimes the propagate break like this and this kind of breakers are called spelling breaker, sometimes sometimes the wave, sometimes the wave they break like this and this is called a collapsing breaker plunging breaker, plunging breaker there is something called the wavelength. Some of the wavelength when the waves propagate is called collapsing breaker, nice pictures will be available in text book. Then there is something called (( )) breaker, where not clear, so these are the three major types of wave breakers and better examples can be framed in the book somewhere suppose in the engineering, any handbook of ocean engineering or coastal engineering, this can be detailed.

Nice pictures one can easily get it here in handbooks, various types of working. All these breaking criteria depend on the slope and there is something called slipe similarity parameter. In fact when the vapor propagate to the shore line, friction also takes place at the bottom, because of the friction along with the changing the depth. These are certain kind of sloppy bed or sometimes there is an appropriate change in the water depth, so there also waves can break. There are other phenomenon will be associated with this that is phenomenon like refraction, reflection, those phenomenon will associated, when there is certain changes in the water depth that will (( )). Now, with this understanding about wave breaking I will just workout another example, what will happen to the waves when the wave interacts with current?

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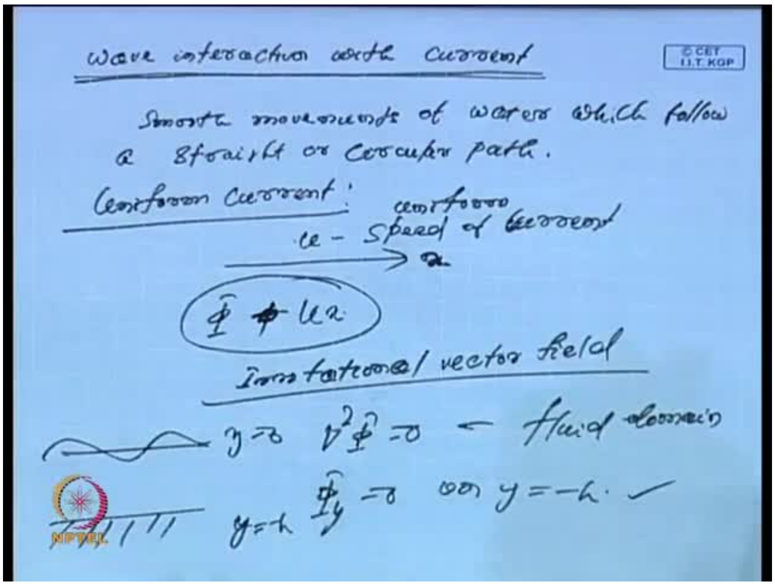

Wave interaction with current, that I will, I started with very simple examples, but slowly slowly we want to understand much more, although everything cannot be covered in this (( )) I just try to introduce. To some other basic things, which are which occurs in day to day affair a little understanding will help in later understanding in detail about the wave phenomenon and the ocean, various aspect of the related to the ocean waves or that to related to the ocean waves or related to the marine environment. But this will give us very fundamental understanding about various things that is associated with the ocean. Suppose I have a current, when I say currents, basically currents are steady generated smooth movement they are kind of there is kind of water, water which follow which follow a straight way path or circular path.

So, I will consider here today in today's discussion will consider of uniform current. You consider I consider (( )) currents flowing in the x axis, let the speed be u, speed of current and basically this is uniform function. So, what will happen when there is a current? What will happen to the corresponding wave? So, are you still in the potential because it is in uniform speed? So, if I say that initial fluid, then my in presence of current my velocity potential will exist, so phi will be phi plus u x, because I am talking of uniform current along x axis, this is along x axis. So, phi can be, this is the speed of current, so my new velocity potential phi will be phi plus u x.

Now, then this is also, the vector field is also, velocity field is also rotational, if I, since I consider the water as a rotational in the presence of a uniform current irrotational vector field. If it is a rotational vector field, then this also satisfy, automatically this will also satisfy del square phi is equal to 0. Again we all know that, in the presence of current, how the, initially I know that when we have the free surface we have bottom condition phi y, this phi y will also satisfy this and then we have phi y, this is also 0 on y is equal to minus h, this is satisfied in the fluid domain, whereas phi y is 0 and y is equal to minus h, that is basically on the bottom and that is there is a wave with current and here y is equal to minus h, so here this is will be satisfied.

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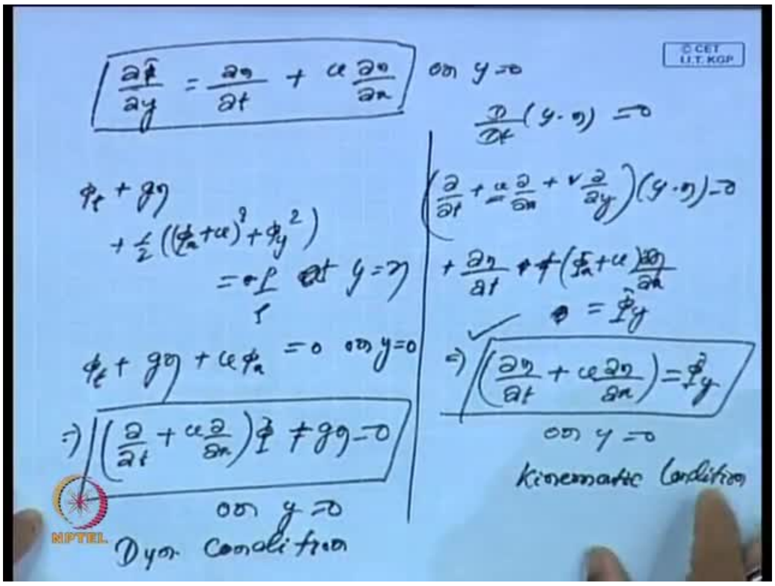

Now, what will happen in the free surface? In the free surface we will have two conditions like we had the dynamic and kinematic condition on a free surface, so in the presence of current also same condition will be satisfied with certain modification. We can see the two free surface condition should be, in the presence of current it will be del phi by del y is equal to del eta by del t, eta is the free surface elevation plus u del eta by del x and that will give us. This comes from the same condition, because we have D by D t, y minus eta, this is on y is equal to 0.

This is the linearized kinematic condition, like this comes from here y minus eta is equal to 0 and you have this is nothing but del by del t plus u del by del x plus v, v will not be there, because, take it v del by del y into y minus eta, here what is happening? I am putting phi is equal to phi plus u x, I put so this will be minus del eta by del t, this side will be del eta by del t and then if I take with these, then you have u is horizontal velocity and that is nothing but this equal to phi x plus u as well as horizontal velocity in presence of current is u is the speed of current and phi is the phi say the fluid motion that comes the horizontal velocity of the fluid particles plus speed of current into phi x plus u phi x plus u into eta and then plus v del by del y, so plus b, so this is del eta this is plus and this is equals phi y, b is phi y.

So, that becomes del by phi x eta that term will go, because of we are looking at small amplitude waves, so we have del eta by del t plus phi x u del eta by del x, sorry del eta by del x, so plus u del eta by del x, this becomes is equal to phi y, this is capital phi. So, this is becomes the kinematic condition, it becomes y is equal to 0, this becomes this is linear kinematic condition. If you look at the dynamic condition what will happen? The dynamic condition will be, because we have phi t plus g eta plus half phi x plus u square plus phi y square is equal to 0, basically minus p atmosphere and this is minus p by row.

But phi is equal to eta and this if you simplify on y is equal to 0, then you get phi t, you get phi t plus g eta, from this from this linearized and you will get plus u phi x and that is equal to 0, that is 0 on y is equal to 0, that becomes the or we can write it in a simpler form del by del t plus u del by del x into phi is equal to plus g eta is equal to 0. This is on y is equal to 0 and in fact this is the kinematic condition, in the presence of the current becomes the dynamic condition, when we have wave with current. So, here if you put u is equal to 0, then you get del phi by del t plus g eta is 0 that is the dynamic linearly dynamic condition and here you get del eta by del t is phi y that is the…

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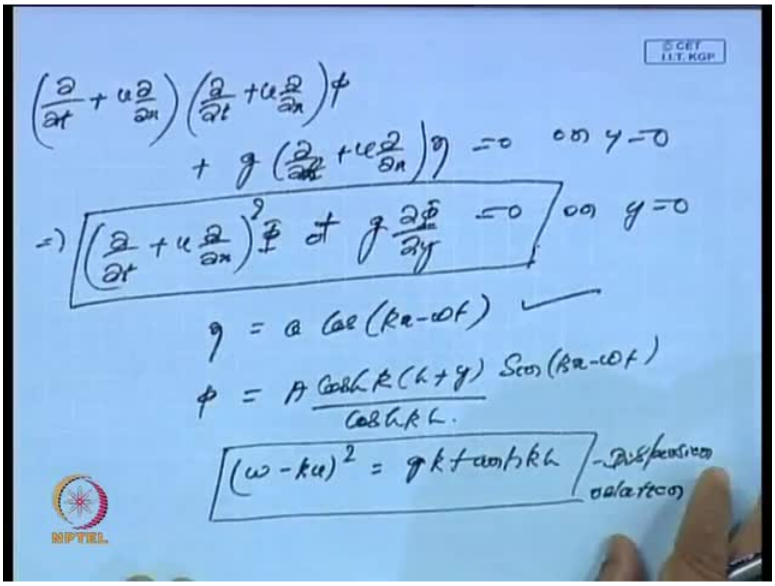

Now, if you what will do here, I will just little simplification, suppose I operate both sides by del or the dynamic condition or I will take del by del t plus u del by del x, this operator I apply on the dynamic condition. If I apply in the dynamic condition then I have already del by del t plus u del by del x, that is phi, this becomes plus g del by del eta del t plus u del by del x. This is one of the interesting result into eta, this is 0, this is on y is equal to 0, but we know del by del t plus u del by del x eta is equal to phi y, so this becomes phi y, so that gives me.

So, this will give me del by del t plus u del by del x square phi equal to g plus g del phi by del y equal to 0 on y is equal to 0. In fact this becomes the free surface boundary condition in the presence of uniform current in one dimensional one dimensional. Then if I start with a simple wave, suppose I start with eta is equal to a cos k x minus omega t, then I can easily see my I can get phi which is of the form A cos hyperbolic k is plus y by cos hyperbolic k h sine k x minus omega t and from this. If I substitute what this, if I have a similar profile of this, then phi y will be this can be easily seen that omega minus k u square will give me g k tan hyperbolic k h and that will give me the dispersion oscillation in the presence of current. So, if I simplify further this one, this is the lesson, because, so in the presence of a current there is a shift of the frequency here omega by k u times.

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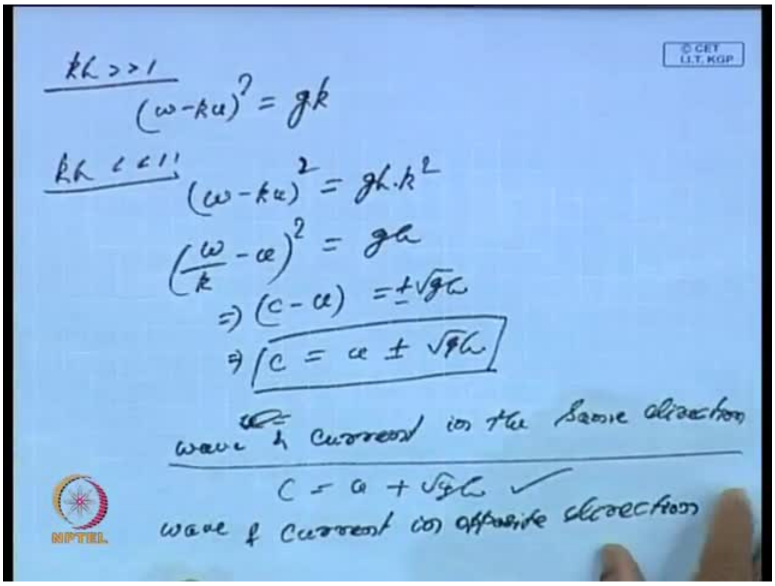

In fact if I go for simple two cases that when k h is large, that means omega minus k u square is g k. On the other hand when k is small small, then we have omega minus k u square g h into, g k into k h. So, that if you just bring k this side, so this will give me omega by k minus u square is g h, which implies omega by k is my C minus u equal to root g h. This can be plus or minus because this is square part and that will give me C is equal to u plus or minus root g h. This one of the interesting result, particularly what will happen to C? What is C? C is a speed of proportional to which wave is propagating, when u is equal to, if I take minus sign, so the positive sign means here two things.

The positive sign is, when there is a positive sign that wave and current co linear currents, when wave and current are in the same wave and current in the same direction, wave and current in the same direction, then when wave and current are in same direction, then C will be u plus root g h. That means, speed of wave propagation will increase in the presence of current, on the other hand when they are in the opposite wave current in the opposite direction, in opposite direction if they move in opposite direction, that means (( )), then there is a retardation in the speed of the wave propagation.

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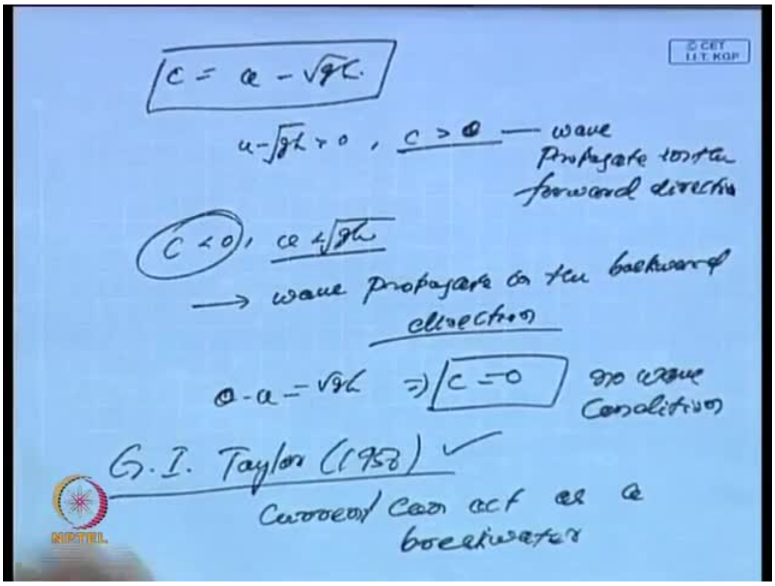

Again it will see the here, that means in that case your C is u minus g h u minus g h and if u minus g h is greater than 0 u minus g h is greater than 0, that means C is greater than 0. So, in that case the wave propagates in the opposite direction in the forward direction. On the other hand, if C is less than 0, that means u is less than g h root g h, if this is this, then we have C less than 0 and once C is less than 0, the wave propagation, wave propagate in the opposite direction, backward direction. So, that means, again there is a situation here we have when u is root g h, which implies C is 0 and this situation gives no wave condition. Same analysis can, we can think of the same analysis in case of deep water and also in any water intermediate depth.

So, in fact this was G I Taylor in 1956 in proceeding, I think it was the proceedings at London, so he pointed out that current can was, current can act as break water. It can always retard that, if there is a current, it can retard the direction of the wave propagation and it can modify the impact of the waves. This is one of the very interesting observations which were meant by G I Taylor.

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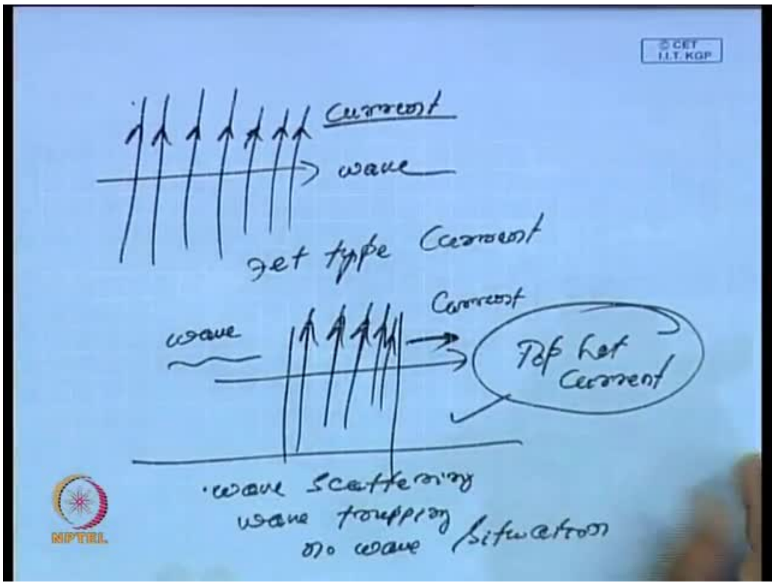

Now, in a same way there are currents which I can think of, there are situation where the waves is propagating in this direction, this is the direction of wave propagation, where as the current propagates along this way, this is the speed of propagation current may be moving, this is the direction of current and this is direction of wave wave propagation direction, this is the direction of current. Sometimes we call this as. Then this is called jet type of current, there is another way that wave is propagating like this, there is a current which is a jet type current, but it is only propagating over along these trip, there is a current, just in a particular strip, there is wave is wave and this is the directional wave propagation, this is the directional current, this is the wave propagate.

In such situation, in this situation that is called we call it sometimes top hat current and in this situation there is a, there are three things occurs. One is a wave scattering, wave trapping and no wave situation. That means, when I say no wave situation that means there is no wave which propagate, in this region, there is a situation when because of the current the waves will be trapped. Another time you will have a wave which will propagate to the other side, there is a scattering and in case of trapping, it should be trapped and there will be not any wave which will propagate to the other side. Another situation is that, there is not at all any wave whether trapping or scattering will take place and these are very interesting phenomenon. Basically (( )) this wave current interaction, in fact one of the major area of today’s what we have make is wave current interaction.

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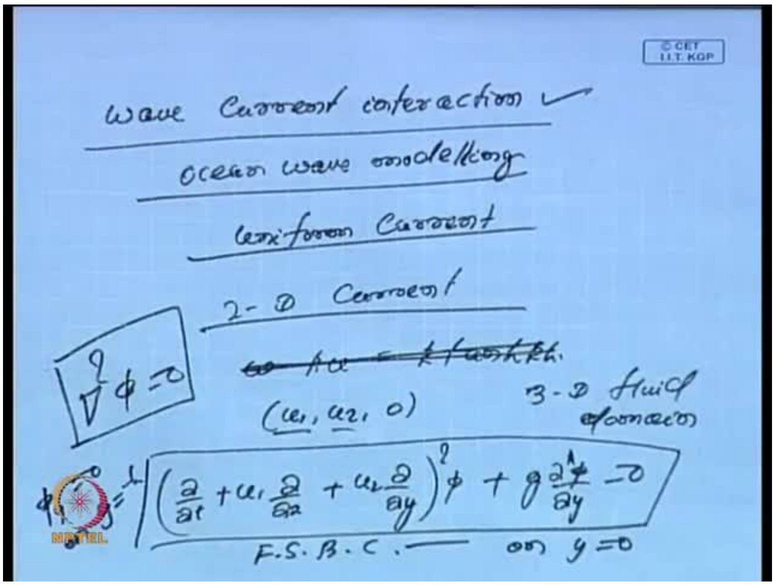

It is very interesting area when it comes to (( )), because you have particularly navigation, whereas navigation and also understanding various aspect of ocean circulation and modeling, but particularly today will go for ocean wave modeling on various regions. One of the major emphasizes is given on waves for an interaction and again I have talked about today, today I have talked about we have uniform current, one dimensional current, uniform current. But always you can have a current which is suppose, I consider a two determinant, 2 Dimensional current. In that case like I say omega minus k u k tan hyperbolic k h and this omega minus k u.

Then, what happen in case of sorry 2 d current? I may I may have the component c 1 u 2 0. Suppose, I consider the 3 D determinant, 3 Dimensional fluid domain, in case of 3 dimensional fluid domain, I may have a current and which propagate both in the x and y direction. In that case my equation of t surface condition will be, what (( )) were by del t plus u 1 del by del x plus u 2 del by del y square phi plus g del square phi by del t square, g del phi by del y is equal to 0. For the same Laplace equation, where this Laplace equation is 3 Dimensional equations and my t surface boundary condition can be of this nature. Whereas, my bottom condition even if we have a uniform bed my bottom condition remain as phi by 0 and is equal to minus h.

This becomes the linearization free surface condition y is equal to 0, this is the free surface boundary condition. So, this is the wave modified and accordingly the dispersion will also change. In fact I only talked about to what happen in a particular case, in case of shallow water it can also be thought in case of deep water or in case intermediate water depth. So, these are some of the details which can be found in various textbook, but this is just a brief introduction to wave. Even if there will be interaction of linear and non-linear interaction of current like I am considering here today on linear interaction.

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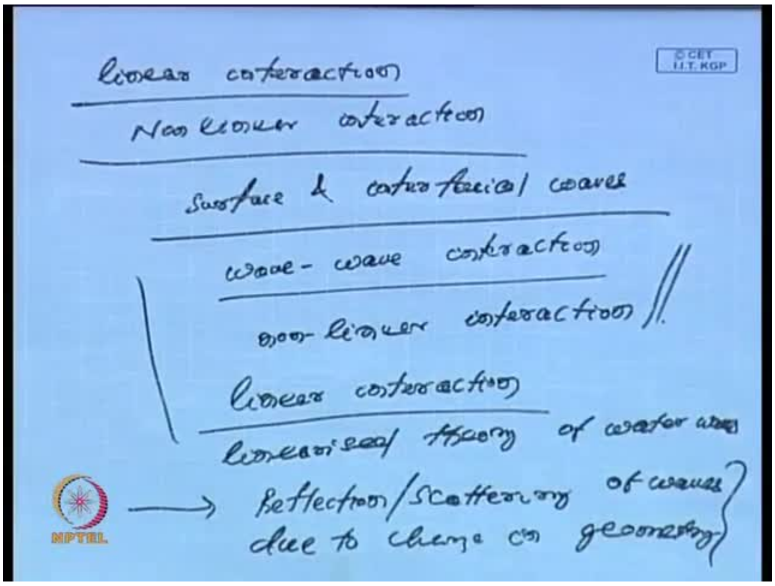

Similarly, we have like in the last linear interaction, we have non-linear interaction, because all these things, like again I have talked about interaction of, we have talked about surface and interfacial waves, in one of the last lecture I talked about surface and interfacial waves. In fact, there can be wave of interaction that means interaction of the internal waves to the surface waves, surface and inter again there will be, there can be there can be non-linear interaction. Today as far as the science is concerned major application associated with wave propagation problems, where the physical, realistic physical problems are handled is based on the linear theory, linear interaction or linearised theory of water waves, I say linearsed theory of water waves.

In the next class, I will tell you some of the two phenomena, how the the concept of reflection and scattering comes (( )) replacing of the scattering of waves, due to abrupt change of water depth, reflection or scattering of waves due to change in due to change in geometry. This will come in the next class I will talk about this and that again I will become passaging on inner interaction. So, this linear Interaction, in fact many problems of coastal engineering or ocean engineering today are handled by the linear, based on the linearised theory of water waves.

I will not go to the details about wave loads and abrupt structure here, because that is a different course all together or anything detailed about coastal engineering, coastal hydrodynamics. But here as a basic emphasizes, I will just say that how the various, the wave, how the wave propagation problem changes? How the wave transformation takes place in various places in brief, because the details we can discuss in other courses like ocean engineering or (( )) free surface hydrodynamics or in courses like coastal hydrodynamics, ship hydrodynamics. But this course gives a very brief idea about what is happening in several cases, when the wave propagates. So, with this background today, I will stop here.

Thank you very much.