Elements of Ocean Engineering
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Lecture - 10
Waves – I

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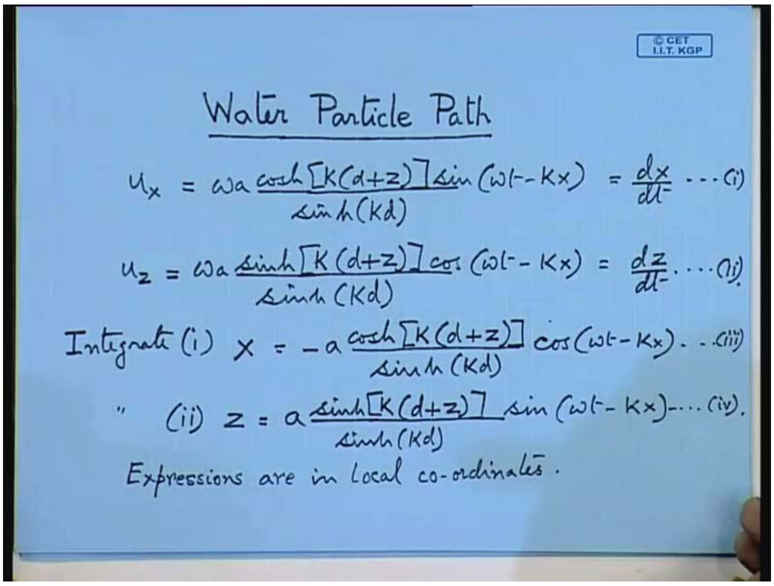

So, today let us calculate the water particle path. So, previously you have obtained the velocity equations, so that was u x, so this is equal to omega a cos hyperbolic term is there. So, there will be 2 time K into d plus z and sin hyperbolic K d, so now, in this is your sin term this is omega t minus K x, so this is equals to you d x by over d t. Now, you integrate this with respect to time you get, the distance x, so we have to calculate the values of x z, so you integrate this and you said was this is the velocity in the x direction, and velocity in the z direction that is in our x z plane.

So, this was omega a this is the cost hyperbolic you to sin hyperbolic term d plus z sin hyperbolic K d, and this cos of omega t minus K x. Now, this is equal to d z over d t, now you find the values of x and z by integrating these equations, so this is you equation 1 this is equation 2. So, integrate 1 what is value, so x will be other hyperbolic you are integrating with respect to time, so only in the sin we are the omega t minus correct t is coming in the sin term.

So, this will be an as it is, so and omega will be divided by omega and this will be minus a, so this is minus a. So, you do not have to bother about the hyperbolic terms, where the hyperbolic terms will come when we study deep water and shallow water, so this is sin hyperbolic K d, so this is you have much cost omega minus t minus K x. Now, you integrate to you get the value of z, so z is equal’s to a sin hyperbolic this will be as it is K d plus z and sin hyperbolic K d were become in this and this will be sin term will come sin omega t minus K x, this is your expression for x and z. Now, you find out the path, now this you write all these expressions are in local coordinates, not global, now you can find out the path.

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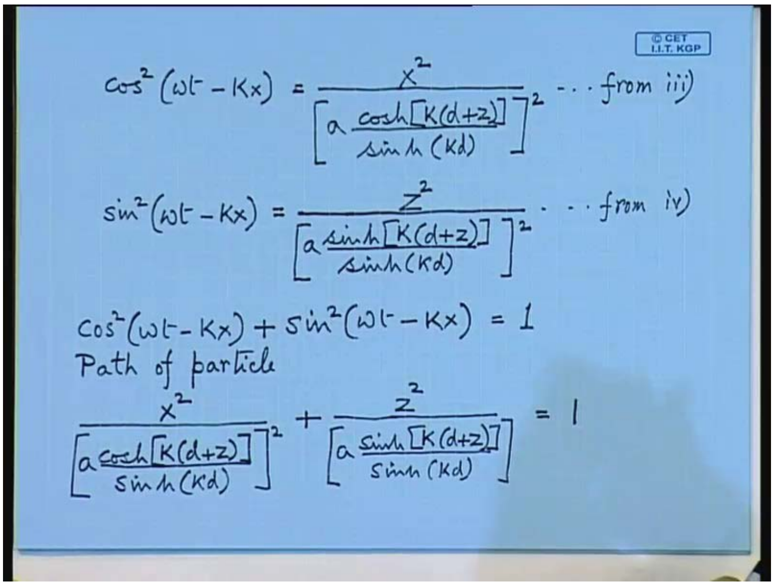

Now, you calculate the find out the path you squared this term cos square omega t minus k x, now what is that. So, this you can find from and you have integrated the find from equation 3, so you have find out x and z these two, so now you can find out you square these terms. So, cost square omega t minus K x is how much, so that is equal to K into d plus z this is sin hyperbolic K d square, now similarly you find out the expression for sin square omega t minus K x you do not integrate the 1 and 2 expression, when you would not get.

But, if you integrate 3 and 4 you will get the path of the particle x and z that is the first 1 you will find out from your having x square in the numerator, and the second 1 you will have z square in the numerator. So, now, we can guess what is the particle path, so this is the first one the cos square we have getting it from, from 3 we are getting, so you are write from 3 and this is from this is from 4. So, you get a interesting relationship now you calculate this.

So, what is the value of this cost square omega t minus K x plus sin square omega t minus K x, so that is equal to 1, so now you are getting the equation. So, you are equation is, so what does have give, so this gives us this single equation, now you can recognize this what is the particle path K d plus z this is sin hyperbolic K d. So, the recognized this equation what is the, so that gives us the equation of ellipse, so path elliptic in nature.

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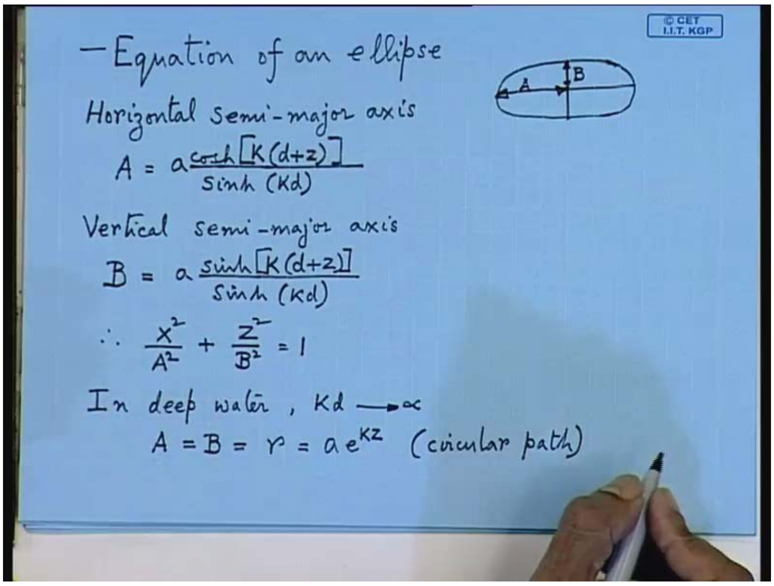

So, this is equation of a ellipse with horizontal semi major axis, what is the semi major axis horizontal ellipse you draw a ellipse, this is you are horizontal axis, this is you are vertical axis. So, now, the semi major axis you marked this as A this is called horizontal semi major axis, and other one the vertical one you marked this as B, this is the vertical semi major axis. So, what is the value of A, so A is small a what is small a, if you do not remember you just look at the equation for eta this surface elevation and there you will find small a is the amplitude of the wave.

So, do not mix of between the capital A and small a, so this is the length of appear what is that horizontal semi major axis. Now, you find out what is vertical semi major axis, now these two expression we find out this not perform there some significance into this which, we will shot we see what is the significance, so B is your vertical semi major axis. So, this is given as small a is the amplitude of the wave, and this is the other term that is you are the hyperbolic term. So, this is your sin hyperbolic K into d plus z and this is sin hyperbolic K d.

So, are equation if you short and het, so this becomes X over A square plus Z over B square is equal to 1 sorry this will be X square plus Z square. So, here also I think the squire terms for what, so now, let us have a look what is happening in deep water what is the value of K d. So, d is in finite your K d value is also infinite right, now in this case you are the value of A will be same as the value of B, and this will be the radius of the circle, so this will be a e rise to the power K z, so you write circular path.

So, always in the ellipse if A is equal to B you get a circular path is not, so the deep water you have getting the circular path, and A is coming up from A and B are the hyperbolic term are reducing to e rise to the power K z. So, that is what is happening, now in shallow waters what is happening?

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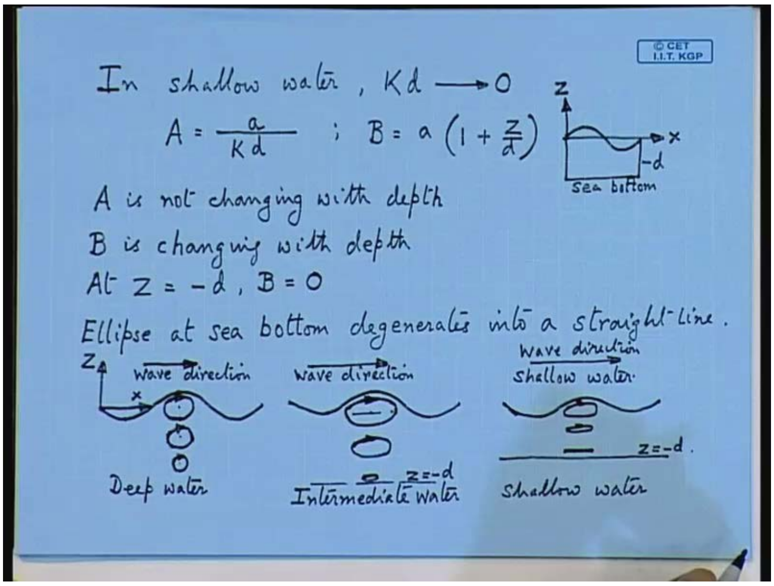

Now, in shallow water what is the value of K d, so K d will always 10 to 0 for small values of d, than what is the value of A. Now, A will reduced to now this you can walkouts in your hyperbolic expressions trigonometric hyperbolic expressions if you work out you will find. So, this is the value of the A and B will be the reduced to this value, now we are taking the value of z to be 0 at the free surface.

So, in the I think the diagram that you are drawn on earlier you will find your z is positive upwards from the free surface, and x is positive it was the right follow. And this is your web profile, you are studying in the near wave and this is the value of minus d, minus d is sea bottom. Let us sea bottom what is happening, now A if you look at this expression for A we will find that A is not changing with depth, but seen K d is tending to the some large value listen it.

So, k is not changing with depth why you are just know z term, were as a you find this B is changing with z, now you tell me what is the value of B at z is equal to minus d. So, at Z is equal to minus d, B is equal to 0, so that is there is no vertical axis. So, ellipse at sea bottom degenerates into the straight line, so there is no value of B whereas, there is some large value of A.

Now, if you look at this diagram you will find this is how it be, so this is you are in deep water diagram. So, the path of the particle you will find, now there is one interesting analysis from here, now if you standard the crest of a wave supposed you are starting in this region. What will be your future can I guess, suppose you are the top of the wave crest, suppose if you are person is the top, now you your deep water actually. So, in my diagrams drawn it the diameter of the circumstance keeps on decreasing.

Now, we propagation of wave is from left to right, so this is your wave direction, so this is your positive direction for x, this is your positive direction for Z. So, what will happen to this person this I thing wave crest, so you will get knocked away by this, whereby this listen it because this is going the clockwise direction. Now, similarly related diagram will find, suppose we are the top of the wave you are likely to be more towards the crust and then you will get thrown away.

So, that is read dynamics now you can see this is a in deep water, so here we can distance to see that, the deepwater the paths us circular, but there radii a cost will be decreasing. Now, next let us come to the intermediate waters, so again you wave direction is still the same I am not altering the wave direction, so here instead of circulate you can get. So; that means, the water is becoming more and more shallower, so we are going from deepwater to the shallow water region.

So, here you get a distinct ellipse, now the size of the ellipse that is you are A and B is going to decrease with depth. So, this is your intermediate waters and our sea bottom this is sea bottom, so the value of z here is minus d, now next let us come to the shallow water. Shallow water we have still you get and ellipse at the surface, but there will be no ellipse at z equal to minus d. So, let us tried to decrease the depth, so here you get rip tic shape and this is come like this, but this is of this short value of your get a straight line like this. So, this is z minus d and this is shallow water sorry this is wave direction, so this is the particle path.

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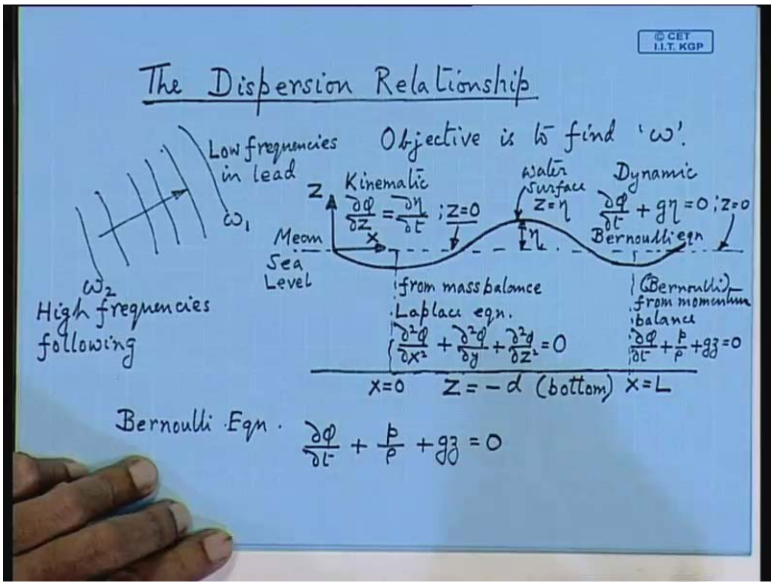

Now, next let us come to the another important relationship which is called the dispersion relationship. Now, what is the meaning of this the term dispersion, so if I tell you get disperse than what do I mean, now if you look at a waved field, suppose in a sea there is the wave field, the number of waves are there. There is going in this direction is a propagating wave, now you find the low frequencies are in the lead and this of course, there is further study which are not telling right now.

So, low frequencies is in lead you can write this has, so let us say omega 1, so what is the value of omega, omega is that 2 pi by t. And here you find high frequencies high frequencies following, so; that means, particular wave field in the sea in where different frequencies, this is low frequencies in the lead leading the pack and added by the high frequencies. So, this is called a dispersion that is the waves have been scattered on dispersed according to the frequencies.

Now, if you look at this diagram if you find out what is dispersion relationship, you have to calculate this value of omega did you find out this value of omega. Now, if you look at the diagram that is your boundary conditions, so what is the boundary conditions, so in the dispersion objective is to find this term omega. So, this is our local axis now in a hydrodynamic class you will come across these conditions.

Now, let us take one wavelength this means of surface, so you take these distance from trap to trap. So, this is one trap and this is another trap, so what is the distance between this two trap, so here let us say this is sea water, so sea bottom is characterized by what boundary Z equal to minus d. So, this is the bottom boundary condition, and seems you are taking one wavelength, so this is at the position x equal to 0, and this is the petition x equal to one wavelength is L x equal to L. So, what is your surface elevation with which is started this is eta.

Now, you want to find out the this was in that is the value of omega you write down what are these conditions at the free surface, this is called the kinematic frees surface condition, will the kinematic free surface condition. Kinematic means do with the velocity, so; that means, delta phi over delta z you have velocity potential will be equal to delta of your surface region that is eta will respect to delta time. So, this is the condition at Z equal to 0 is called the kinematic surface condition of the wave.

So, your wave mechanics class you come to know how there are derived this, and this is your water surface elevation. So, what is the value of Z at this region is now Z equal to 0 is this the dash line follows, and here the value of Z is water surface is Z equal to eta, now on the other extend you will find what is called the dynamic condition, dynamic free surface condition. So, what is that, now the dynamic condition comes from the Bernoulli's equation, so that is del pi over del t plus g of the surface multiplied by eta equal to 0.

So, this is you are Bernoulli's equation, so here to know all this equation you have to find out the value of the dispersion, now you write this expression is that z equal to 0. So; that means, at this line, so these are the equation we have got, now inside this you write down the equation for the Laplace equation. Now, this you are got from mass balance, so what we have got from mass balance that is equation of continuity.

So, from the equation of continuities last class we have got, the Laplace equation that is del square phi over del x square plus del square phi over del y square and plus del square phi over to del square is equal to what. So, this is ever famous Laplace equation from equation are continuity, now you get on the other side you will get the Bernoulli's equation here. So, but this below the mean these called the means sea surface this is your mean sea level.

The same Bernoulli's equation from momentum balance, but slightly different in form only thing you will get, now at the free surface what is the p is equal to 0. But, here actually there will be the water pressure will be there, so Bernoulli's equation will change because of that. So, you will get del phi over del t plus p by rho plus g z equal to 0, so in brocket I am writing in top do not have any space this is your Bernoulli's equation, with is like the change for another parameters p is coming because of hydrostatic pressure. Now, let us see how to use this equation, so you come to this Bernoulli's equation, so Bernoulli's equation gives us del phi over del t plus p by rho plus g z equal’s to 0.

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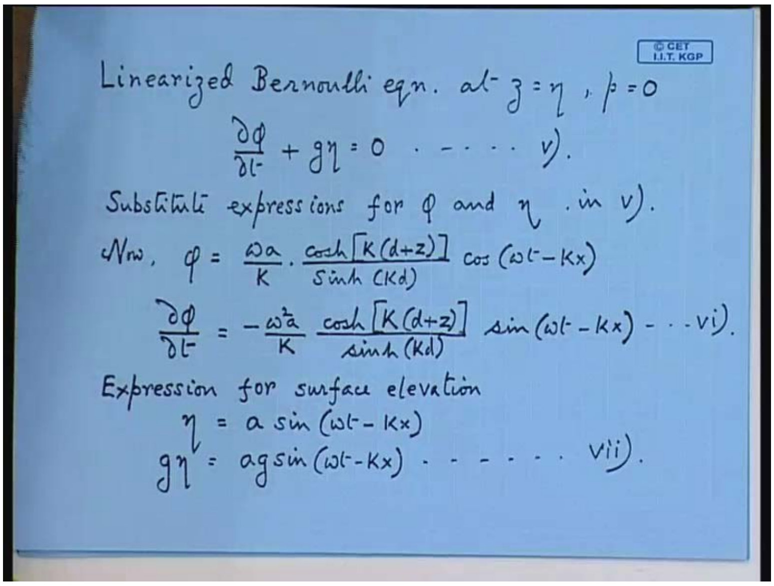

Now, if you put p is equal to 0 then you get a much more simplified equation, so that is called a linearized Bernoulli's equation. Now, this to be at z equal to eta and p equal to 0, p 0 the hydrostatic pressure, so at; obviously, at this surface p is equal to 0, now you get p is equal to 0 in this expression you will get del phi over del t plus g into eta equals to 0.

Now, in this equation, so this is let us say this is equation number I think this will come to 5 and something like this corrective now.

Substitute expressions for phi and eta, now you do you see what you get phi and eta in phi, so what was the expression for phi. So, this is the condition at free surface listen it, so phi is the expression for velocity potential, so which if you turn the pages you find out this expression that you are getting. So, this will be omega a over K this is the multiplied by cost hyperbolic of k multiplied by d plus z, and this is sin hyperbolic K d.

Now, what other term is there, so the cost term is there, so this is omega t minus K x is called a surface angle remember that. Now, this expression you will find out del phi by del t, so differentiate this expression with respect to time, so how much you get, so this is simple. So, this will be minus omega square a over K, this is the cost hyperbolic of K multiplied by d plus z, now what is this is there, this is we sin hyperbolic of K d and this is the caused this will be sin and omega t minus k x.

Now, we all are expression will be what, now you first will be the others, so here the let us get this has this is your getting 6, now you tell me the expression for surface elevation. So, this is simple wave of eta, so this is even amplitude a is the amplitude of the wave, and you are giving a harmonic single, so this is omega t minus K x are the surface angle. So, g eta will be simply a g multiplied by sin of omega t minus K x now you add this to 6 and 7 seeing see what you get, so from this equation you add 6 and 7 you get the dispersion relationship will come.

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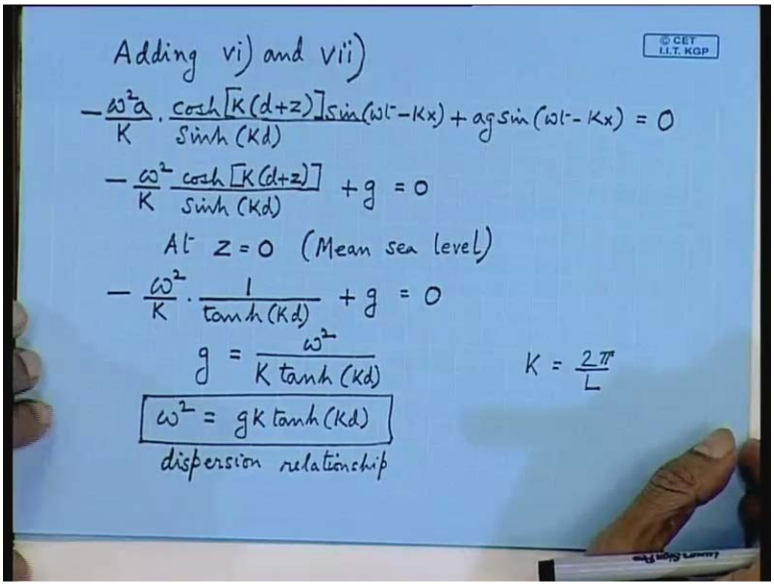

So, this is quite simple, but is simply have to follow the mechanics, so here we get the largest special. So, this is omega square a over K multiplied by, so this factor will be there cos hyperbolic K into d plus z and the sin hyperbolic this is K d, now you are simply fining this expression, your sin term will go this sin omega t minus K x that you can drop up and even drop up also we have an problem. So, this is equal to what is equal to 0.

So, our Bernoulli's equation was these two equal to 0 for getting 0, so a sin omega t minus K x will go and they are getting a rather simple equation omega square. So, this is how you get the dispersion relationship, now here actually this term it is still as to simplify the hyperbolic term. Now, you put at z is equal to 0, now z equal to 0 means free surface listen it, but the value of z you are getting a z where you out here, so this is a pure mean sea level at z equal to 0 you are expression is still simplified phi further.

So, this is will be equal to minus omega square over K what is this expression cos hyperbolic K d divided by sin hyperbolic K d your z is vanishing right. So, this will be 1 over tan hyperbolic of K d plus g equal to 0, so we are getting g equal to, so there are a lot of significations. So, g is equal’s to omega square over K tan hyperbolic K d, so now, from this equation you can get omega square.

So, we have dispersion relationship is coming as omega square is equal’s to g k tan hyperbolic K d, so this expression you should remember this is called a dispersion relationship. Now, in this dispersion relationship there are two things which are important, now I tell you that your omega square actually you have to calculate, if you have to find out the differencing frequencies of the wave field. And if you want to know that g is constant, now what is the things will be constant g will be constant you have d is to some extent to I thing you take it constant.

But, it will vary with this K value and K is dependent round what, what is this value of K, K is nothing but 2 pi over L K is dependent K is equal to wave number of wave number dependent on the wavelength. So, here, so we get a important relationship this is called a dispersion relationship, now from the dispersion relationship we can study.

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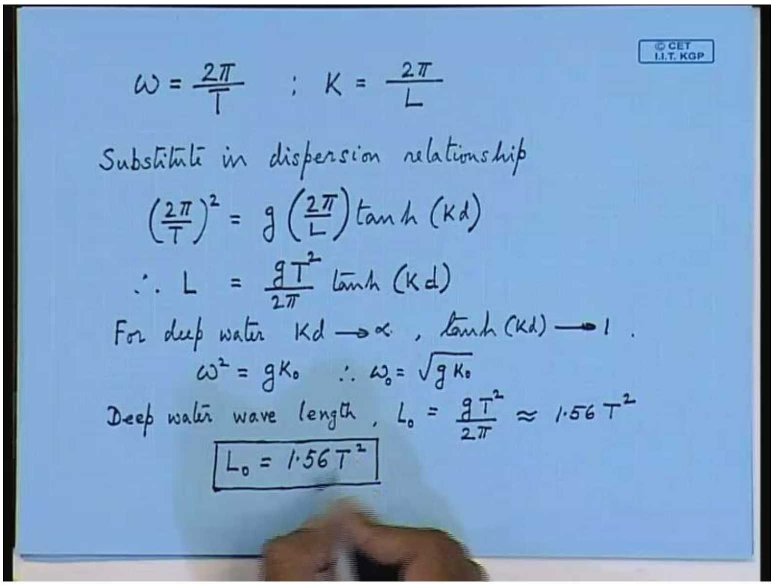

If you put omega equals to in this expression omega what is the value of omega, the omega is related to your the time period of the wave. So, that is it is inversely proportional to T, and this value of K is related to you are inversely proportional to 2 pi over L. Now, substitute in the dispersion relationship, so what is happening, so and the left hand side you will get the omega is 2 pi by t, so this will be you will get a finely the expression with the wavelength, so 2 pi by t whole square, so this will be g.

What is the value of K, K is a wave number you should remember that is always is equal to 2 pi over the wavelength that is 2 pi by L multiplied by tan hyperbolic K d, over. So, from this equation we are getting the relationship between the wavelengths that is given by L, the right hand side we are getting your time period of the wave. So, that is g T square over 2 pi tan hyperbolic K d.

Now, in wave actually you should remember there are two important parameters given by the wavelength and the time period. So, this is you are expression, now for the deep water, deepwater what is the value of K d, K d will tend to infinity, and tan hyperbolic term will we equal to 1, so this will tend to 1. So, the our expression for the dispersion of the frequency will be omega square is equal to g multiplied by K naught, you write k naught is in deep water.

So, therefore, you will get omega is equal to root over g K naught, now from this find out the deep water wavelength. Rather you can write instead of omega since omega is dependent on K naught you can write this omega naught also, so deep water wavelength. So, that will be characterized by L is 0, now what is these value your tan hype ring this expression tan hyperbolic K d is reducing to 1, so this will be g T square over 2 pi and this is approximate the equal's to 1.56 times your T square value.

So, deep water equation you can get from this relationship this is 1.56 over T square, so whenever you are asked a question and all this wavelength and time period, in this 0.06 you can ignore that is your interviewer ask the what is the deep water wavelength, and what is the time period. You simply in your mind you just calculate the 1 and half time is the T square. Now, normally your what is your time period for waves deepwater waves, I thing in the next class I will give you a diagrams to that 20 seconds that type. So, immediately you can guess the wavelength anywhere.

Thank you.