

Thermodynamics
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Lecture 87
Tutorial problem (1 number)

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An ideal Brayton cycle, operating between the pressure limits of 1 bar and 8 bar, has minimum and maximum temperature of 300 K and 1500 K. The ratio of specific heats of the working fluid is 1.4. Find out the final temperatures in at the end of compression and expansion process

$\gamma = \frac{C_p}{C_v} = 1.4$
 $T_1 = 300 \text{ K}$
 $T_3 = 1500 \text{ K}$
 $T_2 = ?$
 $T_4 = ?$

$p_2 = p_3 = 8 \text{ bar}; p_1 = p_4 = 1 \text{ bar}$

$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$
 $T_2 = 300 \left(\frac{8}{1}\right)^{0.4}$
 $T_2 = 543.4 \text{ K}$

$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{8}\right)^{0.4}$
 $\Rightarrow T_4 = 1500 \times \left(\frac{1}{8}\right)^{0.4/1.4} = 828 \text{ K}$

Figure 1.

Solution of the problem in Fig. 1:

The Brayton cycle is shown on a T-s diagram in Fig. 1.

$$p_2 = p_3 = 8 \text{ bar}, p_1 = p_4 = 1 \text{ bar}, T_1 = 300 \text{ K}, T_3 = 1500 \text{ K}, \gamma = 1.4$$

We have, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ and $\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$

Hence, $T_2 = 300 \left(\frac{8}{1}\right)^{0.4} = 543.4 \text{ K} = \text{temperature at the end of the compression process}$

$$T_4 = 1500 \left(\frac{1}{8}\right)^{0.4} = 828 \text{ K} = \text{temperature at the end of the expansion process.}$$

We can also find turbine work output (w_t), compressor work input (w_c), amount of heat added (q_{in}), amount of heat rejected (q_{out}) and efficiency (η).

$$w_t = h_3 - h_4 = C_p(T_3 - T_4) = 1223 \frac{\text{kJ}}{\text{kg}}$$

$$w_c = h_1 - h_2 = C_p(T_1 - T_2) = -244.5 \frac{kJ}{kg}$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = 960 \frac{kJ}{kg}$$

$$q_{out} = h_1 - h_4 = C_p(T_1 - T_4) = -530 \frac{kJ}{kg}$$

$$\eta = 1 - \frac{T_1}{T_2} = 44.8 \%$$

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$$\begin{aligned} w_T &= \\ w_c &= \\ q_{in} &= \\ q_{out} &= \\ \eta &= \end{aligned}$$



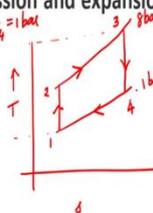
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An ideal Brayton cycle, operating between the pressure limits of 1 bar and 8 bar, has minimum and maximum temperature of 300 K and 1500 K. The ratio of specific heats of the working fluid is 1.4. Find out the final temperatures in at the end of compression and expansion process

$$\begin{aligned} \gamma &= \frac{C_p}{C_v} = 1.4 \\ T_1 &= 300 \text{ K} \\ T_3 &= 1500 \text{ K} \\ T_2 &=? \\ T_4 &=? \end{aligned}$$

$$\begin{aligned} p_2 = p_3 = 8 \text{ bar}; \quad p_1 = p_4 = 1 \text{ bar} \\ \frac{T_2}{T_1} &= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \\ T_2 &= 300 \left(\frac{8}{1}\right)^{0.4} \\ T_2 &= 543.4 \text{ K} \end{aligned}$$



$$\begin{aligned} \frac{T_4}{T_3} &= \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{8}\right)^{0.4} \\ \Rightarrow T_4 &= 1500 \times \left(\frac{1}{8}\right)^{0.4/1.4} = 828 \text{ K} \end{aligned}$$



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$$\begin{aligned}
 w_T &= h_3 - h_4 = C_p(T_3 - T_4) = 1223 \text{ kJ/kg} \\
 w_c &= h_1 - h_2 = C_p(T_1 - T_2) = -244.5 \text{ kJ/kg} \\
 q_{in} &= h_3 - h_2 = C_p(T_3 - T_2) = 960 \text{ kJ/kg} \\
 q_{out} &= h_1 - h_4 = C_p(T_1 - T_4) = -530 \text{ kJ/kg} \\
 \eta &= \frac{w_{net}}{q_{in}} = \frac{\Sigma W}{q_{in}} = \frac{w_T - |w_c|}{q_{in}} = \frac{\Sigma W}{q_{in}} = \frac{q_{in} + q_{out}}{q_{in}} \\
 &= \frac{q_{in} - |q_{out}|}{q_{in}} = 44.8\% \quad C_p = 1004.5 \frac{\text{J}}{\text{kg K}} \\
 T_1 &= 300 \text{ K} \\
 T_2 &= 543.4 \text{ K} \\
 T_3 &= 1500 \text{ K} \\
 T_4 &= 828 \text{ K} \\
 \eta &= 1 - \frac{T_1}{T_2} = 44.8\%
 \end{aligned}$$



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An ideal Brayton cycle, operating between the pressure limits of 1 bar and 8 bar, has minimum and maximum temperature of 300 K and 1500 K. The ratio of specific heats of the working fluid is 1.4. Find out the final temperatures in at the end of compression and expansion process

$p_2 = p_3 = 8 \text{ bar}; p_1 = p_4 = 1 \text{ bar}$
 $\gamma = \frac{C_p}{C_v} = 1.4$
 $T_1 = 300 \text{ K}$
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$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$
 $T_2 = 300 \left(\frac{8}{1}\right)^{\frac{0.4}{1.4}}$
 $T_2 = 543.4 \text{ K}$

$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{8}\right)^{\frac{0.4}{1.4}}$
 $\Rightarrow T_4 = 1500 \times \left(\frac{1}{8}\right)^{\frac{0.4}{1.4}} = 828 \text{ K}$

