

**Thermodynamics**  
**Exergy – Part 3**  
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## Irreversibility

$$\dot{I} = \dot{W}^{rev} - \dot{W}_{cv,act} = \dot{Q}^{rev} = T_0 \dot{S}_{gen,act}$$



We studied the extent of irreversibility before. For a control volume, irreversibility is the difference between the work transfer rates (power) for a reversible process and an irreversible process. Mathematically,  $\dot{I} = \dot{W}_{rev} - \dot{W}_{cv,act}$ . As we saw in the last lecture,  $\dot{I} = \dot{W}_{rev} - \dot{W}_{cv,act} = \dot{Q}_{rev} = T_0 \dot{S}_{gen,act}$ .  $\dot{I}$  is the loss in the work output because of the process being irreversible.

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## Unsteady control mass process

No flow: equation reduces to

$$\Rightarrow \dot{W}^{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) \dot{Q}_j - \left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right)$$

Integrating from state 1 to state 2, we get

$${}_1W_2^{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) Q_{2j} - (E_2 - E_1) + T_o(S_2 - S_1)$$



As we saw in the last lecture, for a control volume, we have,

$$\dot{W}_{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{v_i^2}{2} + gZ_i - T_o s_i\right) - \sum \dot{m}_e \left(h_e + \frac{v_e^2}{2} + gZ_e - T_o s_e\right) - \left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right)$$

For a control mass, there is no flow of mass in or out. Hence,

$$\dot{W}_{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) \dot{Q}_j - \left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right)$$

For the control mass undergoing a process from state 1 to 2, integrating the above equation from state 1 to 2 gives,

$${}_1W_2^{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) Q_{2j} - (E_2 - E_1) + T_o(S_2 - S_1).$$

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## Dead state

- Equilibrium
  - Mechanical
  - Thermal
  - Chemical
  - No KE for system, no KE at exit for CV
  - No PE for system, no PE at exit for CV



As discussed before, if a system is in equilibrium with the reference surroundings, it has no potential to give work as output. For extracting the maximum work from a system or a control volume which are not in equilibrium with the surroundings, the system or the control volume needs to reach a dead state. When a system or a control volume achieves a dead state, they are in mechanical, thermal and chemical equilibrium with the surroundings. At a dead state, a system is at rest. It doesn't have kinetic energy. For a control volume at a dead state, the substance flowing through the control volume does not have kinetic energy at the exit of the control volume. It is not practically possible because, then, the substance will not move out of the control volume. However, at a dead state, the kinetic energy at the exit of the control volume should be as minimum as possible because we want to extract the maximum work from the control volume. At a dead state, a system has 0 potential energy. For a control volume at a dead state, the potential energy at the exit of the control volume is 0. The exit of the control volume should be on the ground (local ground level), ideally. We get the maximum work from the system or the control volume when they reach the dead state at the end of the process.

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## Flow exergy

- CV with single inlet, no heat transfer, exit = dead state (vel, entropy, enthalpy, height):

$$w^{rev} = \frac{\dot{W}^{rev}}{\dot{m}} = \sum \left(1 - \frac{T_o}{T_j}\right) q_j + \left(h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i\right) - \left(h_e + \frac{V_e^2}{2} + gZ_e - T_o s_e\right)$$

$$\psi = \left(h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i\right) - \left(h_o + \frac{0^2}{2} + gZ_o - T_o s_o\right)$$

We can use differences in flow exergy: reversible work from a single inlet-outlet steady state flow equals the decrease in exergy between inlet and outlet

$$w^{rev} = \psi = \psi_i - \psi_e$$



Let's look at flow exergy.

Consider a control volume with a single inlet and outlet. There is no heat transfer. The exit of the control volume is at dead state. We have the expression for the maximum possible work from a control volume,

$$w_{rev} = \frac{\dot{W}_{rev}}{\dot{m}} = \sum \left(1 - \frac{T_o}{T_j}\right) q_j + \left(h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i\right) - \left(h_e + \frac{V_e^2}{2} + gZ_e - T_o s_e\right)$$

Since there is no heat transfer and the exit is at a dead state,

$\varphi = w_{rev} = \left(h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i\right) - \left(h_o + \frac{0^2}{2} + gZ_o - T_o s_o\right)$ , where  $h_o$  is the reference enthalpy,  $Z_o$  is the local ground level,  $s_o$  is the local reference entropy (entropy of the local surroundings). This expression represents the flow exergy for a control volume with single inlet and outlet and having no heat transfer. The expression  $h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i$  can be termed as exergy at the inlet,  $\varphi_i$ , of a control volume, whereas the expression  $h_e + \frac{V_e^2}{2} + gZ_e - T_o s_e$  can be termed as the exergy at the outlet,  $\varphi_e$ , of the control volume. If the exit is at a dead state then  $\varphi_e = \varphi_o = 0$ . Hence, we can write  $\varphi = w_{rev} = \varphi_i - \varphi_e$  if the exit is not at a dead state. Hence, reversible work from a single inlet-outlet steady state flow device with no heat transfer equals the decrease in exergy between its inlet and outlet.

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## Exergy for system

- Similarly, for a control mass, starting with the CV equation, and considering that the volume can change to do work against the surroundings  $\dot{W}_{surr} = p_o \dot{V}$

$$\dot{W}_{avail}^{max} = \dot{W}_{storage}^{rev} - \dot{W}_{surr} \quad \Rightarrow \dot{W}_{avail}^{max} = -\left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right) - p_o \dot{V}$$

Integrating,

$$\Phi = -(E_o - E) - T_o(S_o - S) + p_o(V_o - V)$$

$$\Rightarrow \Phi = (E - T_o S + p_o V) - (E_o - T_o S_o + p_o V_o)$$

We can use differences in exergy and also differences in specific exergy to get work between two states

$$\phi = \Phi/m = (e - T_o s + p_o v) - (e_o - T_o s_o + p_o v_o)$$



We know the expression for the maximum available rate of work from a control volume,

$$\dot{W}_{rev} = \sum \left(1 - \frac{T_o}{T_j}\right) \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i - T_o s_i\right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e - T_o s_e\right) - \left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right).$$

The last term,  $\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}$ , represents storage terms for a control volume. The reversible work from a storage effect due to a change of state in a control volume can also be used to find an exergy. The expression we obtain can also be treated as the exergy of a system/control mass.

Here, the volume may change, and work is done against the surroundings. This work is represented as  $\dot{W}_{surr} = p_o \dot{V}$ , where  $p_o$  is the pressure of the surroundings and  $\dot{V}$  is the rate of change of volume. The maximum possible rate of work from the storage terms is,

$$\dot{W}_{avail}^{max} = \dot{W}_{storage}^{rev} - \dot{W}_{surr} = -\left(\frac{dE_{cv}}{dt} - T_o \frac{dS_{cv}}{dt}\right) - p_o \dot{V}$$

Integrating between the initial state to a dead state gives exergy (maximum possible work) as,

$$\phi = -(E_o - E) - T_o(S_o - S) + p_o(V_o - V) \rightarrow \phi = (E - T_o S + p_o V) - (E_o - T_o S_o + p_o V_o)$$

Specific exergy is represented as,

$$\phi = \frac{\varphi}{m} = (e - T_0s + p_0v) - (e_0 - T_0s_0 + p_0v_0)$$

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## Note

- Exergy: for system + environment together
- Cannot be negative
- Not conserved
- Maximum work obtainable or minimum work input



Few important points regarding exergy are mentioned below:

1. Exergy is defined for a system and environment together. The same system in different environments can have different exergy.
2. Exergy cannot be negative.
3. Exergy is not conserved (as the expression for exergy involves entropy terms, and entropy is not conserved for irreversible processes)
4. Exergy is the maximum work obtainable from a system or a control volume when they achieve a dead state from their current state. It is also the minimum work required to take the system or control volume from a dead state to their original state.

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- Determine the specific exergy of water vapor at 170 °C (with a quality of 0.7), having a velocity of 50 m/s at a height of 10 m, relative to an exergy reference environment at 25 °C and 1 bar.  $g = 9.8 \text{ m/s}^2$ . Properties of liquid water are given below:

Temperature (°C)	Pressure (MPa)	Volume (m <sup>3</sup> /kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg·K)	Phase
15	0.1	0.001001	62.975	63.076	0.2245	liquid
20	0.1	0.001002	83.906	84.006	0.2965	liquid
25	0.1	0.001003	104.82	104.92	0.3672	liquid
30	0.1	0.001004	125.72	125.82	0.4367	liquid

System:

Dead state:  $T_0 = 25 \text{ °C} = 298 \text{ K}$ ,  $p_0 = 1 \text{ bar} = 0.1 \text{ MPa}$ , at rest on ground

State 1: 170 °C,  $x = 0.7$ ,  $V = 50 \text{ m/s}$ ,  $z = 10 \text{ m}$

Substance: steam



Figure 1.

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- State 1 is saturated: from steam tables,

170 0.7922 0.001114 0.2426 718.2 2575.7 719.08 2048.8 2767.9 2.0417 4.6233 6.6650

$$\phi = \Phi/m = (e - T_0 s + p_0 v) - (e_0 - T_0 s_0 + p_0 v_0)$$

$$e_1 = u + ke + pe = (718.2 + 0.7 * (2575.7 - 718.2)) * 1000 + 0.5 * 50^2 + 9.81 * 10 = \text{_____ J/kg} = \text{m}^2/\text{s}^2$$

$$s_1 = 2.0417 + 0.7 * 4.6233 =$$

$$v_1 = 0.001114 + 0.7 * (0.2426 - 0.001114) =$$

- $e_0$  (from liquid tables) =  $104.82 * 1000 + 0 + 0 =$

$$s_0 = 0.3672 * 1000 =$$

$$v_0 = 0.001003 =$$



### Solution of the problem in Fig. 1:

Dead state:  $T_0 = 25 \text{ °C} = 298 \text{ K}$ ,  $p_0 = 1 \text{ bar} = 0.1 \text{ MPa}$ ,  $Z = Z_0$  (local ground level)

State 1:  $T_1 = 170 \text{ °C}$ ,  $x_1 = 0.7$ ,  $V_1 = 50 \frac{\text{m}}{\text{s}}$ ,  $Z_1 = 10 \text{ m}$

Substance: steam

The mass is fixed.

$$\text{Specific exergy, } \phi = \frac{\varphi}{m} = (e_1 - T_0 s_1 + p_0 v_1) - (e_0 - T_0 s_0 + p_0 v_0) \dots (1)$$

$$\text{Also, } e_1 = u_1 + k e_1 + p e_1.$$

Hence, we need to find  $u_1$ ,  $s_1$  and  $v_1$  at state 1 from steam tables.

$$u_1 = [u_f + x_1(u_g - u_f)]_{170^\circ\text{C}} \times 1000 = 2018.45 \times 10^3 \frac{J}{kg}$$

$$\text{Similarly } s_1 = 2.0417 + 0.7(4.6233) = 5.278 \times 1000 \frac{J}{kg \cdot K}$$

$$\text{Similarly, } v_1 = 0.17 \frac{m^3}{kg}$$

$$\text{Now, } e_1 = u_1 + k e_1 + p e_1 = 2018.45 \times 10^3 + 0.5 \times 50^2 + 9.81 \times 10 = 2019.8 \times 10^3 \frac{J}{kg}$$

Similarly, we need  $e_0, u_0, s_0, v_0$ . At a pressure of 1 bar, the saturation temperature  $T_{\text{sat}}=99.6^\circ\text{C}$ .  $T_0 < T_{\text{sat}}$ . Hence, the steam is in liquid state at the reference conditions, and its properties are given in a table in Fig. 1.

$$e_0 = u_0 + k e_0 + p e_0 = 104.82 \times 1000 + 0 + 0 = 104.82 \times 10^3 \frac{J}{kg}$$

$$s_0 = 0.3672 \times 1000 = 367.2 \frac{J}{kg \cdot K}$$

$$v_0 = 0.001003 \frac{m^3}{kg}$$

Equation (1) implies,

$$\phi = \frac{\varphi}{m} = 463956 - (-4505.3) = 468461.3 \frac{J}{kg}$$

Students are encouraged to look at contributions from different terms towards exergy.