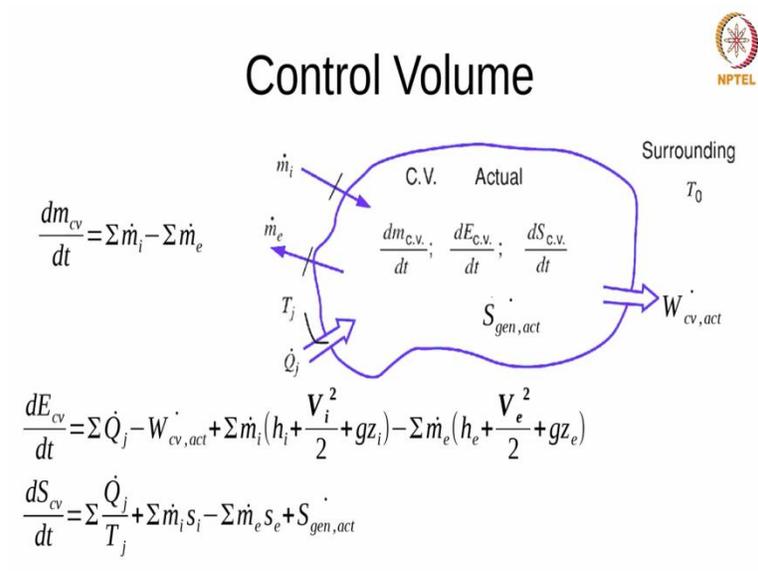(Refer Slide Time 00:15)

Figure 1.

Figure 1 shows an arbitrarily shaped control volume. The process happening inside the control volume is not ideal (which is also true in practice). Let's call this control volume as the actual control volume. The rate equation of mass conservation for the control volume is $\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$ (cv represents control volume). The rate equation of the first law for a control volume is $\frac{dE_{cv}}{dt} = \sum \dot{Q}_j - \dot{W}_{cv,act} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right) \dots (1)$, where $\dot{W}_{cv,act}$ represents the actual work interaction for the non-ideal process. There could be more than one heat sources. Also, there could be different amount of heat transfers through different parts of the boundary at different temperatures. Hence, there is a summation symbol before $\dot{Q}_j$. For the rate equation of entropy, we also need to consider the temperatures of different parts of the boundary. The rate equation of entropy for the control volume is $\frac{dS_{cv}}{dt} = \sum \frac{\dot{Q}_j}{T_j} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen,act} \dots (2)$, where $\dot{S}_{gen,act}$ is the entropy generation rate for the non-ideal process. $\frac{\dot{Q}_j}{T_j}$ can be

positive or negative depending on the direction of heat transfer (i.e., into or out of the control volume). The mass of the substance entering the control volume brings entropy into the control volume, whereas the mass leaving the control volume takes entropy out of the control volume. The surroundings is at temperature T_0.
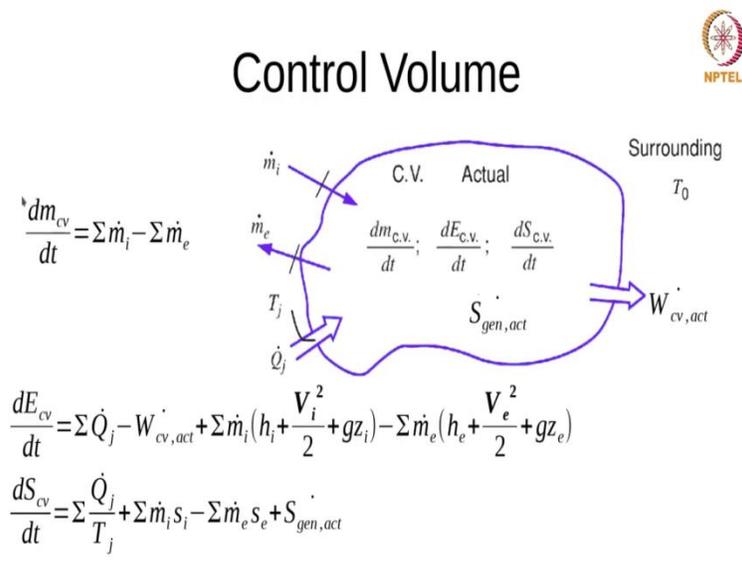
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- Compare with similar ideal CV with same states
  - Mass conservation is same
  - dE/dt is same
  - dS/dt is same: but $\dot{S}_{gen,act}$ is 0.
  - There must be a reversible addition of entropy so that dS/dt is same
    - Heat must have been added reversibly
    - Let this be $\dot{Q}_o^{rev}$ from the ambient at T_o
      - This needs to be accounted for in Energy equation also.
      - Work in energy equation will also be different

Now, consider a similar control volume as shown in Fig. 1. However, the process in this control volume is reversible (ideal process). That is the only difference between the control volume of Fig. 1 (actual control volume) and this ideal control volume. Let's compare conservation equations for this control volume or the ideal control volume with that of Fig. 1 (actual control volume).

The mass conservation equation is the same for both the control volumes. The terms on the left hand side of the energy equation and the entropy equation, i.e., dE/dt and dS/dt are also the same for both the control volumes as the states achieved during the processes in both the control volumes are exactly the same. Time taken for the processes is also the same. The only difference comes through the term $\dot{S}_{gen}$ in the entropy equation. For the actual control volume of Fig. 1, $\dot{S}_{gen,act} > 0$ as the process is irreversible. However, for the ideal control volume $\dot{S}_{gen} = 0$ as the process is reversible.

Control Volume

$$\frac{dm_{cv}}{dt}=\Sigma\dot{m}_i-\Sigma\dot{m}_e$$

$$\frac{dE_{cv}}{dt}=\Sigma\dot{Q}_j-\dot{W}_{cv,act}+\Sigma\dot{m}_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)-\Sigma\dot{m}_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)$$

$$\frac{dS_{cv}}{dt}=\Sigma\frac{\dot{Q}_j}{T_j}+\Sigma\dot{m}_is_i-\Sigma\dot{m}_es_e+\dot{S}_{gen,act}$$

Hence, though the left hand side of the entropy equation (dS/dt) is the same for the actual control volume and the ideal control volume, the right hand sides are different for them because of $\dot{S}_{gen}$. $\dot{S}_{gen}=0$ for the ideal control volume, whereas $\dot{S}_{gen}>0$ for the actual control volume. Hence, if the left hand sides are the same for both the control volume, we need to compensate for $\dot{S}_{gen}=0$ on the right hand side of the entropy equation for the ideal control volume, i.e., we need add something on the right hand side which will add entropy to the control volume. As the processes are reversible in the ideal control volume, we cannot have entropy generation because of irreversibilities. Another way of adding entropy is to add heat. If we are going to add heat, it also needs to be reflected in the energy equation.

We need to add heat reversibly to increase entropy of the ideal control volume as the process in the ideal control volume is reversible. This heat addition can be from the ambient which can be assumed as a constant temperature reservoir. Let's represent the heat added as Q_0^{rev}. This heat transfer happens at temperature T_0 of the ambient. Because of this reversible heat addition, the work term in the energy equation for the ideal control volume changes.

Figure 2.

So, the rate of entropy transfer into the ideal control volume because of reversible heat transfer Q_0^{rev} is $\frac{Q_0^{\dot{r}ev}}{T_0}$ which equals $\dot{S}_{gen,act}$ of the actual control volume (Fig. 2). Hence, $Q_0^{\dot{r}ev} = T_0 \dot{S}_{gen,act}$. After inserting the term of $Q_0^{\dot{r}ev}$ in the energy equation, we get, for the ideal control volume,

$$\frac{dE_{icv}}{dt} = \sum \dot{Q}_j + Q_0^{\dot{r}ev} - \dot{W}_{rev} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right)$$ (icv is ideal control volume. Also, $\frac{dE_{icv}}{dt} = \frac{dE_{cv}}{dt}$).

Comparing this equation with the energy equation for the actual control volume (Fig. 1), we get $- \dot{W}_{cv,act} = Q_0^{\dot{r}ev} - \dot{W}_{rev}$. Hence, $\dot{W}_{rev} = Q_0^{\dot{r}ev} + \dot{W}_{cv,act}$. Hence, the work transfer rate (power) in the ideal control volume is larger than that in the actual control volume by $Q_0^{\dot{r}ev}$. An interesting thing to note here is that the ideal control volume has heat transfer from ambient even if the actual control volume undergoes an adiabatic process.

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Figure 3.

Now, $\dot{W}_{rev} = \dot{W}_{cv,act} + Q_0^{\dot{r}ev} = -\frac{dE_{cv}}{dt} + \sum \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right) + Q_0^{\dot{r}ev} = -\frac{dE_{cv}}{dt} + \sum \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right) +$

$T_0 \dot{S}_{gen,act} = -\frac{dE_{cv}}{dt} + \sum \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right) + T_0 \left(\frac{dS_{cv}}{dt} - \right.$

$\left. \sum \frac{\dot{Q}_j}{T_j} - \sum \dot{m}_i s_i + \sum \dot{m}_e s_e \right)$ ....(taking $\frac{dE_{cv}}{dt}$ from equation 1 and and $\dot{S}_{gen,act}$ from equation 2)

We can rewrite it as,

$\dot{W}_{rev} = \sum \left(1 - \frac{T_0}{T_j} \right) \dot{Q}_j + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i - T_0 s_i \right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e - T_0 s_e \right) - \left(\frac{dE_{cv}}{dt} - T_0 \frac{dS_{cv}}{dt} \right).$

For a steady flow device having single inlet and outlet, we can divide all the terms by $\dot{m}$ ($\dot{m}_i = \dot{m}_e = \dot{m}$). Hence, we get,

$w_{rev} = \frac{\dot{W}_{rev}}{\dot{m}} = \sum \left(1 - \frac{T_0}{T_j} \right) q_j + \left(h_i + \frac{V_i^2}{2} + gZ_i - T_0 s_i \right) - \left(h_e + \frac{V_e^2}{2} + gZ_e - T_0 s_e \right)$

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$$\Rightarrow \dot{W}^{rev} = \Sigma\left(1 - \frac{T_o}{T_j}\right)\dot{Q}_j + \Sigma\dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i - T_o s_i\right) - \Sigma\dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e - T_o s_e\right)$$

$$-\left(\frac{dE_{cv}}{dt} - T_o\frac{dS_{cv}}{dt}\right)$$

For a single inlet-outlet, steady flow device, we can divide all terms by m' as usual, to get

$$w^{rev} = \frac{\dot{W}^{rev}}{\dot{m}} = \Sigma\left(1 - \frac{T_o}{T_j}\right)q_j + \left(h_i + \frac{V_i^2}{2} + gz_i - T_o s_i\right) - \left(h_e + \frac{V_e^2}{2} + gz_e - T_o s_e\right)$$