

Thermodynamics
Professor Anand T N C
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Lecture 80: Entropy Part - 7

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Reversible adiabatic process for an
ideal gas



$$Tds = dh - vdp = du + pdv$$

For a reversible adiabatic process

$$ds = dh/T - v/Tdp = du/T + p/Tdv = 0$$

For an ideal gas,

$$du = C_v dT$$

$$\Rightarrow 0 = C_v dT/T + p/Tdv$$

For an ideal gas, $pv = RT \Rightarrow p/T = R/v$

$$\Rightarrow 0 = C_v dT/T + R/vdv$$

For an ideal gas, $C_v = R/(\gamma - 1)$

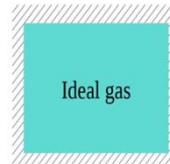


Figure 1.

Let's look at a reversible adiabatic process for an ideal gas.

Figure 1 shows a deformable and insulated chamber containing ideal gas. The volume of the chamber can change. We have the Gibb's relations, $Tds = dh - vdp = du + pdv \rightarrow ds = \frac{dh}{T} - \frac{vdp}{T} = \frac{du}{T} + \frac{pdv}{T}$. For a reversible adiabatic process, $ds=0$. Hence, $\frac{dh}{T} - \frac{vdp}{T} = \frac{du}{T} + \frac{pdv}{T} = 0$. For an ideal gas, $du = C_v dT$ and $pv = RT$. Hence, $\frac{du}{T} + \frac{pdv}{T} = \frac{C_v dT}{T} + R \frac{dv}{v} = 0$. For an ideal gas, $C_v = \frac{R}{\gamma - 1}$.

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$$\Rightarrow 0 = R/(\gamma - 1) \times dT/T + R/v dv$$

$$\Rightarrow 0 = dT/T + (\gamma - 1) dv/v$$

Integrating,

$$\ln(T) + (\gamma - 1) \ln(v) = c$$

$$Tv^{\gamma-1} = c$$

Since $pv = RT$,

$$Tv^{\gamma-1} = c \Rightarrow pv/R * v^{\gamma-1} = c \Rightarrow 1$$

$$\Rightarrow pv^\gamma = k$$

Hence, $\frac{c_v dT}{T} + R \frac{dv}{v} = \frac{R}{\gamma-1} \frac{dT}{T} + R \frac{dv}{v} = 0 \rightarrow \frac{dT}{T} + (\gamma - 1) \frac{dv}{v} = 0$. Integrating, $\ln(T) + (\gamma - 1) \ln(v) = c$, where c is a constant. Using the properties of logarithms, we get, $Tv^{\gamma-1} = c$. We know that for an ideal gas, $T = \frac{pv}{R}$. Hence, $Tv^{\gamma-1} = \frac{pv}{R} v^{\gamma-1} = \frac{pv^\gamma}{R} = c \rightarrow pv^\gamma = c_1$, where $c_1 = c \times R$. Hence, for an ideal gas undergoing a reversible adiabatic process, $pv^\gamma = k$, where k is a constant. We have used this relation before. However, we did not look at the formal proof. For using the expression $pv^\gamma = k$, the process also has to be a quasi-static process.

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Rate Equation of Entropy



$$\frac{dS}{dt} = \frac{1}{T} \frac{\delta Q}{dt} + \frac{\delta S_{gen}}{dt}$$

Unlike in energy equation, here, the temperature at which each heat transfer occurs needs to be considered

$$\frac{dS_{em}}{dt} = \sum \frac{1}{T} \dot{Q} + \dot{S}_{gen}$$

Figure 2.

Let's look at the rate equation of entropy. We have $dS = \frac{\delta Q}{T} + \delta S_{gen}$. Dividing each term by small time interval δt , we get, $\frac{dS}{\delta t} = \frac{1}{T} \frac{\delta Q}{\delta t} + \frac{\delta S_{gen}}{\delta t}$. We saw a similar expression when we looked at the rate equation of the first law for a control mass, $\frac{dE}{\delta t} = \frac{\delta Q}{\delta t} - \frac{\delta W}{\delta t}$. Taking limit as $\delta t \rightarrow 0$, we get the familiar form, $\frac{dE}{dt} = \dot{Q} - \dot{W}$.

In the rate equation of entropy, the temperature at which the heat transfer occurs needs to be considered, unlike the first law. The heat transfer is associated with the temperature at which it happens. A control mass can have heat transfer through its boundary, and the boundary may have different temperatures in different parts along its length. Hence, we need to sum up individual $\frac{\delta Q}{T}$ terms for each part. Hence, the rate equation of entropy for a control mass as $\delta t \rightarrow 0$ is $\frac{dS_{cm}}{dt} = \sum \frac{1}{T} \dot{Q} + \dot{S}_{gen}$ (cm represents control mass).

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Example



A heater in a room takes in 1.65 kW of electric power at steady state and transfer out heat. The temperature of the heater is 550 K. Find the rate of total entropy generation.

Rate equation of first law

$$\frac{dE_{cm}}{dt} = \dot{Q} - \dot{W}$$

Rate equation of second law

$$\frac{dS_{cm}}{dt} = \sum \frac{1}{T} \dot{Q} + \dot{S}_{gen}$$

Figure 3.

Let's solve the problem given in Fig. 3.

Consider heater as our system. The rate equation of the first law is $\frac{dE}{dt} = \dot{Q} - \dot{W}$.

The heater is operating at steady state conditions. Hence, $dE/dt=0$. Hence, $\dot{Q} = \dot{W}$. We know, $\dot{W} = -1.65 \text{ kW}$ (work is being done on the system). Hence, $\dot{Q} = -1.65 \text{ kW}$ (the heater is losing heat). The temperature at which the heater is losing heat is 550 K. This temperature stays constant.

The rate equation of second law (or the rate of equation of entropy) is, $\frac{dS}{dt} = \sum \frac{1}{T} \dot{Q} + \dot{S}_{gen}$.
 $\frac{dS}{dt} = 0$ as steady state conditions have been achieved. Hence, $\dot{S}_{gen} = -\frac{1}{550}(-1.65) = 3 \frac{W}{K}$ = rate of total entropy generation.

So, energy is conserved, i.e., electric power is getting converted into heat, completely. However, entropy is generated in this process which is irreversible.

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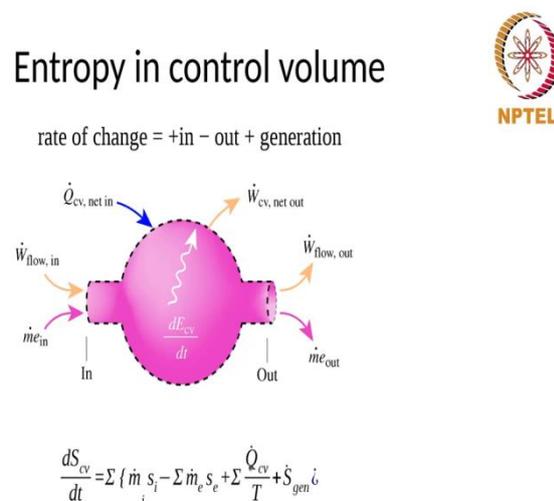


Figure 4.

Let's look at the rate equation of entropy for a control volume.

We have already looked at the rate equation of the first law for a control volume (energy equation). It is basically an equation for energy conservation. However, we know that entropy is not conserved unlike energy. It is generated in irreversible processes. Hence, we need to consider entropy generation while writing the rate equation of entropy for a control volume. The rate equation of entropy for the control volume shown in Fig. 4 is,

$\frac{dS_{cv}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$, where $\sum \dot{m}_i s_i$ represents the rate at which entropy enters the control volume, $\sum \dot{m}_e s_e$ represents the rate at which entropy leaves the control volume, $\sum \frac{\dot{Q}_{cv}}{T}$ represents the entropy transfer because of heat transfer, and \dot{S}_{gen} represents the rate of entropy generation because of irreversibilities in the control volume. There is no term related to work in the equation as the work is an orderly fashion of adding or removing energy. Heat is a disorderly fashion of adding or removing energy which can

change the entropy of a system or control volume as entropy essentially is a measure of disorder. The rate equation of entropy for a control volume is quite different compared to the rate equation of the first law for a control volume. Students are encouraged to compare the two equations and find the differences.

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Entropy change in a steady adiabatic process



$$\frac{dS_{cv}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

Assuming one inlet, one outlet

$$0 = \dot{m}_i s_i - \dot{m}_e s_e + \dot{S}_{gen}$$

$$s_e = s_i + s_{gen}$$

$$s_e \geq s_i$$

For a steady adiabatic process and a control volume with one inlet and one outlet, the rate equation of entropy for a control volume reduces to,

$$0 = \dot{m}_i s_i - \dot{m}_e s_e + \dot{S}_{gen}$$

Since the process is steady, $dm/dt=0$. Hence, $\dot{m}_i = \dot{m}_e = \dot{m}$. Dividing by \dot{m} throughout, $s_e = s_i + s_{gen}$. Hence, $s_e \geq s_i$. The equality holds true for the reversible process, whereas the inequality holds true for an irreversible process. Hence, the entropy at the exit of a control volume is either equal to the entropy at the inlet or greater than it.