Thermodynamics

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Lecture 08
Work - Part III

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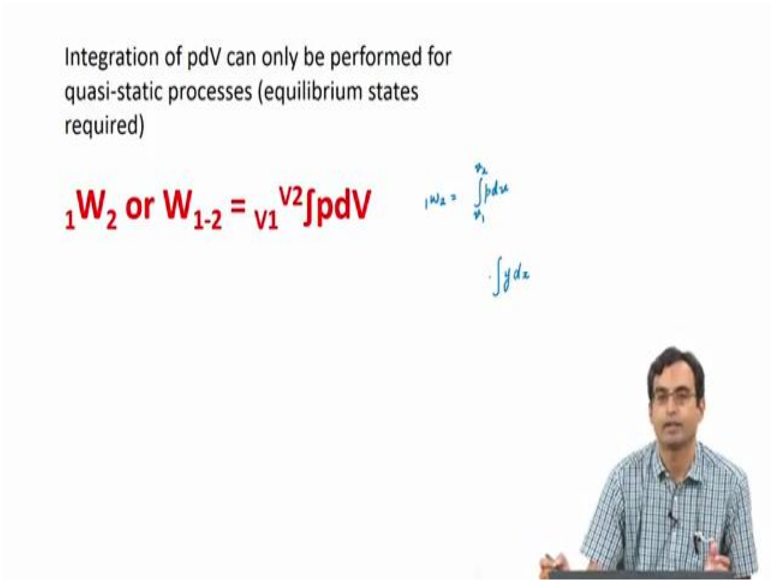

Work done by a system is represented as $_1W_2$ or W_{1-2}, and $W_{1-2} = \int_{V_1}^{V_2} pdV$. Specific work is represented as $_1w_2$ or w_{1-2}, and $w_{1-2} = \int_{v_1}^{v_2} pdv$ (mass of the system is fixed). The integral in the expression for work can be calculated if the functional form of p in terms of V is known or p is constant.

A system may undergo various processes. For different processes, the expressions for work done are different. Let's look at some common processes.

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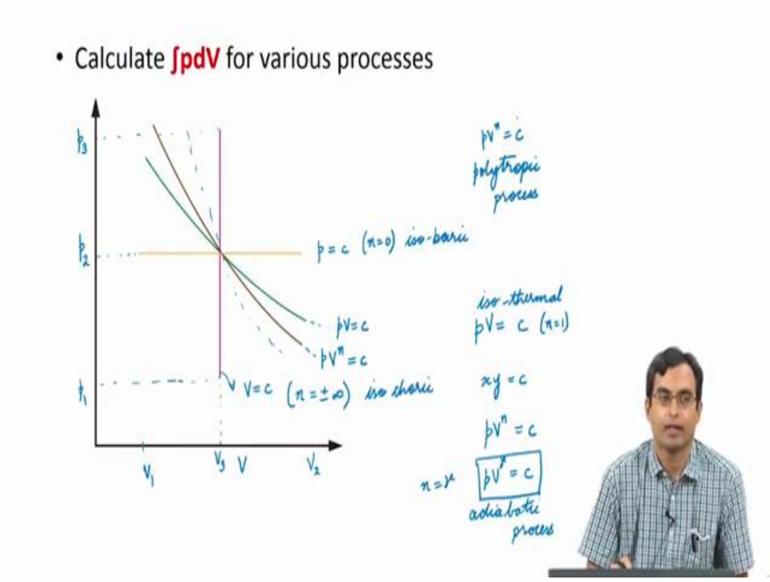

Figure 1

The horizontal line (yellow colored) in Fig. 1 represents a process on a p-V diagram where pressure remains constant for any value of volume. The process can be represented as p = c where c is some constant. All the horizontal lines on such a p-V diagram represent constant pressure processes.

The vertical line in Fig. 1 represents a process where volume remains constant for any value of pressure. All the vertical lines represent constant volume processes.

One of the other processes which is commonly encountered in thermodynamics is an isothermal process. It is represented as $pV = constant$ for an ideal gas. We will look at the concept of an ideal gas a little later. The equation $pV = constant$ on a p-V graph represents a rectangular hyperbola. In Fig. 1, the green curve is rectangular hyperbola, $pV = constant$. It represents an isothermal process for an ideal gas.

The process expressed as $pV^n = constant$ on a p-V diagram is called a polytropic process. We can obtain expressions for different processes by using different values of n. For example, if $n = \gamma$ (γ is the ratio of specific heats), we get an adiabatic process for an ideal gas. Isobaric, isochoric and isothermal processes can also be thought of as special cases of a polytropic process.

n = 1 in $pV^n = constant$ results in the expression for an isothermal process. n = 0 implies p = constant which is an isobaric process. Rewriting $pV^n = constant$ as $V = \frac{constant}{p^{\frac{1}{n}}}$ and setting n = $\pm\infty$, we get isochoric process (V = constant).

Let's see how to calculate work done in each of these processes.

Isobaric process:

- p = constant
- $W_{1-2} = \int_{V_1}^{V_2} pdV = p \int_{V_1}^{V_2} dV = p\,(V_2 - V_1)$

Isochoric process:

- V = constant
- $W_{1-2} = \int_{V_1}^{V_2} pdV = 0$ (dV = 0 as V = constant)

Isothermal process:

- pV = c (c is constant)
- $p_1V_1 = p_2V_2 = p_3V_3 = \ldots = c$
- $W_{1-2} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} c\frac{dV}{V} = c\int_{V_1}^{V_2} \frac{dV}{V} = c\ln(V_2 - V_1) = c\ln\frac{V_2}{V_1} = p_1V_1\ln\frac{V_2}{V_1}$
  (since $p_1V_1 = p_2V_2 = p_3V_3 = \ldots = c$)

Polytropic process:

- $pV^n = c$ (c is constant)
- $p_1V_1^n = p_2V_2^n = p_3V_3^n = \ldots = c$
- $W_{1-2} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} c\frac{dV}{V^n} = c\left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2} = \frac{cV_2^{1-n} - cV_1^{1-n}}{1-n} = \frac{p_2V_2^nV_2^{1-n} - p_1V_1^nV_1^{1-n}}{1-n}$
  $= \frac{p_2V_2 - p_1V_1}{1-n} = \frac{p_1V_1 - p_2V_2}{n-1}$ ....(1)
- Work done in all the other processes discussed in this lecture can be obtained from the expression of the work done in a polytropic process by setting the value of n appropriately. For example, for an isobaric process, n=0 and p=constant. Hence, equation 1 implies $W_{1-2} = p(V_2 - V_1)$.

Adiabatic process:

- $n = \gamma$
- Equation 1 implies $W_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{p_1V_1 - p_2V_2}{\gamma-1}$