

Thermodynamics
Professor Anand T N C
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Lecture 76
Tutorial Problem (1 number)

(Refer Slide Time 00:14)

Figure shows steady state operating data for a well insulated device with air entering at one location and exiting at another with a mass flow rate of 10 kg/s. Neglecting PE changes, determine (a) the direction of flow and (b) the power.



$\dot{m} = 10 \text{ kg/s}$

$\Delta PE = 0$

$\frac{dE}{dt} = \dot{Q} - \dot{W} + \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$

$\dot{W} = 10 \left[1004.5 \times 600 + \frac{1000^2}{2} \right] - 10 \left[1004.5 \times 900 + \frac{5^2}{2} \right]$

$\dot{W} = \pm 1986 \text{ kW}$

$p = 1 \text{ bar}$
 $T = 600 \text{ K}$

$p = 5 \text{ bar}$
 $T = 900 \text{ K}$

$Tds = du + pds = dh - vdp$

$\frac{d\dot{m}}{dt} = \dot{m}_i - \dot{m}_e$

$\dot{m}_i = \dot{m}_e = 10$

air \rightarrow ideal gas

$h = C_p T$

$C_p = \frac{R}{\gamma - 1}$ $R = 287 \text{ J/kgK}$

$\Rightarrow C_p = 1004.5 \text{ J/kgK}$



Figure 1.

$ds = \frac{dh}{T} - \frac{vdp}{T}$ $pv = RT$

$ds = \frac{C_p dT}{T} - \frac{R dp}{p}$

$\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$= 1004.5 \times \ln \frac{900}{600} - 287 \ln \frac{5}{1}$

$= 407.28 - 461.9$

$\Delta s = -54.7 \frac{\text{J}}{\text{kgK}}$ $\Delta S = m \Delta s = -547 \frac{\text{W}}{\text{K}}$

\therefore direction chosen was not correct

Flow must be from B to A

$\dot{W} = -1986 \text{ kW}$



Solution of the problem in Fig. 1:

We are not given the direction of entry and exit of air into the device. Let's assume that the air enters at A and leaves at B.

$$\dot{Q} = 0, \dot{m} = 10 \frac{kg}{s}, \Delta PE = 0$$

The first law for a control volume is,

$$\frac{dE}{dt}_{cv} = \dot{Q} - \dot{W} + \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) - \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right)$$

The device is operating at steady state conditions ($dE/dt = dm/dt = 0$). Hence, $\dot{m}_i = \dot{m}_e = \dot{m}$. $\Delta PE = 0$. Hence,

$$\dot{W} = \dot{m}_i \left(h_i + \frac{V_i^2}{2} \right) - \dot{m}_e \left(h_e + \frac{V_e^2}{2} \right) = \dot{m}_i \left(C_p T_i + \frac{V_i^2}{2} \right) - \dot{m}_e \left(C_p T_e + \frac{V_e^2}{2} \right) \text{ (assuming air as an ideal gas)}$$

$$C_{p,air} = \frac{\gamma R}{\gamma - 1} = 1004.5 \frac{J}{kg \cdot K} \quad (\gamma = 1.4 \text{ assuming air to be a diatomic gas and } R = 287 \text{ J/kg} \cdot \text{K}).$$

Substituting the values,

$$\dot{W} = 10 \left(1004.5 \times 600 + \frac{1000^2}{2} \right) - 10 \left(1004.5 \times 900 + \frac{5^2}{2} \right) = 1986 \text{ kW}$$

$\dot{W} = +1986 \text{ kW}$ if A is the inlet and B is outlet, whereas $\dot{W} = -1986$ if B is the inlet and A is the outlet. We still don't know the inlet and outlet.

To determine the inlet and outlet, we will calculate the entropy change for the air as it enters and leaves the device. Also, the process given in the problem is adiabatic. Hence, if there is going to be entropy change, it is going to be because of irreversibilities. Hence, entropy at the outlet must be larger than the entropy at the inlet. If the entropy change for air between the inlet and outlet comes out to be negative for a choice of inlet and outlet, that choice is then wrong, and the flow happens in the opposite direction.

For an ideal gas, we have, $Tds = dh - vdp \rightarrow ds = \frac{dh}{T} - \frac{v}{T}dp \rightarrow ds = \frac{C_p dT}{T} - \frac{R}{p}dp \rightarrow \Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$.

Hence, $\Delta s = 1004.5 \times \ln\left(\frac{900}{600}\right) - 287 \ln\left(\frac{5}{1}\right) = -54.7 \frac{J}{kg \cdot K}$ (taking A as the inlet and B as the outlet).

Now, $\Delta S = \dot{m}\Delta s = -547 \frac{W}{K}$.

The entropy is decreasing as air enters at A and leaves at B. Hence, our choice of inlet and outlet is wrong. The air enters at B and leaves at A. Hence, the work output $\dot{W} = -1986 kW$.

Work is done on the device.