Thermodynamics

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Lecture 76

Tutorial Problem (1 number)

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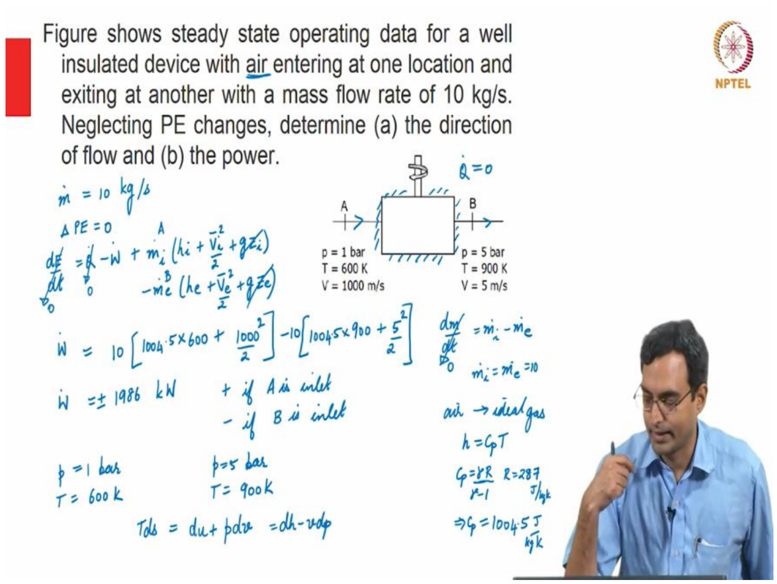
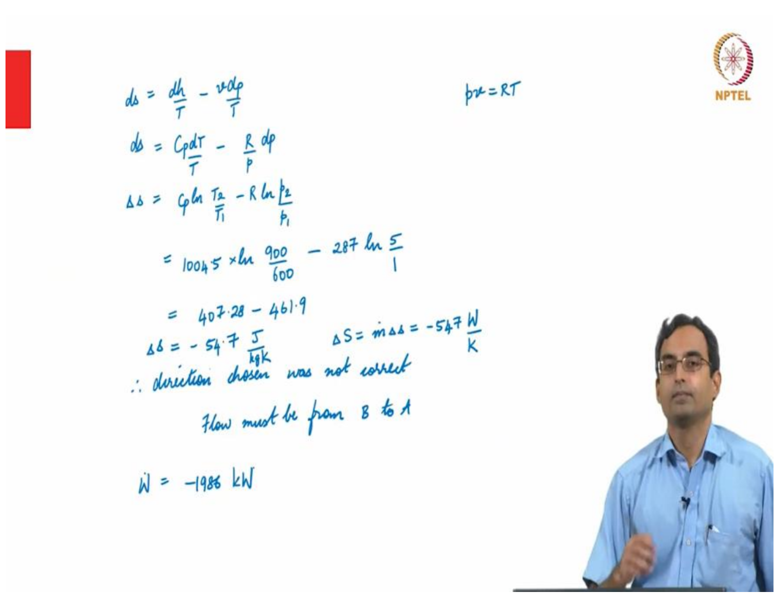


Figure 1.



Solution of the problem in Fig. 1:

We are not given the direction of entry and exit of air into the device. Let's assume that the air enters at A and leaves at B.

$$\dot{Q} = 0, \dot{m} = 10\frac{kg}{s}, \Delta PE = 0$$

The first law for a control volume is,

$$\frac{dE}{dt}_{cv} = \dot{Q} - \dot{W} + \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gZ_i\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gZ_e\right)$$

The device is operating at steady state conditions (dE/dt=dm/dt=0). Hence, $\dot{m}_i = \dot{m}_e = \dot{m}$. $\Delta PE = 0$. Hence,

$\dot{W} = \dot{m}_i\left(h_i + \frac{V_i^2}{2}\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2}\right) = \dot{m}_i\left(C_pT_i + \frac{V_i^2}{2}\right) - \dot{m}_e\left(C_pT_e + \frac{V_e^2}{2}\right)$ (assuming air as an ideal gas)

${C_p}_{air} = \frac{\gamma R}{\gamma - 1} = 1004.5\ \frac{J}{kg\cdot K}$ ($\gamma = 1.4$ assuming air to be a diatomic gas and R=287 J/kg·K).

Substituting the values,

$$\dot{W} = 10\left(1004.5 \times 600 + \frac{1000^2}{2}\right) - 10\left(1004.5 \times 900 + \frac{5^2}{2}\right) = 1986 \text{ kW}$$

$\dot{W} = +1986$ kW if A is the inlet and B is outlet, whereas $\dot{W} = -1986$ if B is the inlet and A is the outlet. We still don't know the inlet and outlet.

To determine the inlet and outlet, we will calculate the entropy change for the air as it enters and leaves the device. Also, the process given in the problem is adiabatic. Hence, if there is going to be entropy change, it is going to be because of irreversibilities. Hence, entropy at the outlet must be larger than the entropy at the inlet. If the entropy change for air between the inlet and outlet comes out to be negative for a choice of inlet and outlet, that choice is then wrong, and the flow happens in the opposite direction.

For an ideal gas, we have, $Tds = dh - vdp \rightarrow ds = \frac{dh}{T} - \frac{v}{T}dp \rightarrow ds = \frac{C_p dT}{T} - \frac{R}{p}dp \rightarrow \Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$.

Hence, $\Delta s = 1004.5 \times \ln\left(\frac{900}{600}\right) - 287 \ln\left(\frac{5}{1}\right) = -54.7 \frac{J}{kg \cdot K}$ (taking A as the inlet and B as the outlet).

Now, $\Delta S = \dot{m}\Delta s = -547 \frac{W}{K}$.

The entropy is decreasing as air enters at A and leaves at B. Hence, our choice of inlet and outlet is wrong. The air enters at B and leaves at A. Hence, the work output $\dot{W} = -1986\ kW$.

Work is done on the device.