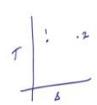


Thermodynamics
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Lecture 74
Entropy Part 4

Let's look at a few other concepts related to entropy.

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$$\begin{aligned}
 dS &\geq 0 & dS &\geq \frac{\delta Q}{T} \\
 dS &= \frac{\delta Q}{T} + \delta S_{gen} \\
 \delta Q_{rev} &= TdS \\
 \rightarrow \delta Q_{irrev} &= TdS - T\delta S_{gen} \\
 \text{First law} \quad \delta Q &= dU + \delta W \\
 \delta Q_{irrev} &= dU + \delta W_{irrev} \\
 \delta Q_{irrev} &= TdS - pdV + \delta W_{irrev} \\
 &= \delta Q_{rev} - \delta W_{rev} + \delta W_{irrev} \\
 \Rightarrow \delta Q_{rev} - \delta Q_{irrev} &= \delta W_{rev} - \delta W_{irrev} = T\delta S_{gen} \\
 \Rightarrow \delta W_{irrev} &= \delta W_{rev} - \underbrace{T\delta S_{gen}}_{\text{lost work}}
 \end{aligned}$$




We know that, for a process, $dS \geq \frac{\delta Q}{T}$. The equality is for a reversible process, whereas the inequality is for an irreversible process. The inequality can be converted to equality as $dS = \frac{\delta Q_{irrev}}{T} + \delta S_{gen}$, where δS_{gen} is the entropy generated in the irreversible process. $\delta S_{gen} = 0$ for a reversible process. For an irreversible process, $\delta Q_{irrev} = TdS - T\delta S_{gen}$. Hence, for a system going from state 1 to 2, heat transferred in an irreversible process is less than the heat transferred in the reversible process by $T\delta S_{gen}$, i.e., $\delta Q_{rev} - \delta Q_{irrev} = T\delta S_{gen}$. Applying the first law to an irreversible process, $\delta Q_{irrev} = dU + \delta W_{irrev}$. Similarly, $\delta Q_{rev} = dU + \delta W_{rev}$. Hence, $\delta Q_{rev} - \delta Q_{irrev} = \delta W_{rev} - \delta W_{irrev} = T\delta S_{gen}$ or $\delta W_{irrev} = \delta W_{rev} - T\delta S_{gen}$, where $T\delta S_{gen}$ is termed as a lost work. Hence, in an irreversible process between the two states of a system, we get less work than the reversible process between the same two states by $T\delta S_{gen}$. Hence, $T\delta S_{gen}$ is called as lost work.

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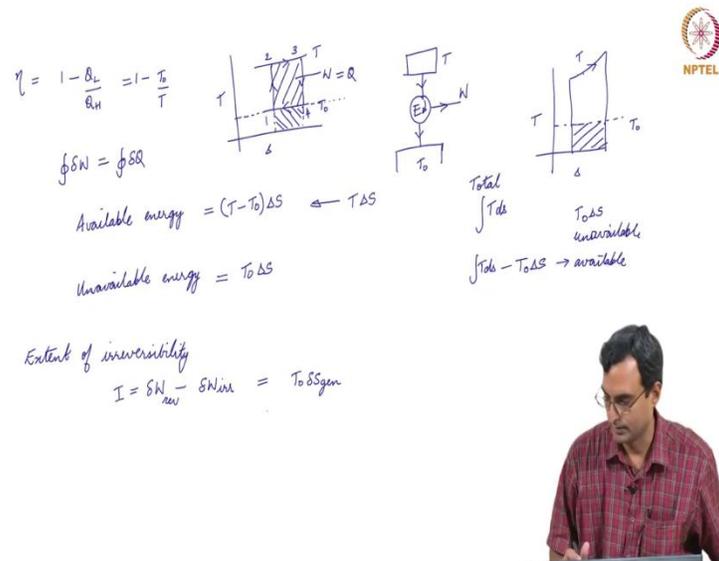


Figure 1.

For a Carnot's engine operating between a temperature reservoir at T (heat source) and T_0 (heat sink) and giving W as work output, the efficiency of the engine is given as $\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_0}{T}$, where Q_L and Q_H are the heat transfers with the temperature reservoirs at T_0 and T , respectively.

A Carnot's cycle is shown on a T-S diagram in Fig. 1. We already know that, on a T-S diagram, the area under a curve represents the heat transfer for that process. The Carnot's cycle involves two heat transfers, Q_L and Q_H . For the Carnot's cycle shown in Fig. 1, $Q_H = T\Delta S_{2-3}$ and $Q_L = T_0\Delta S_{4-1}$. $\Delta S_{2-3} = \Delta S_{4-1}$ (in magnitude). Hence, $Q_H + Q_L = Q_H - |Q_L| = (T - T_0)\Delta S$. According to the first law, $\oint \delta Q = Q_H + Q_L = Q_H - |Q_L| = \oint \delta W$. Hence, $W = (T - T_0)\Delta S$. This is the maximum work output for a reversible heat engine operating between the temperature reservoirs at T and T_0 . For a given T , reducing T_0 increases the work output. Hence, there is always an amount of heat equal to $T_0\Delta S$ which cannot be converted to work. The total energy (heat) available for conversion to work is $T\Delta S$, which is termed as available energy. Out of that, the energy (heat) which cannot be converted into work is $T_0\Delta S$, which is termed as unavailable energy. Usually, T_0 represents the temperature of the ambient. In cold countries, ambient temperature is lower than hotter countries. Hence, the unavailable energy is lower. Here, T and

T_0 are taken as constants. What if we have a constant pressure heat source with varying temperature and heat sink at constant temperature T_0 ?

The variation of T with S is shown in Fig. 1 on a T - S diagram (right top corner). If the variation of T with S is known, we can operate a series of Carnot's engines between the heat source with varying temperature and heat sink at T_0 . For all the engines, heat sink temperature remains at T_0 . However, for every subsequent engine, the temperature of the heat source increases as shown on the T - S diagram in Fig. 1. In this case, the total available energy for the conversion into work is $\int TdS$ and the unavailable energy is T_0dS . Hence, the available energy for conversion into work is $\int TdS - T_0dS$.

We know that we get the maximum work output in a reversible process. However, in reality, the processes are irreversible. We have already seen that $\delta W_{rev} - \delta W_{irrev} = T\delta S_{gen}$. The extent of irreversibility I is defined as $I = \delta W_{rev} - \delta W_{irrev} = T\delta S_{gen}$.

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$\frac{dE}{dt} = \dot{Q} - \dot{W} + \dot{m}_i h_i + \frac{\dot{m}_i V_i^2}{2} + \dot{m}_i g z_i - [\dot{m}_e h_e + \frac{\dot{m}_e V_e^2}{2} + \dot{m}_e g z_e]$
 Steady state
 Adiabatic
 No work
 Isentropic
 $T ds = du + v dp$
 $T dh = dh - v dp$
 $dh = v dp$
 $\int dh = \int v dp \Rightarrow h = \int v dp$
 Incompressible $v = \frac{1}{\rho}$
 $\frac{V_i^2}{2} + g z_i = (h_e h_i) + \frac{V_e^2}{2} + g z_e$
 $\frac{V_i^2}{2} + g z_i = \frac{V_e^2}{2} + g z_e$
 $\left[\frac{p_i}{\rho} + \frac{V_i^2}{2} + g z_i = \frac{p_e}{\rho} + \frac{V_e^2}{2} + g z_e \right]$
 Bernoulli's equation
 $\dot{m}_i = \dot{m}_e = \dot{m}$
 $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e = 0$
 One inlet, one outlet

Figure 3.

Consider a flow device with a single inlet and outlet. The device is operating at steady state conditions ($dE/dt = 0$, $dm/dt=0$). Hence, the mass flow rates at the inlet and the outlet are equal, i.e., $\dot{m}_i = \dot{m}_e = \dot{m}$. The device is adiabatic ($\dot{Q} = 0$) and there is no work transfer ($\dot{W} = 0$). Hence, the first law for a control volume in the case the device considered above reduces to,

$$h_i + \frac{v_i^2}{2} + gZ_i = h_e + \frac{v_e^2}{2} + gZ_e \rightarrow \frac{v_i^2}{2} + gZ_i = h_e - h_i + \frac{v_e^2}{2} + gZ_e \dots (1)$$

We know that $Tds = dh - vdp$. Assuming the process to be isentropic, $dh = vdp$. Integrating, $\int dh = \int vdp$. Assuming the substance incompressible, $\Delta h = \frac{1}{\rho} \Delta p$ ($v = \frac{1}{\rho}$). For the flow device considered above, $\Delta h = \frac{1}{\rho} (p_e - p_i) = h_e - h_i$. Hence, equation (1) implies, $\frac{p_i}{\rho} + \frac{v_i^2}{2} + gZ_i = \frac{p_e}{\rho} + \frac{v_e^2}{2} + gZ_e$. This is also known as Bernoulli's equation. This equation can be used for devices such as nozzles, diffusers, pipes, etc. to get property values. However, keep in mind the assumptions we made to arrive at this equation.