

**Thermodynamics**  
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**Lecture 73**  
**Entropy Part 3**

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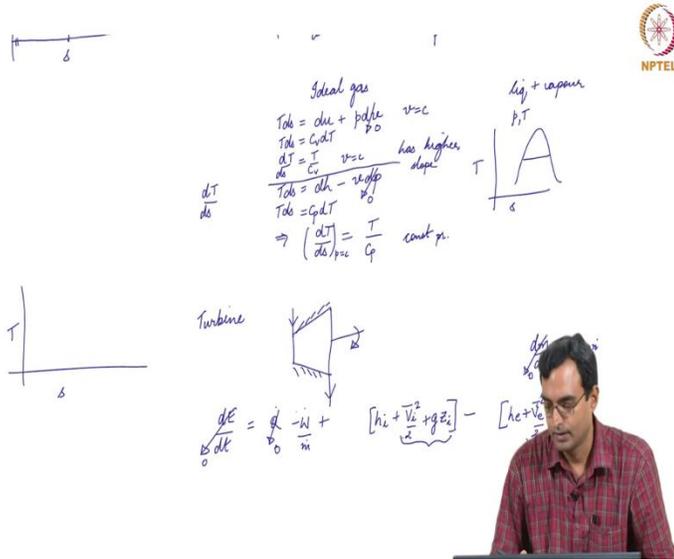


Figure 1.

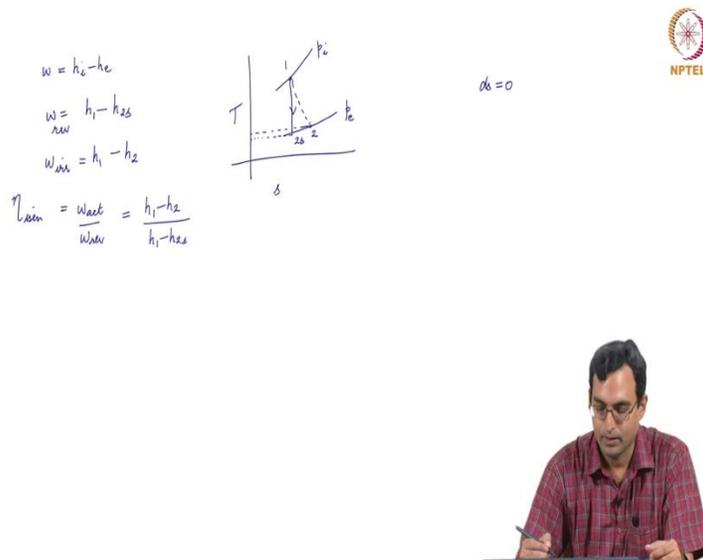


Figure 2.

Let's define isentropic efficiency for different engineering devices.

For a turbine, after simplifications, the first law for a control volume, as discussed in previous lectures, reduces to,  $\dot{W} = \dot{m}(h_i - h_e) \rightarrow W = h_i - h_e$ . In addition to assuming the gas expansion process in a turbine adiabatic, if we assume that the process is also reversible, then the change in entropy ( $dS$ ) for a gas would be zero. The process would be isentropic. The isentropic process (1-2s) for a gas expanding from the initial pressure  $p_i$  to the final pressure  $p_e$  in a turbine is shown by a line parallel to the T axis on a T-S diagram in Fig. 2. However, if the process is not reversible, the entropy would increase during the process. The process takes place between the same two pressures  $p_i$  and  $p_e$ . The temperature at the exit for an irreversible process would be higher than the temperature at the exit for the reversible process. The irreversible process is shown by dotted line joining the states 1 and 2 on a T-S diagram in Fig. 2. For the reversible process,  $W = h_1 - h_{2s}$ , whereas for the irreversible process,  $W = h_1 - h_2$ . Real life processes are irreversible. Hence,  $W = h_1 - h_2$  represents the actual work output of the turbine.  $W = h_1 - h_{2s}$  represents the maximum possible work output from a turbine. Isentropic efficiency for a turbine is defined as  $\eta_{isen,turbine} = \frac{W_{act}(smaller\ quantity)}{W_{rev}(larger\ quantity)} = \frac{h_1 - h_2}{h_1 - h_{2s}}$ . Though we mentioned that for an irreversible process, the entropy increases, we have not yet proved it. Let's look at the proof.

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The figure contains several handwritten mathematical derivations and diagrams:

- Top Left:**  $\oint \frac{\delta Q}{T} \leq 0$
- Top Middle:** A T-S diagram showing a reversible cycle (1-A-2-B-1) with a solid line and an irreversible cycle (1-A-2-C-1) with a dotted line. The area under the dotted line is shaded, indicating entropy increase.
- Top Right:** A T-S diagram showing a cycle with  $dT = T_2 - T_1$  and  $dT = T_1 - T_2 = 0$ .
- Middle Left:**  $\oint \frac{\delta Q}{T} = 0 = \int \frac{\delta Q_A}{T_A} + \frac{\delta Q_B}{T_B} = 0 = \oint dS$  reversible cycle
- Middle Right:**  $\oint dS = \oint \frac{\delta Q}{T} > 0$  irreversible
- Bottom Left:**  $\oint \frac{\delta Q}{T} < 0 \Rightarrow \int \frac{\delta Q_A}{T_A} + \frac{\delta Q_C}{T_C} < 0$  irreversible
- Bottom Middle:**  $dS = 0$   
 $\oint dS > \oint \frac{\delta Q}{T}$   
 $dS > \frac{\delta Q}{T}$  irrev.  
 $dS \geq \frac{\delta Q}{T}$
- Bottom Right:** A video frame of a lecturer with handwritten notes:  $Q=0, m=0$  for an insulated system,  $dS \geq 0$  isolated system.
- Top Right Logo:** NPTEL logo.

Figure 3.

According to Clausius, for a reversible cycle,  $\oint \frac{\delta Q}{T} = 0$ , and for an irreversible cycle,  $\oint \frac{\delta Q}{T} < 0$ . Consider a system undergoing a cyclic process. States 1 and 2 of the system are shown on some property diagram (say p-v diagram) in Fig. 3. Paths A and B between states 1 and 2 are reversible, whereas the path C is irreversible. For the path 1-A-2-B-1,  $\oint \frac{\delta Q}{T} = \oint dS = \int \frac{\delta Q_A}{T_A} + \int \frac{\delta Q_B}{T_B} = 0$ . For the path 1-A-2-C-1,  $\oint \frac{\delta Q}{T} = \int \frac{\delta Q_A}{T_A} + \int \frac{\delta Q_C}{T_C} < 0$ . For a cyclic process,  $\oint dS = 0$  as S is a property. However, if the cycle is irreversible as 1-A-2-C-1,  $\oint \frac{\delta Q}{T} < 0$ . Hence,  $\oint dS > \oint \frac{\delta Q}{T}$ . So, for an irreversible cycle, entropy change over a cycle is not 0. It is greater than 0, i.e., entropy increases in an irreversible cycle. We can also write  $dS > \frac{\delta Q}{T}$  if the process is irreversible. For the reversible process,  $dS = \frac{\delta Q}{T}$ . So, we can write,  $dS \geq \frac{\delta Q}{T}$  in general for a process. Hence, for an irreversible process, the entropy increases.

It can be proved in another way. For the path 1-A-2-C-1,  $\oint \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q_A}{T_A} + \int_2^1 \frac{\delta Q_C}{T_C} < 0$ .  $\int_1^2 \frac{\delta Q_A}{T_A} = \Delta S = S_2 - S_1$  as the path A is reversible. Hence,  $S_2 - S_1 + \int_2^1 \frac{\delta Q_C}{T_C} < 0$ . Multiplying by -1,  $S_1 - S_2 > \int_2^1 \frac{\delta Q_C}{T_C}$ . If the path C were reversible,  $S_1 - S_2 = \int_2^1 dS = \int_2^1 \frac{\delta Q_C}{T_C} > \int_2^1 \frac{\delta Q_C}{T_C}$ . Hence, for the same states 1 and 2, the difference in entropy between those two states for a reversible process equals  $\int_1^2 \frac{\delta Q}{T}$ . However, the difference in entropy between those states for an irreversible process is greater than  $\int_1^2 \frac{\delta Q}{T}$ . It means that the entropy difference between the same two states for an irreversible process is larger than the entropy difference between the same two states for a reversible process. In general, for a process,  $dS \geq \frac{\delta Q}{T}$ .



$1-A = A^{-1}$

$$\oint \frac{\delta Q}{T} < 0 \Rightarrow \int \frac{\delta Q_A}{T_A} + \frac{\delta Q_C}{T_C} < 0$$

irreversible

$$dS = 0$$
$$\oint dS > \oint \frac{\delta Q}{T}$$
$$dS > \frac{\delta Q}{T} \text{ irrev.}$$
$$dS \geq \frac{\delta Q}{T}$$
$$dS = 0$$

For an isolated system  $Q=0$   $m=0$

$$dS \geq 0$$

Isolated system:  $dS \geq 0$

For the universe

$$dS \geq 0$$

Principle of increase of entropy  $\Delta S = S_2 - S_1$



For an isolated system, there is no heat transfer ( $\delta Q = 0$ ). Hence,  $dS \geq 0$ . The universe is an isolated system. Hence, for the universe,  $dS \geq 0$ , i.e., the entropy can remain constant or increase. This is the principle of increase of entropy. If the processes are reversible, then  $dS=0$ . If the processes are irreversible, which is the case in reality,  $dS > 0$ . Here, we see that there is a directionality associated with a process, i.e., a process happens such that the entropy increases or remains constant, but it doesn't decrease. This is unlike other physical laws which cannot determine anything regarding the direction or the feasibility of the process. For example, we can use the first law for calculating the heat transfer for hot coffee getting cooled in a cold environment. We can also calculate the heat transfer if the hot coffee gets hotter in the cold environment spontaneously. However, this process does not happen as it violates the principle of increase of entropy. Hence, the processes always happen in a direction where the entropy remains constant or increase. The principle of increase of entropy is a consequence of the second law.

If we transfer heat into the system reversibly ( $dS = \delta Q/T$ ), its entropy increases. If the process undergone by a system is adiabatic but irreversible, then also, the entropy of the system increases (as  $dS > \delta Q/T$  for an irreversible process). For a closed system (fixed mass), the only way of reducing the entropy is to remove heat. However, for an open system, where there could be a transfer of mass also across its boundaries, the entropy can also be reduced by taking mass out of the system. However, the specific entropy does not change by mass transfer.

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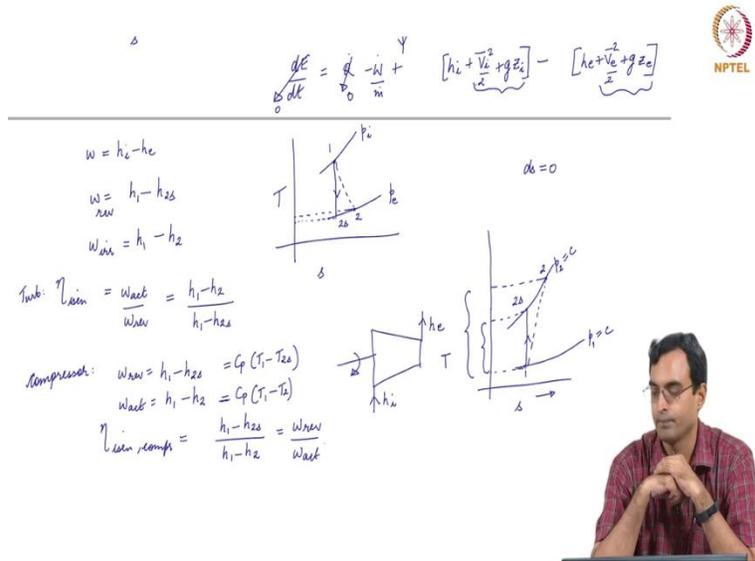


Figure 4.

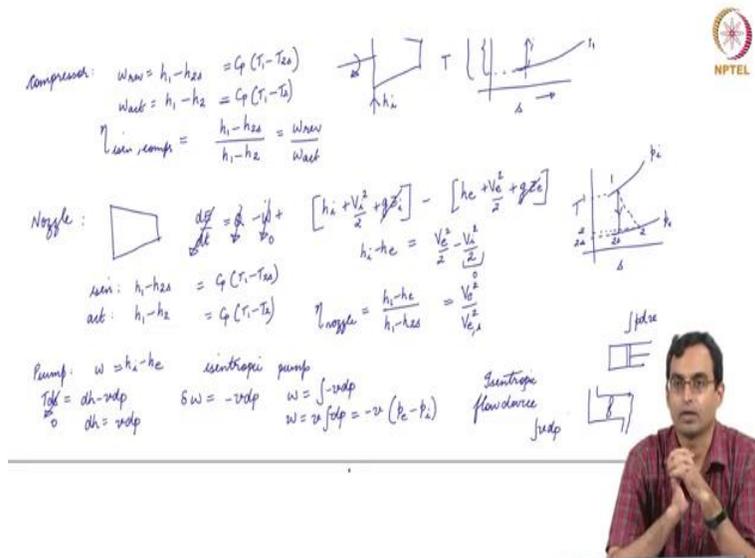


Figure 5

Let's define isentropic efficiencies for engineering devices.

For a compressor, after simplifications, the first law for a control volume gives a similar expression as that for a turbine,  $W = h_i - h_e$ . However, here, work is done on the compressor ( $h_i < h_e$ ). The adiabatic reversible (isentropic) compression process 1-2s is shown by a line parallel to the T axis on a T-S diagram (Fig. 4) between the inlet pressure  $p_1$  and the outlet

pressure  $p_2$ .  $W = h_1 - h_2$  represents the minimum work input that the compressor needs if the process is isentropic. However, as mentioned before, in reality, the processes are irreversible and the entropy increases. Irreversible compression process is shown by a dotted line 1-2 between the same pressures  $p_1$  and  $p_2$  (Fig. 4) on a T-S diagram. The temperature and the entropy at state 2 is larger than the temperature and the entropy at  $2s$ . The isentropic efficiency of a compressor is defined as  $\eta_{isen,compressor} = \frac{W_{rev}}{W_{act}} = \frac{h_1 - h_{2s}}{h_1 - h_2}$ . Hence, for an irreversible compression, we need to give in more work than the reversible compression. For an ideal gas,  $h_1 - h_{2s} = C_p(T_1 - T_{2s})$  and  $h_1 - h_2 = C_p(T_1 - T_2)$ .

Similarly, for a nozzle, we have  $h_i - h_e = \frac{V_e^2}{2} - \frac{V_i^2}{2}$  (see a lecture on the first for a control volume for the derivation of this expression). In a nozzle, the gas expands from some initial pressure  $p_i$  to  $p_e$ . Reversible adiabatic (isentropic) expansion of a gas in a nozzle from pressure  $p_i$  to  $p_e$  is shown by a line 1- $2s$  parallel to T axis on a T-S diagram in Fig. 5. In the irreversible expansion, the entropy increases. The irreversible expansion in a nozzle between pressure  $p_i$  and  $p_e$  is shown by a dotted line joining 1 and 2 on a T-S diagram in Fig. 5. The isentropic efficiency of a nozzle

is defined as  $\eta_{isen,nozzle} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\frac{V_2^2}{2} - \frac{V_1^2}{2}}{\frac{V_{2s}^2}{2} - \frac{V_1^2}{2}}$ . Sometimes, the inlet kinetic energy is very small

compared to that at the exit. Then  $\eta_{isen,nozzle} = \frac{V_2^2}{V_{2s}^2}$ . Again, for an ideal gas,  $h_1 - h_{2s} = C_p(T_1 - T_{2s})$  and  $h_1 - h_2 = C_p(T_1 - T_2)$ .

For a pump, the expression for the isentropic efficiency is the same as the compressor. For a pump compressing a liquid  $W = h_i - h_e$ . This is the work input. We have  $Tds = dh - vdp$ . If the pump works isentropically,  $ds=0$ . Hence,  $dh = vdp$ . As the work input to a pump is the difference between enthalpies at the inlet and outlet,  $dh = \delta W = -vdp$  ( $h_i < h_e$ ). Total work input to the pump can be obtained by integrating,  $W = \int -vdp = -v(p_e - p_i)$ , where  $p_e$  and  $p_i$  are the pressure values at the exit and inlet of the pump. As liquids are considered incompressible, the specific volume  $v$  can be taken out of the integral for calculating work input to the pump. However, we cannot do this for a compressor as the gases are not incompressible.  $v$  changes with pressure. In that case, we need to know the variation of  $v$  with pressure to calculate  $W = \int -vdp$ .

We know that the displacement work for a system like a piston-cylinder arrangement is given by  $W = \int p dv$ . For an isentropic flow device such as a pump,  $W = - \int v dp$ .