Thermodynamics

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Lecture 07

Work - Part II

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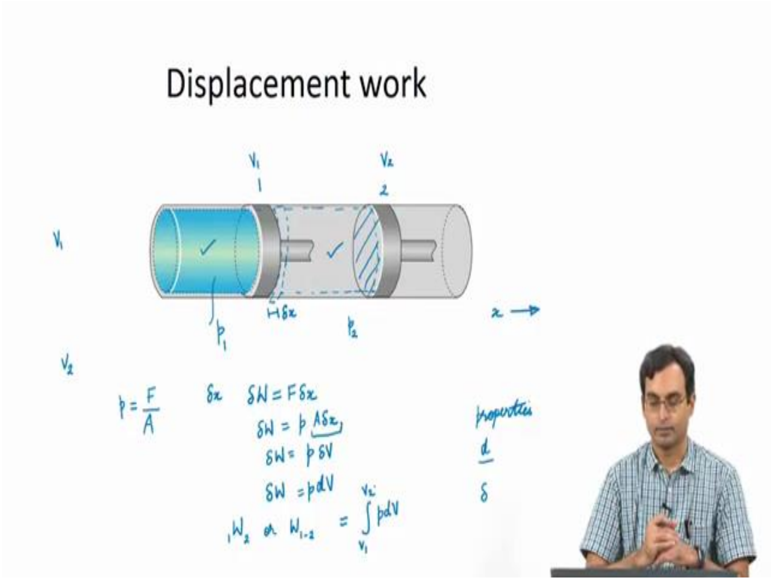

Figure 1

Consider a piston-cylinder arrangement as shown in Fig. 1. At any spatial point, p represents the pressure. There is atmospheric pressure on the right side of the piston.

Initially, the piston is at location 1. The volume on the left hand side of the piston is V_1 and the pressure is p_1. Suppose the contents on the left hand side of the piston-cylinder arrangement (the system) underwent a process during which the piston moved out and reached point 2. Here, the volume is V_2 and the pressure is p_2. The pressure acting on the piston at any given point of time, as usual, is given as the force acting on the piston at that point in time divided by the piston cross-sectional area, A.

Initially, since the system is in equilibrium, the pressure values inside and outside are equal. Let's say pressure inside is p_1. Assume that there is a slight imbalance in the forces on the two sides of the piston and the piston moves a small distance dx in the x-direction (Fig. 1) (we use d to represent exact differentials). There is an expansion of the system. In this case, the work done is Fdx where F is the force acting on the piston and dx is the small displacement. Over the small time interval during which the piston moved by dx, the pressure can be taken as p_1. Now, $\delta W = Fdx = p_1 Adx$ where A represents the cross-sectional area of the piston and δW represents the small amount of work done by the system (we use δ to represent inexact differentials).

The term Adx represents the volume swept by the piston as it moved. It is the volume of the surroundings that was displaced. So, $\delta W = pdV$, where dV represents the volume swept by the piston. dV is infinitesimally small.

The displacement of the piston from location 1 to 2 can be divided into small parts such as dx. As the volume of the system increases from V_1 to V_2, the total work done by the system is equal to the sum of all such δWs which can be obtained by integrating pdV. Hence, $\int_{V_1}^{V_2} pdV$ equals the total work done by the system as it expands from V_1 to V_2.

In reality, the pressure inside the system is changing. The volume of the system is also changing. By considering small time intervals during each of which the piston moves dx, and assuming that the pressure stays constant over that time interval, we can obtain the total work done by the system by adding the work done over such small time intervals. The cost of assuming the pressure to be constant over those small time intervals for the accurate calculation of work can be reduced by choosing even smaller time intervals.

The work done is represented as ${}_1W_2$ or W_{1-2}. So, displacement work, $W_{1-2} = \int_{V_1}^{V_2} pdV$. The system expands quasi-statically from V_1 to V_2.

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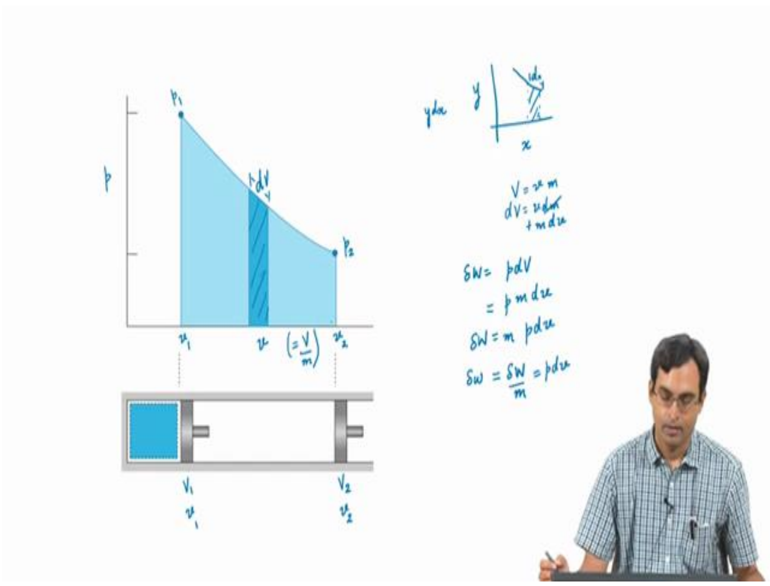

Figure 2

We can represent a process on a graph, e.g., a p-v graph for a system consisting of some gas where v represents specific volume (specific volume = total volume/mass). Figure 2 shows a process undergone by a piston-cylinder system as it expands from V_1 (corresponding pressure is $p_1$and the specific volume is v_1) to V_2 (corresponding pressure is p_2 and the specific volume is v_2) on a p-v diagram. The process is quasi-static. The piston is moving slowly.

For this process, the total work done by the system is $W = \int_{V_1}^{V_2} pdV$. pdV is essentially the area under the curve for a small change dV. Hence, $\int_{V_1}^{V_2} pdV$ gives the area under the curve joining points (v_1, p_1) and (v_2, p_2). Also, $W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} pd(mv) = m\int_{V_1}^{V_2} pdv$ as the mass of the system is constant. Hence, the work done per unit mass i.e. specific work is $w = \int_{V_1}^{V_2} pdv$. Thus, the area under a curve on p-v graph gives specific work. The area under the curve can be found out if we know the equation of the curve. The curve can be plotted by experimentally measuring the values of p and v.

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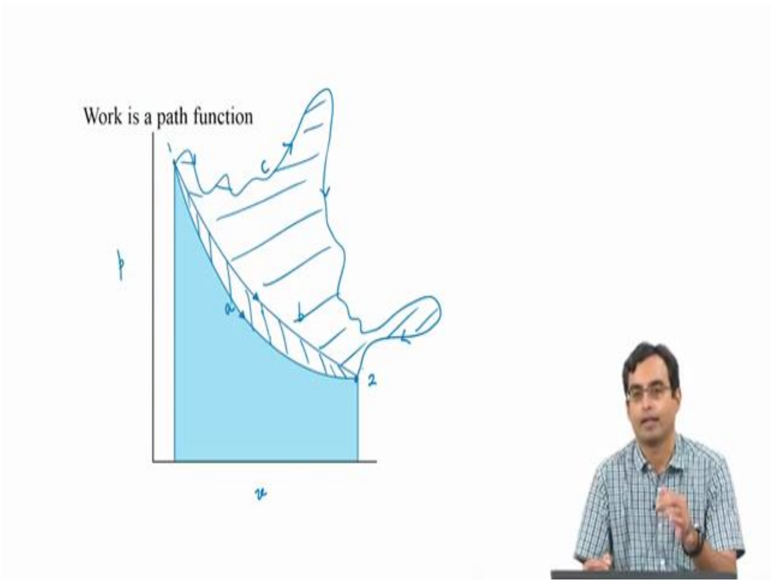

Figure 3

Since the area under the curve depends on the curve, different curves will have different areas. Hence, the work done also would be different for different curves. In Fig. 3, the system changes its state from 1 to 2 through three different paths, a, b and c. In all the three processes, the work done would be different. Hence, work is a path function. It is analogous to the distance travelled between two points on a map since distance travelled depends on the path taken to travel. However, the displacement is independent of the path. Since work is a path function, we cannot talk of work on a point. We consider work transfer along a path the system takes when it goes from state 1 to some other state 2.