Thermodynamics
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Lecture 67
Tutorial problems (1 numbers)

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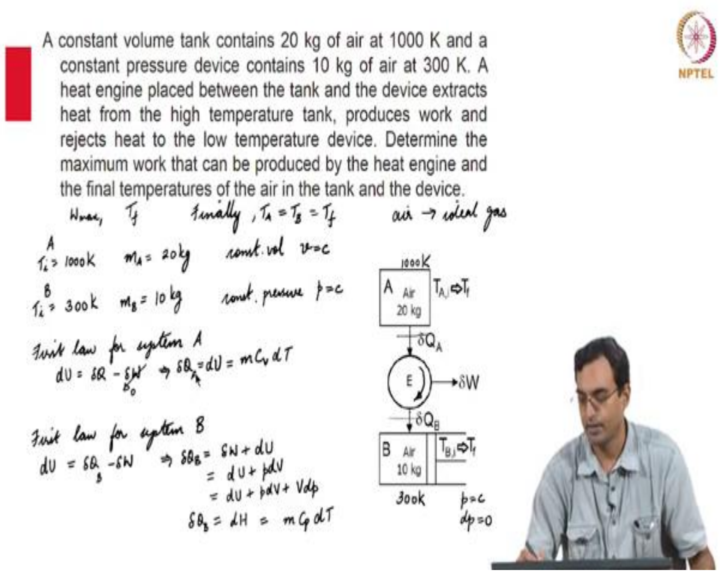

Figure 1.

Solution of the problem in Fig. 1:

Constant volume tank A,

$T_{i,A} = 1000\ K, m_A = 20\ kg, v = constant$

Constant pressure device B,

$T_{i,B} = 300\ K, m_B = 10\ kg, p = constant$

Air is assumed to be an ideal gas.

Here, the temperature of the temperature reservoirs is changing, unlike the problems we saw previously, because the reservoirs are finite in size.

The heat engine operating between the temperature reservoirs will give maximum work if the engine runs reversibly.

The engine will stop working when the temperature of both the reservoirs become equal, i.e., the final temperature T_f of both the reservoirs is the same.

We can apply the first law to the constant volume tank A and the constant pressure device B separately. The tank A and the constant pressure device B are closed systems.

The first law for the tank A, $dU = \delta Q - \delta W$ (there are no changes in the kinetic and potential energy of the system). As the tank is rigid, the process is a constant volume process for tank A. Hence, $\delta W = 0$. Thus, $\delta Q_A = dU = m_A C_v dT_A$. δQ_A is negative for the tank A.

The piston in the constant-pressure device moves as the device receives heat from the engine. The first law for the constant pressure device B, $\delta Q_B = \delta W + dU = pdV + dU = dU + pdV + Vdp = dH = m_B C_p dT_B$. δQ_B is positive for constant pressure device B.

Now, apply the first law to the heat engine.

$\oint \delta Q = \oint \delta W \quad \rightarrow \quad \delta Q_A + \delta Q_B = \delta W \quad \rightarrow \quad |\delta Q_A| - |\delta Q_B| = \delta W$ (considering the sign convention)

Integrating,

$W = |\int m_A C_v dT_A| - |\int m_B C_p dT_B| = |m_A C_v (T_f - T_{i,A})| - |m_B C_p (T_f - T_{i,B})|$…..(1) ($C_v$ and C_p are assumed constant. $\delta Q_A$is positive for the engine whereas $\delta Q_B$is negative)

We are asked to find the maximum work. Hence, the heat engine must be reversible.

Hence, $\frac{\delta Q_A}{T_A} = \frac{\delta Q_B}{T_B} \rightarrow -\frac{m_A C_v dT_A}{T_A} = \frac{m_B C_p dT_B}{T_B}$ (The temperature of the tank A goes down. Hence, dT_A would be negative. To make the numerator positive on the left hand side, minus sign is added.)

Integrating,

$m_A C_v [\ln T_A]_i^f = -m_B C_p [\ln T_B]_i^f$ (i – initial, f - final)

Substituting the values,

$\ln T_f - \ln(1000) = -0.7[\ln T_f - \ln(800)]$ $\left(C_v = 717.5 \frac{J}{kg \cdot K}\right.$ and $C_p = 1004.5 \frac{J}{kg \cdot K}$ assuming $R_{air} = 287 \frac{J}{kg \cdot K}$ and $\gamma_{air} = 1.4$ (diatomic gas))

Hence, $T_f = 609$ K.

Equation (1) implies,

$W = 20 \times 717.5 \times (1000 - 609) - 10 \times 1004.5 \times (609 - 300) = 2.5\ MJ$

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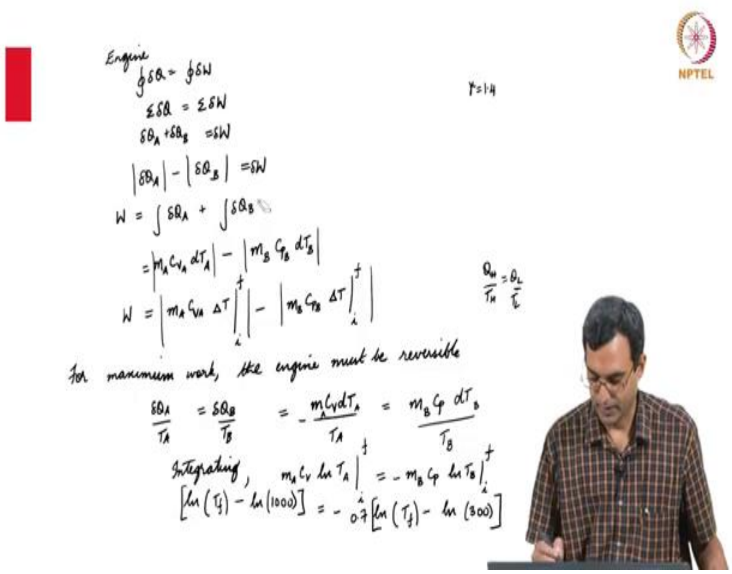

$\Rightarrow 1.7 \ln(T_f) = \ln(1000) + 0.7\ln(300) = \sout{12.77}\ 10.9$

$\ln(T_f) = \sout{7.51} = 10.9/1.7 = 6.41$

$\Rightarrow T_f = \sout{1826.6}\ K = 609\ K$ ✓

$W = \left|\int \delta Q_A\right| - \left|\int \delta Q_B\right|$ $\quad \Sigma\, \delta Q$

$= 20 \times 717.5 \times (1000 - 609) - 10 \times 1004.5 \times (609 - 300)$

$= 5.6\ MJ - 3.1\ MJ$

$W = 2.5\ MJ$ ✓

$\gamma = 1.4$

$C_v = \frac{R}{\gamma - 1}$

$R = \frac{\bar{R}}{M}$

air $R = 287$ J/kgK

$C_v = 717.5$ J/kgK

$C_p = 1004.5$ J/kgK

A constant volume tank contains 20 kg of air at 1000 K and a constant pressure device contains 10 kg of air at 300 K. A heat engine placed between the tank and the device extracts heat from the high temperature tank, produces work and rejects heat to the low temperature device. Determine the maximum work that can be produced by the heat engine and the final temperatures of the air in the tank and the device.

Here, T_f $\quad$ finally, $T_A = T_B = T_f$ $\quad$ air → ideal gas

$T_i^A = 1000\,K \quad m_A = 20\,kg \quad$ const. vol $v = c$

$T_i^B = 300\,K \quad m_B = 10\,kg \quad$ const. pressure $p = c$

First law for system A

$dU = \delta Q - \underset{0}{\delta W} \Rightarrow \delta Q_A = dU = m C_v\, dT$

First law for system B

$dU = \delta Q_B - \delta W \Rightarrow \delta Q_B = \delta W + dU$

$= dU + p dV$

$= dU + p dV + V dp$

$\delta Q_B = dH = m_B C_{p_B} dT_B$