Thermodynamics
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Lecture 63
Second law of Thermodynamics: Carnot's cycle and theorems

We will now look at the Carnot's cycle. Sadi Carnot was an engineer in the USA. He worked in the 1800s and came up with a thought experiment of what would be the most efficient engine or a refrigerator.

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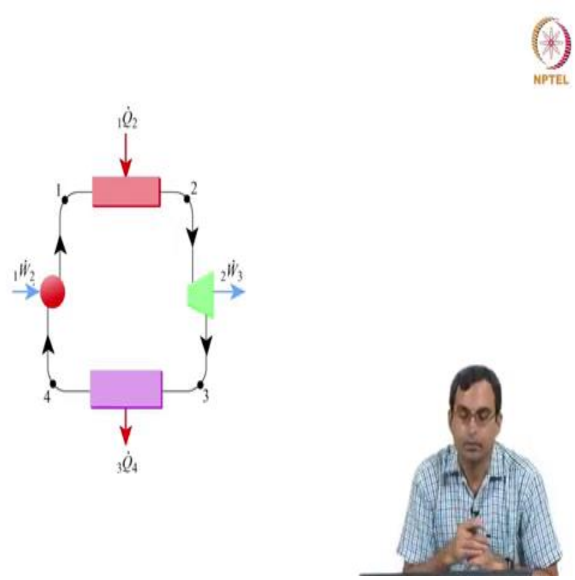

Figure 1.

A system shown in Fig. 1 undergoes a cyclic process. It receives and rejects heat isothermally (reversibly). Also, it receives work from the surroundings and does work on the surroundings, reversibly. There are 4 reversible processes. Two of the processes involve heat transfer and the other two processes involve work transfer. According to Carnot, such a system would have the highest efficiency $\left(\eta = \frac{W_{net}}{Q_{in}}\right)$ of all.

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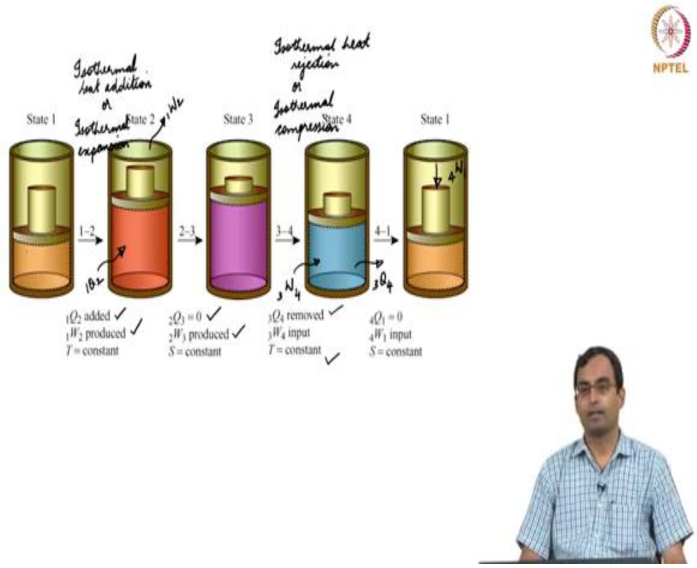

Figure 2.

Consider a piston-cylinder arrangement shown in Fig. 2. Add heat isothermally. The piston moves and produces work. The state of the system changes from 1 to 2. This process is isothermal heat addition or isothermal expansion. From the state 2 to 3, the piston moves (the system expands) and produces work, adiabatically. The process from the state 2 to 3 is an adiabatic reversible process (it is an isentropic process which we will study later). From the state 3 to 4, heat is rejected isothermally. The piston moves in the opposite direction (the system gets compressed, i.e. work is done on the system). The process from 3 to 4 is isothermal heat rejection or isothermal compression. In the process 4 to 1, the piston moves again compressing the system. Hence, work is done on the system. The process 4-1 is reversible and adiabatic. All the processes are reversible. Two of them are isothermal and the other two are adiabatic. These 4 processes form the Carnot’s cycle.

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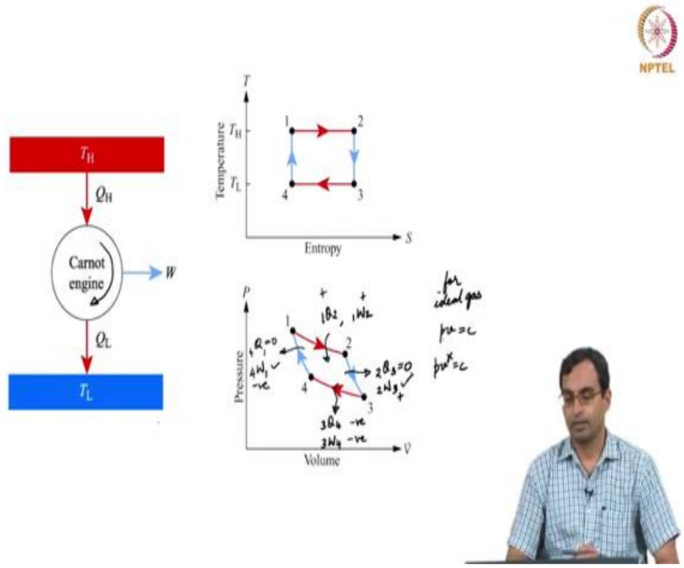

Figure 3.

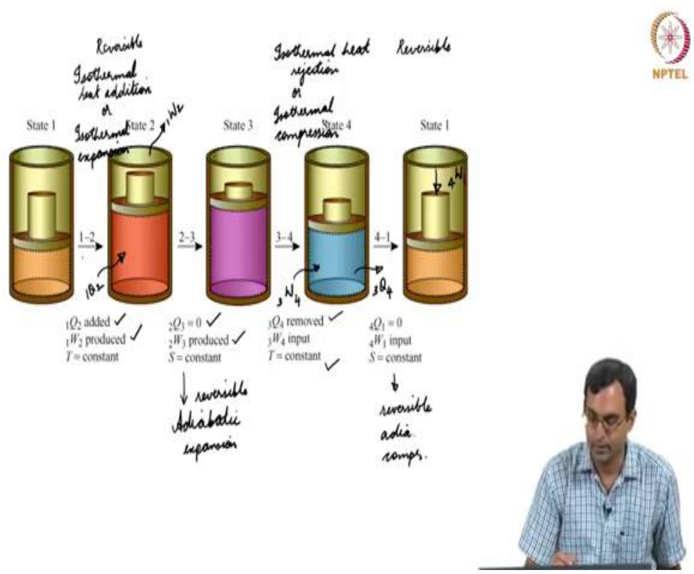

Figure 3 shows a schematic of the engine running on the Carnot't cycle. It takes in heat Q_H from the high temperature source at T_H and rejects heat Q_L to the low temperature source at T_L. It generates net work W. Figure 3 also shows a T-S (temperature-entropy) and a p-V diagram of the Carnot's cycle. We will look at the T-S diagram later on.

The process 1-2 is an isothermal heat addition/isothermal expansion (for an ideal gas, it would follow pV = constant). For the process 1-2, the heat added is represented as ${}_1Q_2$ and the work done is represented as ${}_1W_2$. The process 2-3 is reversible adiabatic expansion (for an ideal gas, it would follow $pV^{\gamma} = constant$). The work done for 2-3 is represented as ${}_2W_3$. The heat interaction for 2-3 is 0, i.e., ${}_2Q_3 = 0$. The process 2-3 is steeper than the process 1-2 on the p-V diagram. The process 3-4 is isothermal heat rejection/isothermal compression. The heat rejected and the work done in the process 3-4 are represented as ${}_3Q_4$ and ${}_3W_4$. The process 4-1 is reversible adiabatic compression. Heat and work interactions for the process 4-1 are represented as ${}_4Q_1$ and ${}_4W_1$. For 4-1, ${}_4Q_1 = 0$.

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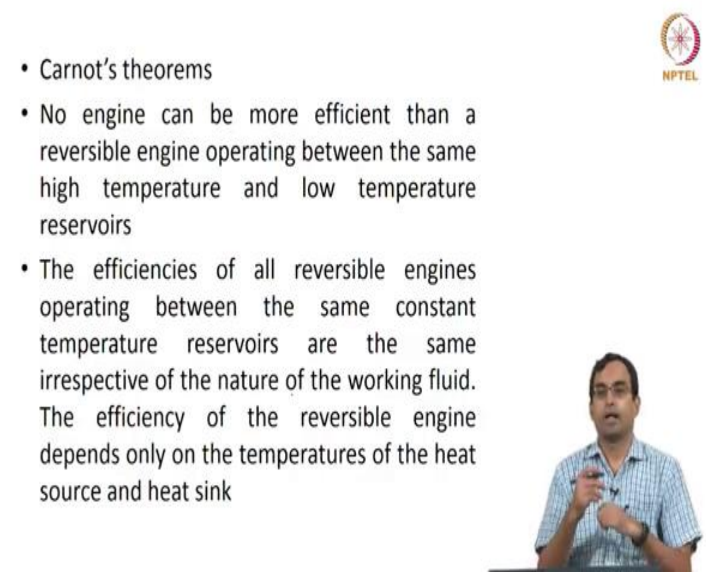

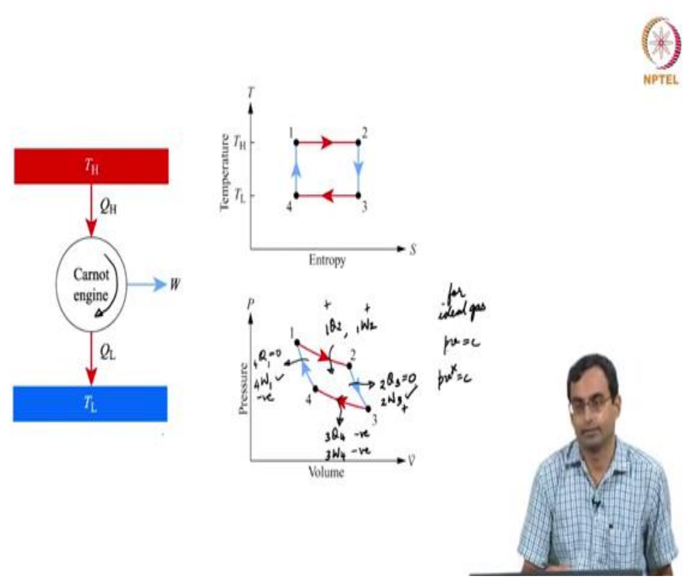

Carnot also came up with 2 theorems:

(1) No engine can be more efficient than a reversible engine (which is essentially the Carnot engine) operating between the same high temperature and low temperature reservoirs.

(2) The efficiency of all reversible engines operating between the same constant temperature reservoirs are the same irrespective of the working fluid. The efficiency of the reversible engine depends only on the temperature of the heat source (high temperature reservoir) and the temperature of the heat sink (low temperature reservoir).

According to the Carnot's theorem, a reversible engine using an ideal gas and another reversible engine using a non-ideal gas operating between the same high temperature and low temperature reservoirs have the same efficiency. The efficiency of the reversible engine is independent of the working fluid. Hence, it depends only on the temperature of a heat source and a heat sink.

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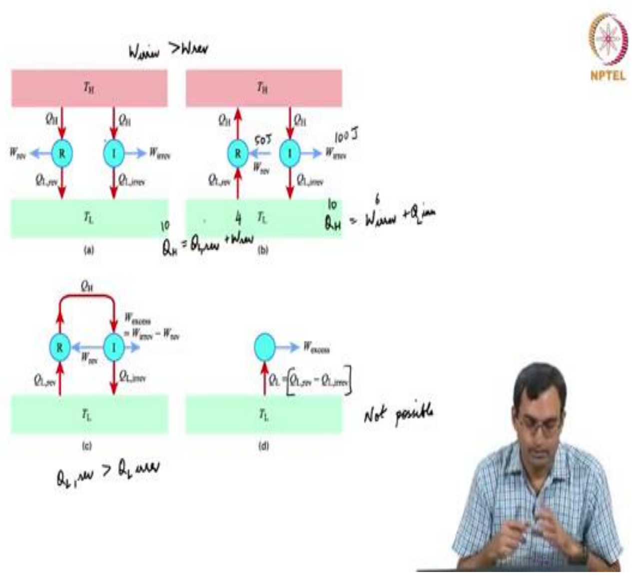

Figure 4.

The violation of the Carnot's theorems violates the second law of thermodynamics. It can be proved in many ways. Let's look at one of those ways.

Consider a reversible (R) and irreversible (I) heat engine between the same heat source (at T_H) and the heat sink (at T_L) (see the top left corner of Fig. 4). Both the engines, the reversible as well as the irreversible one, take in Q_H amount of heat from the heat source. The reversible engine rejects heat $Q_{L,rev}$ to the heat sink, whereas the irreversible engine rejects $Q_{L,irrev}$ to the same heat sink. The work done by the reversible heat engine is W_{rev}, whereas the work done by the irreversible heat engine is W_{irrev}. Now, run the reversible heat engine in a reverse fashion so that it acts as a heat pump or a refrigerator. It takes in $Q_{L,rev}$ from the heat sink at T_L and rejects heat Q_H to the heat source at T_H. It also takes in work W_{rev} (see the top right corner of Fig. 4). We cannot do such process reversal with the irreversible engine (I).

Assume that the irreversible engine (I) is more efficient than the reversible engine (R) (this assumption is in violation of the Carnot's theorem). As the heat pump or a refrigerator R gives out Q_H to the reservoir at T_H and the irreversible heat engine I takes in heat Q_H from the same reservoir, we can remove that reservoir from the analysis (see bottom left of Fig. 4). Since, $W_{irrev} > W_{rev}$ (we assumed that the irreversible engine I is more efficient than the reversible engine R), $Q_{L,irrev} < Q_{L,rev}$ (according to the first law). From W_{irrev}, W_{rev} can be used to drive the heat pump R. Now, we combine the heat pump R and the irreversible heat engine I into one system which takes in heat $Q_{L,rev} - Q_{L,irrev}$ from the reservoir at T_L and does work $W_{irrev} - W_{rev}$. So, this system, operating in a cycle, gives out work while taking heat from a single heat reservoir. This violates the Kelvin-Planck statement of the second law which says that it is impossible for a system running in a thermodynamic cycle to produce work with heat transfer from a single reservoir. It means our assumption that the irreversible engine (I) is more efficient than the reversible engine (R) (which violates the Carnot's theorem) is not true as it violates the second law. Hence, the violation of the Carnot's theorem violates the second law.

The Carnot's heat engine or the Carnot's heat pump (or a refrigerator) are the most efficient heat engine or a heat pump between the given heat source and heat sink. All the reversible heat engines between the same two temperature reservoirs must have the same efficiency. It can also be proved in the same way as we discussed above, by assuming that one of the engines have more efficiency than the other.