

Thermodynamics
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Lecture 57
Tutorial Problems - Part 2

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A tank of volume 0.3 m^3 is initially filled with air at a pressure and temperature of 3.5 MPa and 400 degrees C. The air is now allowed to discharge slowly through a turbine into the atmosphere until the pressure in the tank falls to the atmospheric pressure of 0.1 MPa . Determine the work developed by the turbine. Neglect friction, heat loss, KE & PE changes.



$$\frac{dm}{dt} = \frac{m_i}{V_i} - m_e \Rightarrow m_e = -\frac{dm}{dt}$$

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \frac{m_i}{V_i} \left[h_i + \frac{V_i^2}{2} + gZ_i \right] - m_e \left[h_e + \frac{V_e^2}{2} + gZ_e \right]$$

$$\frac{dE}{dt} = -\dot{W} - m_e h_e$$

$$\dot{W} = -\frac{dE}{dt} - m_e h_e = -\frac{dE}{dt} + \frac{dm}{dt} h_e$$



Figure 1.

$$\dot{W} = -\frac{dE}{dt} + \frac{dm}{dt} h_e$$

$$\dot{W} = -\frac{d(mu)}{dt} + \frac{dm}{dt} h_e$$

$$\dot{W} = -\frac{d[mC_v T]}{dt} + \frac{dm}{dt} C_p T_e$$

$$\dot{W} = -[m_1 C_v T_1] + m_1 C_p T_2$$

$$= C_v [m_1 T_1 - m_2 T_2] + [m_2 - m_1] C_p T_2$$

$$= 77.5 [543 \times 673 - 0.428 \times 243.7] + [0.428 - 543] \times 104.5 \times 243.7$$

$$= 2.57 \times 10^6 - 1.224 \times 10^6$$

$$\dot{W} = 1.3 \times 10^6 \text{ J}$$

$$E_{cv} = U + KE + PE$$

$$E_{cv} = U$$

$$u = m$$

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$m = \frac{pV}{RT}$$

$$m_1 = \frac{3.5 \times 10^6 \times 0.3}{287 \times 673}$$

$$m_1 = 543 \text{ kg}$$

$$T_2 = ?$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_2 = \frac{T_1}{\left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}}}$$

$$T_2 = 243.7 \text{ K} = \frac{673}{\left(\frac{3.5 \times 10^6}{0.1 \times 10^6} \right)^{\frac{1.4-1}{1.4}}}$$





$$m_2 = \frac{0.1 \times 10^3 \times 0.5}{287 \times 2437}$$

$$m_2 = 0.928 \text{ kg}$$

$$R = 287 \text{ J/kgK}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{287}{0.4}$$

$$C_v = 717.5 \text{ J/kgK}$$

$$C_p = C_v + R$$

$$= 1004.5 \text{ J/kgK}$$



Solution of the problem in Fig. 1:

The tank and the turbine form a control volume as shown in Fig.1. Here, there is no inlet to the control volume. There is only one outlet. It is an unsteady process. The control volume loses mass with time.

The rate equation for the mass conservation is $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e$. Since there is no inlet, $\frac{dm}{dt} = -\dot{m}_e$.

The first law for a control volume,

$$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{1}{2} \bar{V}_i^2 + gZ_i \right) - \sum \dot{m}_e \left(h_e + \frac{1}{2} \bar{V}_e^2 + gZ_e \right) \dots (1)$$

Neglecting heat loss, KE and PE changes, setting $\dot{m}_i = 0$,

$$\frac{dE}{dt} = -\dot{W}_{cv} - \dot{m}_e h_e \rightarrow \dot{W} = -\frac{dE}{dt} + \frac{dm}{dt} h_e \dots (2)$$

Since there are no changes in kinetic and potential energy, $E = U$. Hence,

$$\dot{W} = -\frac{d(mu)}{dt} + \frac{dm}{dt} h_e \rightarrow \dot{W} = -\frac{d}{dt} (mC_v T)_{cv} + \frac{dm}{dt} C_p T_e \dots (3)$$

where T_e is the temperature at the exit and air is assumed as an ideal gas.

We need to know the mass and the temperature of the control volume before and after the process took place.

Initially, $m_1 = \frac{p_1 V_1}{RT_1} = \frac{3.5 \times 10^6 \times 0.3}{287 \times 673} = 5.43 \text{ kg}$ = mass in the control volume before the process took place.

Here, the air (ideal gas) undergoes an adiabatic process. Hence, $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$. p_2 is the pressure inside the control volume after the process took place and $p_2 = 0.1 \text{ MPa}$. Substituting all the values, $T_2 = 243.7 \text{ K} = T_e$ = temperature at the exit of the control volume. T_2 is lower than T_1 as the air expands adiabatically. Now, we can calculate m_2 .

$$m_2 = \frac{p_2 V_2}{RT_2} = \frac{0.1 \times 10^6 \times 0.3}{287 \times 243.7} = 0.428 \text{ kg}$$

Now, we can integrate (3) with respect to time,

$$\dot{W} = -[m C_v T]_1^2 + [m]_1^2 C_p T_2 = C_v (m_1 T_1 - m_2 T_2) + (m_2 - m_1) T_2 \quad (C_v \text{ and } C_p \text{ are constant})$$

$$\text{Now, } C_v = \frac{R}{\gamma-1} = \frac{287}{1.4-1} = 717.5 \frac{J}{\text{kg}\cdot\text{K}} \text{ and } C_p = C_v + R = 1004.5 \frac{J}{\text{kg}\cdot\text{K}}.$$

Substituting all the values,

$$\dot{W} = 1.3 \text{ MJ}.$$