Thermodynamics
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Lecture 57
Tutorial Problems - Part 2

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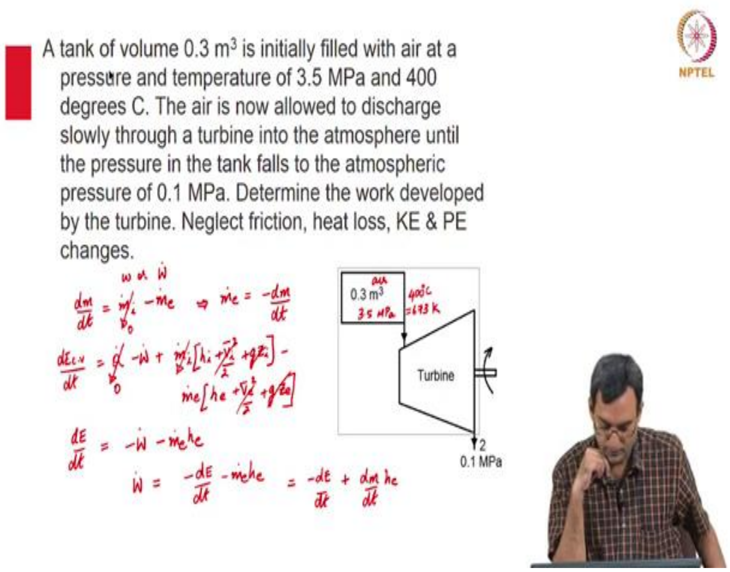
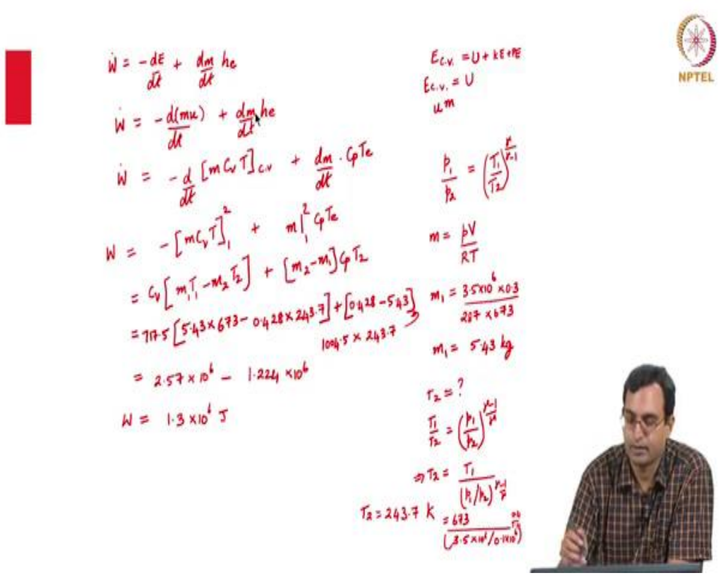
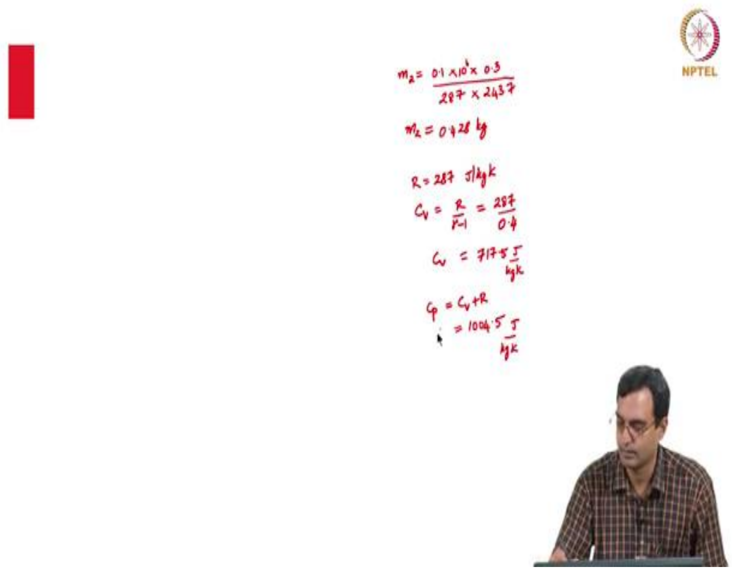


Figure 1.





Solution of the problem in Fig. 1:

The tank and the turbine form a control volume as shown in Fig.1. Here, there is no inlet to the control volume. There is only one outlet. It is an unsteady process. The control volume loses mass with time.

The rate equation for the mass conservation is $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e$. Since there is no inlet, $\frac{dm}{dt} = -\dot{m}_e$.

The first law for a control volume,

$$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \sum \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right) \ldots (1)$$

Neglecting heat loss, KE and PE changes, setting $\dot{m}_i = 0$,

$$\frac{dE}{dt} = -\dot{W}_{cv} - \dot{m}_e h_e \quad \rightarrow \quad \dot{W} = -\frac{dE}{dt} + \frac{dm}{dt} h_e \ldots\ldots (2)$$

Since there are no changes in kinetic and potential energy, E = U. Hence,

$$\dot{W} = -\frac{d(mu)}{dt} + \frac{dm}{dt} h_e \quad \rightarrow \quad \dot{W} = -\frac{d}{dt}\,(mC_vT)_{cv} + \frac{dm}{dt} C_p T_e, \ldots\ldots\ldots\ldots (3)$$

where T_e is the temperature at the exit and air is assumed as an ideal gas.

We need to know the mass and the temperature of the control volume before and after the process took place.

Initially, $m_1 = \frac{p_1 V_1}{R T_1} = \frac{3.5\times10^6\times0.3}{287\times673} = 5.43\ kg =$ mass in the control volume before the process took place.

Here, the air (ideal gas) undergoes an adiabatic process. Hence, $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$. p_2 is the pressure inside the control volume after the process took place and $p_2 = 0.1\ MPa$. Substituting all the values, $T_2 = 243.7\ K = T_e =$ temperature at the exit of the control volume. T_2 is lower than T_1 as the air expands adiabatically. Now, we can calculate m_2.

$$m_2 = \frac{p_2 V_2}{R T_2} = \frac{0.1\times10^6\times0.3}{287\times243.7} = 0.428\ kg$$

Now, we can integrate (3) with respect to time,

$\dot{W} = -[mC_vT]_1^2 + [m]_1^2 C_p T_2 = C_v(m_1T_1 - m_2T_2) + (m_2 - m_1)T_2$ (C_v and C_p are constant)

Now, $C_v = \frac{R}{\gamma-1} = \frac{287}{1.4-1} = 717.5\frac{J}{kg\cdot K}$ and $C_p = C_v + R = 1004.5\frac{J}{kg\cdot K}$.

Substituting all the values,

$\dot{W} = 1.3\ MJ$.