Thermodynamics

Professor Divya A
Department of Mechanical Engineering
Indian Institute of Technology Madras

Lecture 54

Energy equation for a steady-state

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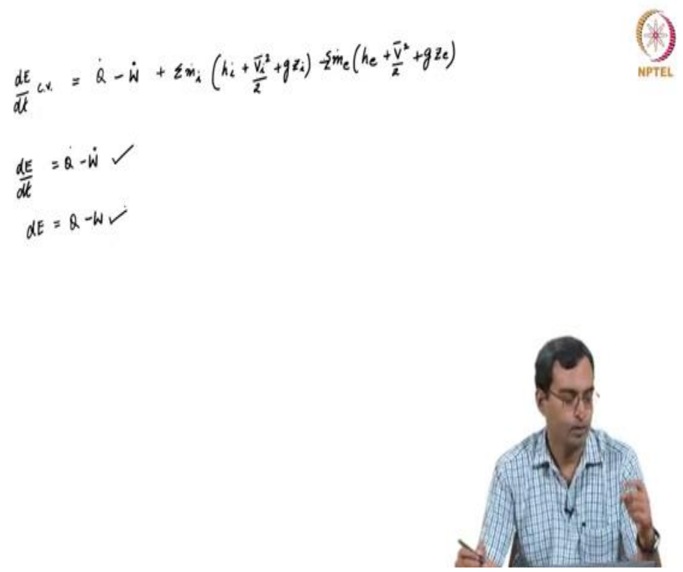

The first law for a control volume is written here:

$$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \sum \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right) \ldots.(1)$$

This expression can be simplified based on the application.

If the mass flow rates are 0, equation (1) reduces to the expression of the rate equation of the first law for a control mass which is $\frac{dE}{dt} = \dot{Q} - \dot{W}$. By integrating with respect to time (assuming we know the variation of $\dot{Q}$ and $\dot{W}$ with respect to time), we can obtain the expression for the first law, $\Delta E = Q - W$. So, we can obtain all the previous expressions we used from the expression of the first law for a control volume (equation (1)).

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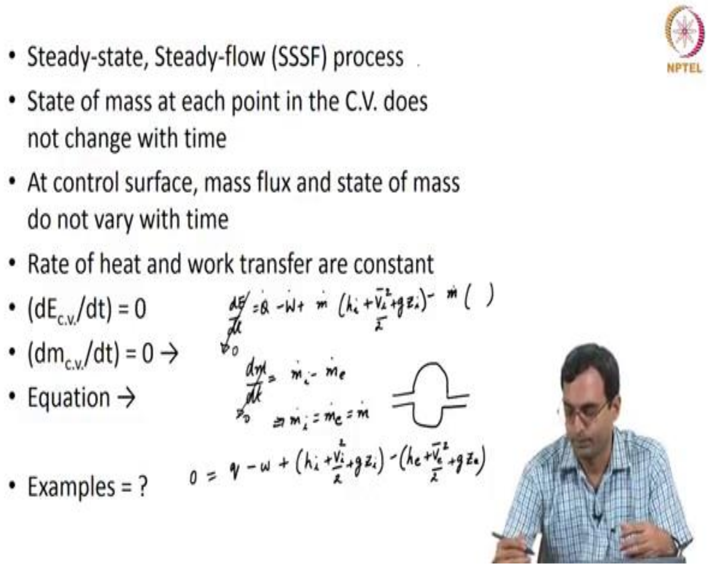

Let’s look at some other simplifications of the first law for a control volume.

For a steady state steady flow process, (1) state of mass at each point in the control volume does not change with time, (2) mass flux and the state of mass at the control surface do not vary with time, (3) rate of heat and work transfer are constant with time. In essence, in a steady process, nothing changes with time. Hence, in equation (1), $\frac{dE}{dt} = 0$. $\dot{Q}$ and $\dot{W}$ do not change with time (they may not be 0 individually). The term $\frac{dm}{dt}$ in the rate equation of mass conservation $\left(\frac{dm}{dt} = \sum \dot{m}_i - \sum \dot{m}_e\right)$ becomes 0. Hence, $\sum \dot{m}_i = \sum \dot{m}_e = \sum \dot{m}$. If we have only one inlet and outlet, which is common in many practical applications, then $\dot{m}_i = \dot{m}_e = \dot{m}$. Dividing by $\dot{m}$, equation (1) reduces to $0 = q - w + \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right)$, where q and w are specific heat and work interactions. It is recommended to remember only the general form of the first law for a control volume.

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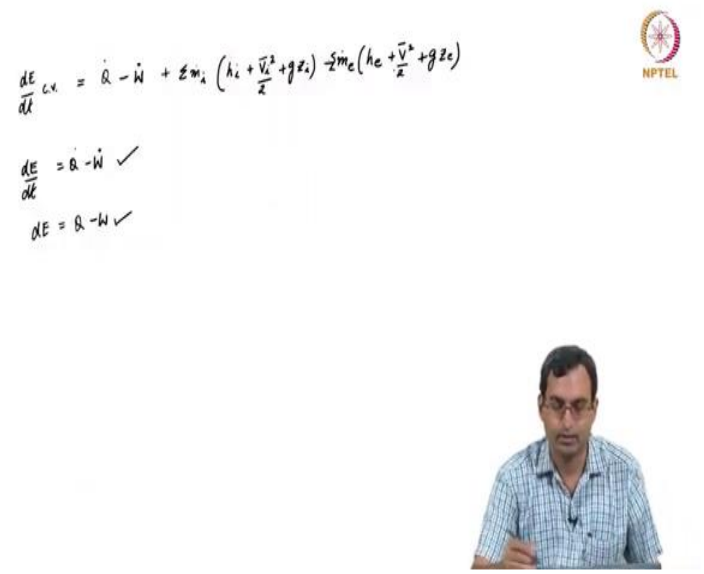

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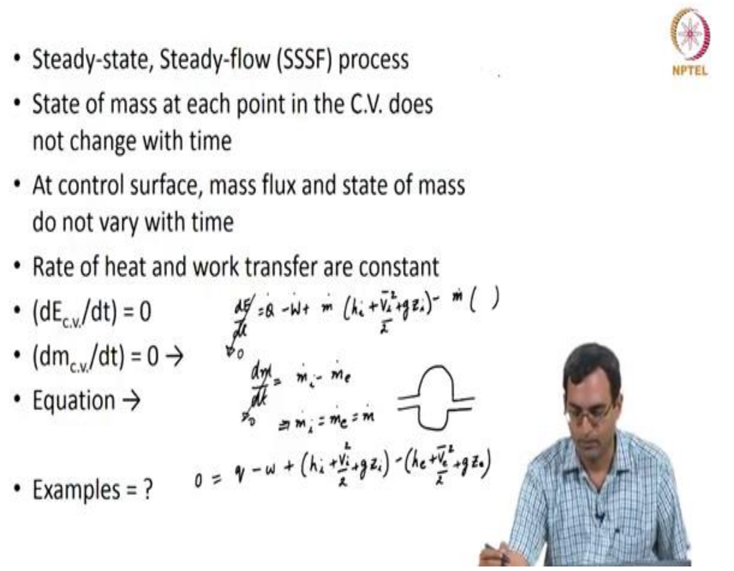

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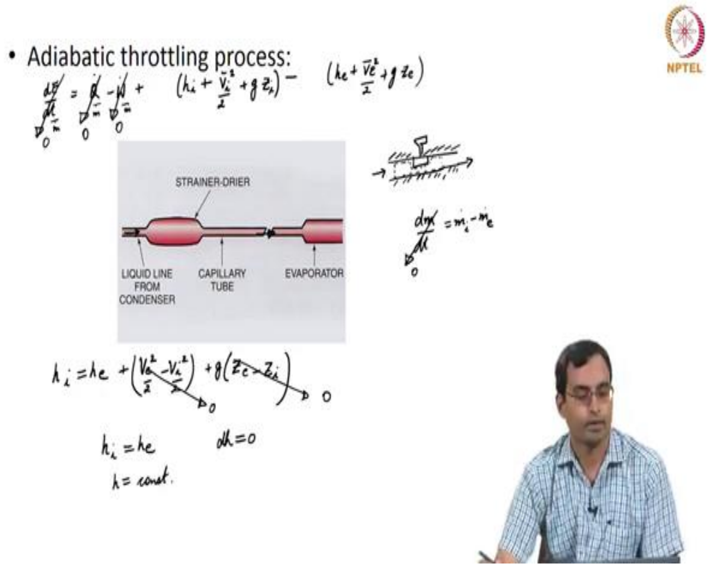

Figure 1.

Let's apply the first law for a control volume to an adiabatic throttling process.

A capillary tube is shown in Fig. 1 (which is also used in refrigerators). If a fluid is pushed through such a long capillary tube (it is not shown as long in the Fig. 1, but it is usually quite long) having a small cross-section, pressure drops significantly. This process is called throttling. If the fluid is a non-ideal gas, the temperature also drops. This process is implemented in the refrigerators. Similarly, a valve in a pipe carrying some fluid can block the fluid flow and there is a pressure drop across the valve. This is also a throttling process. Let's apply the first law to a control volume around a valve in a pipe (Fig. 1). It has one inlet and one outlet (hence, $\dot{m}_i = \dot{m}_e = \dot{m}$). This process is assumed to be a steady process $\left(\frac{dE}{dt} = 0\right)$. Usually, these pipes and tubes are insulated. Hence, the process is also adiabatic ($\dot{Q} = 0$). In this process, we do not have any work interaction ($\dot{W} = 0$). The only work interaction for pushing the fluid in and out of the control volume is included in the enthalpy term in the equation (h_i, h_e). Hence, the first law for a control volume (Eq. 1) undergoing an adiabatic throttling process is,

$$0 = 0 - 0 + \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right)$$

Hence, $h_i = h_e + \left(\frac{V_e^2}{2} - \frac{V_i^2}{2}\right) + g(Z_e - Z_i)$. Usually, for such tubes and pipes, the changes in Z are very small, i.e., $Z_e - Z_i$ is very small. Hence, there is no appreciable change in specific potential energy of the fluid, i.e., $g(Z_e - Z_i) \approx 0$. If we assume that the change in specific kinetic energy $\left(\frac{V_e^2}{2} - \frac{V_i^2}{2}\right)$ is also small compared to the enthalpy values, the equation reduces to $h_i = h_e$. However, if the velocity values at the inlet and the outlet are given and they are significantly different, the change in specific kinetic energy should not be neglected.

In an ideal adiabatic throttling process, the enthalpy does not change, i.e., $h_i = h_e$.

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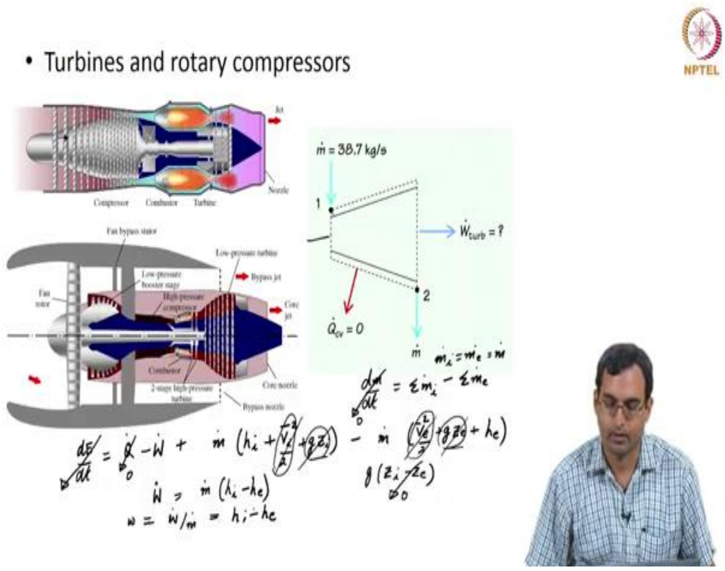

Figure 2.

Figure 2 shows turbines. In a turbine, high pressure high temperature gases expand. While expanding, they rotate blades connected to a shaft, doing work. A turbine has a single inlet and outlet (hence, $\dot{m}_i = \dot{m}_e = \dot{m}$). A turbine is represented by a diverging section as shown in Fig. 2. 1 represents inlet and 2 represents outlet. It gives out power, $\dot{W}_{turbine}$. The turbine runs at steady state except when it starts or stops $\left(\frac{dE}{dt} = 0\right)$. For an ideal turbine, the process is adiabatic ($\dot{Q} = 0$). The turbine produces power ($\dot{W} \neq 0$). The first law (Eq. 1) for a turbine reduces to,

$$0 = 0 - \dot{W} + \dot{m}_i\left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \dot{m}_e\left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right)$$

$$\dot{W} = \dot{m}(h_i - h_e) + \dot{m}\left(\frac{V_i^2}{2} - \frac{V_e^2}{2}\right) + \dot{m}g(Z_i - Z_e)$$

The terms $\frac{V_i^2}{2}, \frac{V_e^2}{2}$ and Z_i, Z_e individually are not zero. But, their differences are not appreciable. Hence,

$$\dot{W}_{turbine} = \dot{m}(h_i - h_e) \;\rightarrow\; \dot{w}_{turbine} = (h_i - h_e)$$

The expression for the power in the case of a compressor is exactly the same as that for the turbine. Hence,

$$\dot{W}_{compressor} = \dot{m}(h_i - h_e) \;\rightarrow\; \dot{w}_{compressor} = (h_i - h_e).$$

However, in the case of turbine, $h_i > h_e$ and $\dot{W}_{turbine}$ is positive, whereas in the case of a compressor, $h_i < h_e$ and $\dot{W}_{compressor}$ is negative. In the case of a turbine, the work is done by the control volume (gases expand rotating the shaft), while in the case of a compressor, the work is done on the control volume (a fluid is pressurized). A compressor is represented by a converging section as opposed to a turbine.

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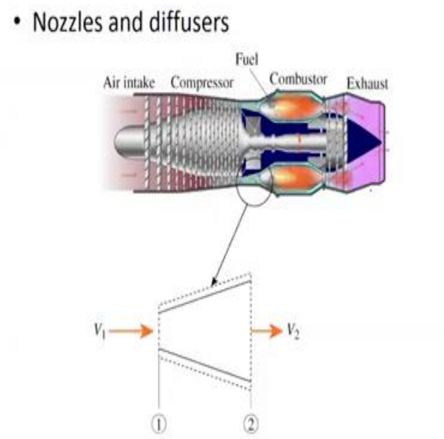

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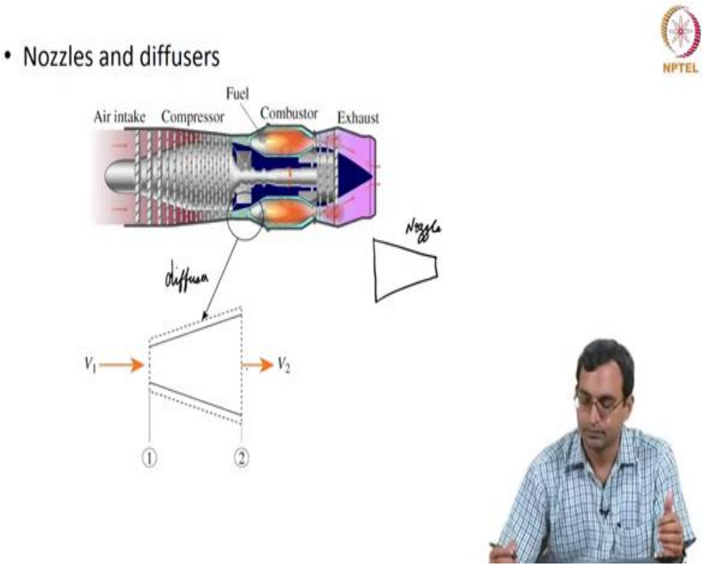

Figure 3.

Figure 3 shows a cross-section of a turbine. A turbine contains a nozzle and a diffuser. A nozzle has a reducing cross-sectional area, while a diffuser has an increasing cross-sectional area. In the nozzle, the fluid velocity increases, while in the diffuser, the fluid velocity decreases. The nozzle and diffusers are not power producing devices ($\dot{W} = 0$). They have a single inlet and outlet ($\dot{m}_i = \dot{m}_e = \dot{m}$). These devices operate at steady state conditions except at the start and the stop $\left(\frac{dE}{dt} = 0\right)$. These devices are insulated ($\dot{Q} = 0$). Hence, for nozzles and diffusers, equation (1) reduces to,

$0 = 0 - 0 + \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right)$. Hence,

$0 = \dot{m}(h_i - h_e) + \dot{m}\left(\frac{V_i^2}{2} - \frac{V_e^2}{2}\right) + \dot{m}g(Z_i - Z_e)$. The change in specific potential energy is not appreciable in these devices. Hence, $h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2}$. Hence, $h_i - h_e = \frac{V_e^2}{2} - \frac{V_i^2}{2}$. If the specific kinetic energy at the exit is larger than the inlet (which happens in a nozzle), $h_i - h_e$ would be positive, whereas $h_i - h_e$ would be negative if the specific kinetic energy at the exit is smaller than the inlet (which happens in a diffuser).

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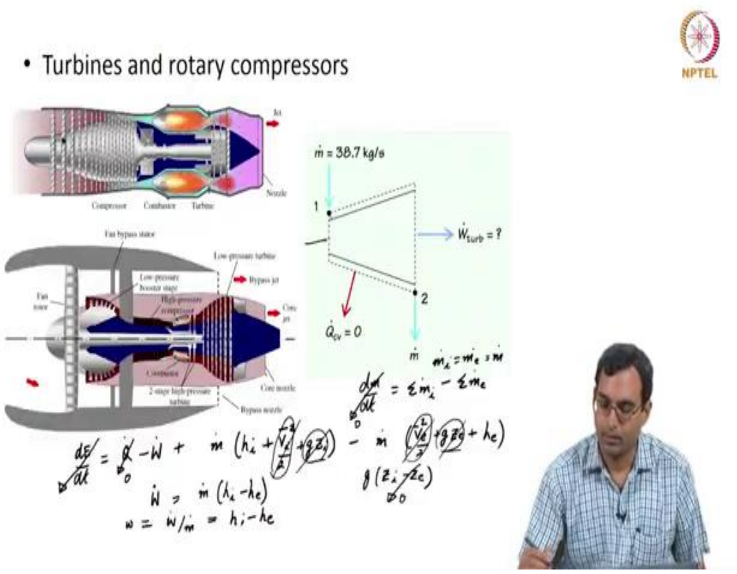

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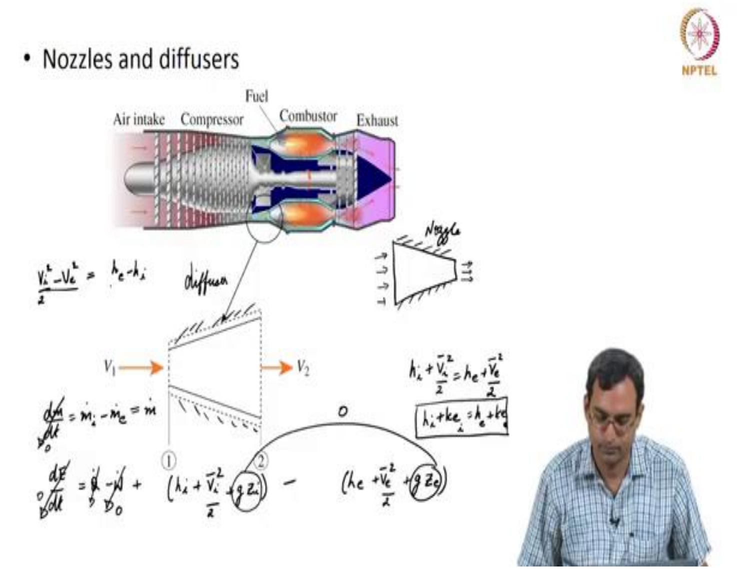

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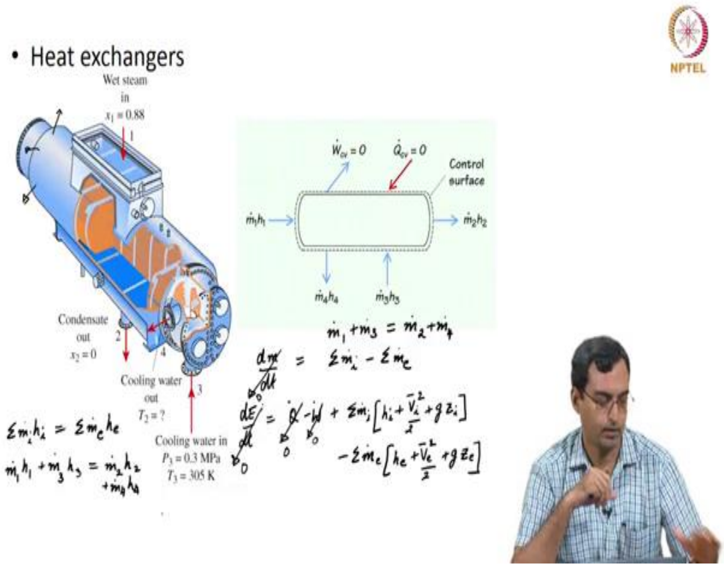

Figure 4.

Figure 4 shows a cut section of a heat exchanger. Heat exchangers are used in many applications such as cars, refrigerators, power plants, etc. A schematic of a heat exchanger is shown in Fig. 4. It has more than one inlet and outlet. For a steady state operation of the heat exchanger (shown in the schematic), $\frac{dm}{dt} = 0$. Hence, $\sum \dot{m}_i = \sum \dot{m}_e$ or $\dot{m}_1 + \dot{m}_3 = \dot{m}_2 + \dot{m}_4$. Inside a heat exchanger, heat transfer happens between a hot and cold fluid. Hence, there are separate inlets and outlets for a cold fluid and a hot fluid. The heat transfer across the boundaries of the heat exchanger (which appears in the equation of the first law for control volumes) is 0, ideally (hence, $\dot{Q} = 0$). There is no work interaction for a heat exchanger ($\dot{W} = 0$). As the operation is at steady state conditions, $\frac{dE}{dt} = 0$. Hence, equation (1) reduces to,

$0 = 0 - 0 + \sum \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\,2} + gZ_i\right) - \sum \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\,2} + gZ_e\right)$. The changes in kinetic and potential energy are not usually appreciable. Hence, $\sum \dot{m}_i\, h_i = \sum \dot{m}_e\, h_e$. Thus, $\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$.

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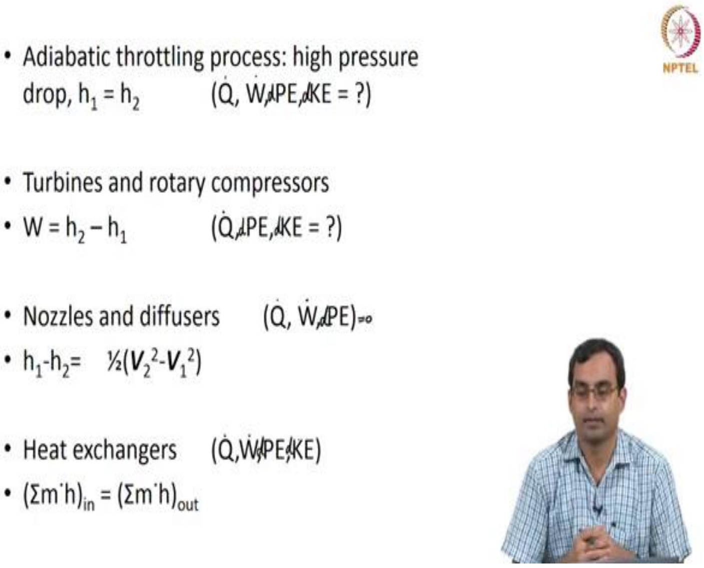

Figure 5.

Figure 5 summarizes the simplified versions of the first law for a control volume in the cases we discussed above.

(1) For an adiabatic throttling process and high pressure drop,

$h_1 = h_2 \; (\dot{Q} = \dot{W} = \Delta KE = \Delta PE = 0)$

(2) For turbines and rotary compressors,

$\dot{W} = h_i - h_e \; (\dot{Q} = \Delta KE = \Delta PE = 0)$

(3) For nozzles and diffusers,

$h_i - h_e = \frac{V_e^2}{2} - \frac{V_i^2}{2} \; (\dot{Q} = \dot{W} = \Delta PE = 0)$

(4) For heat exchangers,

$\sum \dot{m}_i \, h_i = \sum \dot{m}_e \, h_e \; (\dot{Q} = \dot{W} = \Delta KE = \Delta PE = 0)$