Thermodynamics
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Lecture 53
Rate equation of the first law of thermodynamics

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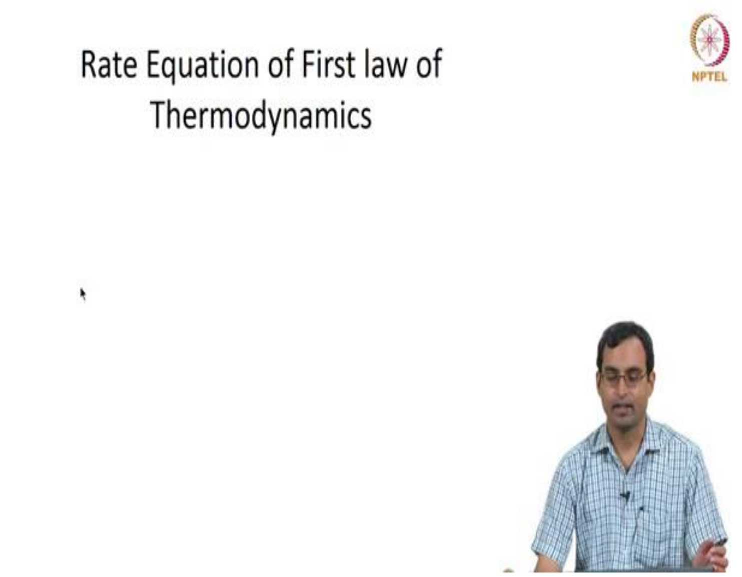

Today we look at the rate equation of the first law of thermodynamics and then go on to the first law for the control volume.

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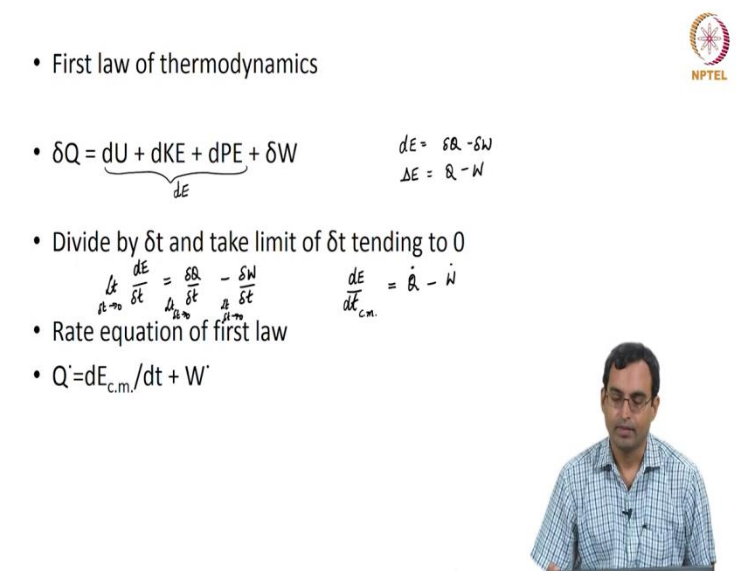

We have been writing the first law of thermodynamics as $dE = \delta Q - \delta W$. Dividing by δt (t is time) and taking limit as $\delta t \rightarrow 0$, we get

$\lim_{\delta t \to 0} \frac{dE}{\delta t} = \lim_{\delta t \to 0} \frac{\delta Q}{\delta t} - \lim_{\delta t \to 0} \frac{\delta W}{\delta t} \quad \rightarrow \quad \frac{dE}{dt}_{cm} = \dot{Q} - \dot{W}$, where $\frac{dE}{dt}$ is the change in energy with time, $\dot{Q}$ is the rate of heat transfer, $\dot{W}$ is the rate of doing work (power), cm represents control mass. This is the rate equation of the first law for a control mass. We have been using this equation without specifically mentioning that it is the rate equation of the first law.

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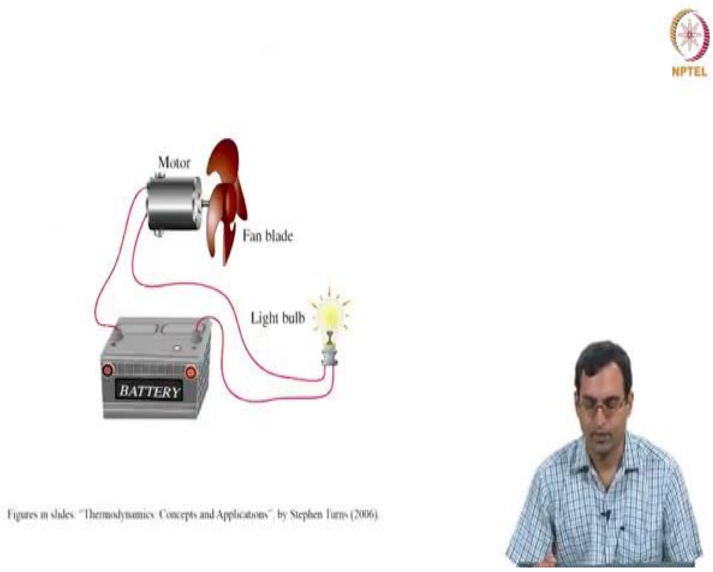

Figure 1.

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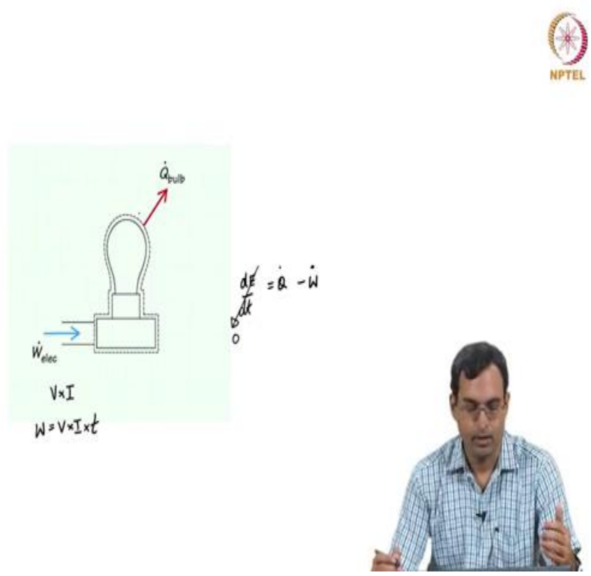

Figure 2.

If we take the bulb as shown in Fig. 2 as our system, the work done on the bulb by a battery is $W = VIt$. The power is $\dot{W} = VI$. Assuming steady state conditions, the rate of change of energy of the system (bulb) equals 0, i.e., $\frac{dE}{dt} = 0$. Hence, $\dot{Q} = \dot{W} = VI$, i.e., the rate at which heat (electromagnetic radiation) is emitted by the bulb equals the rate at which the work is done on the bulb. $\frac{dE}{dt}$ can be replaced by $\frac{dU}{dt}$ in this case as there is no change in kinetic and potential energy of the system during the process. A similar analysis can be done in the case of the fan shown in Fig. 1.

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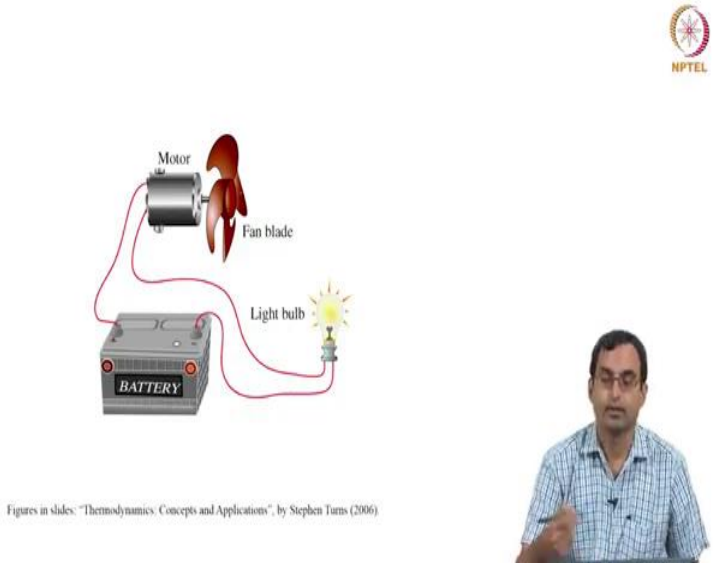

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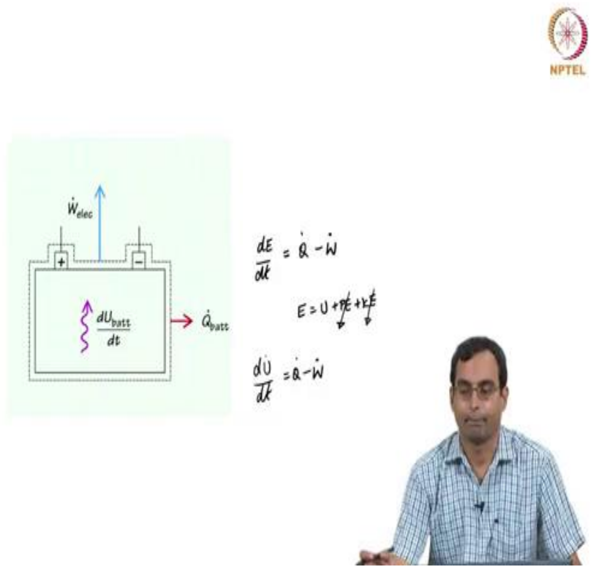

Figure 3.

Let's look at the battery shown in Fig. 3.

The battery is the system. The battery does work on the surroundings (bulb, fan, etc.) by pushing current across its boundary. Hence, we have $\dot{W}$ across the system boundary. We also have $\dot{Q}$ across the system boundary as the battery loses heat. For the battery, the rate equation of the first law is $\frac{dU}{dt} = \dot{Q} - \dot{W}$ (there are no changes in kinetic and potential energy of the system). Since the battery is doing work on the surroundings and losing heat, its internal energy will go down with time. After some time, the battery will stop working and losing heat. This is an unsteady process. Total work done or the heat lost or the total change in internal energy can be found out by integration. If the $\dot{Q}$ and $\dot{W}$ are constant over a small interval of time Δt, the work done and heat lost in that time interval is $W = \dot{W}\Delta t$ and $Q = \dot{Q}\Delta t$.

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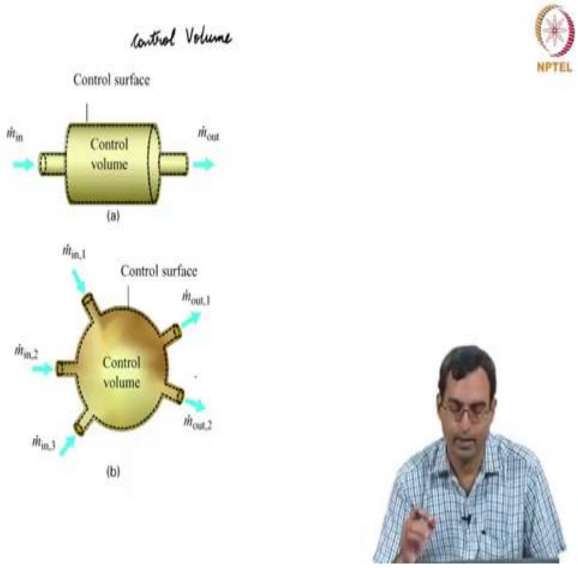

Figure 4.

Let's do the control volume analysis.

Consider a control volume with a single inlet and outlet as shown in Fig. 4. The rates at which the mass comes in and goes out of the control volume are $\dot{m}_{in}$ and $\dot{m}_{out}$. Similarly, we can have control volumes with multiple inlets and outlets as shown in Fig. 4.

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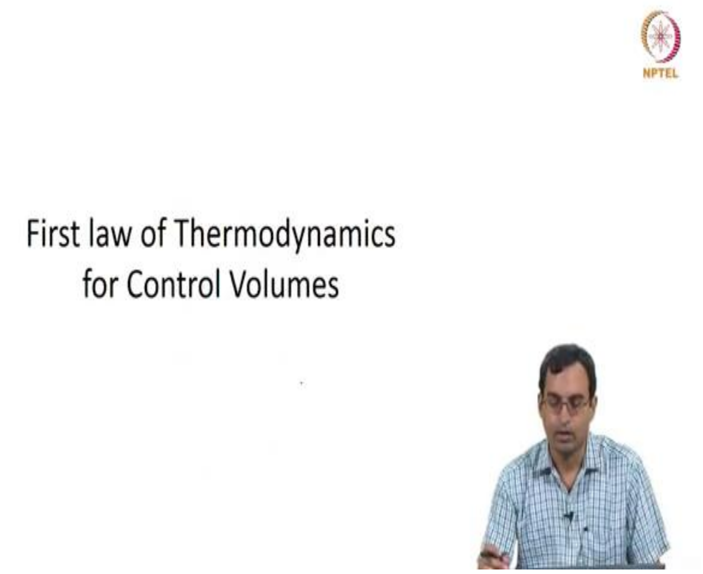

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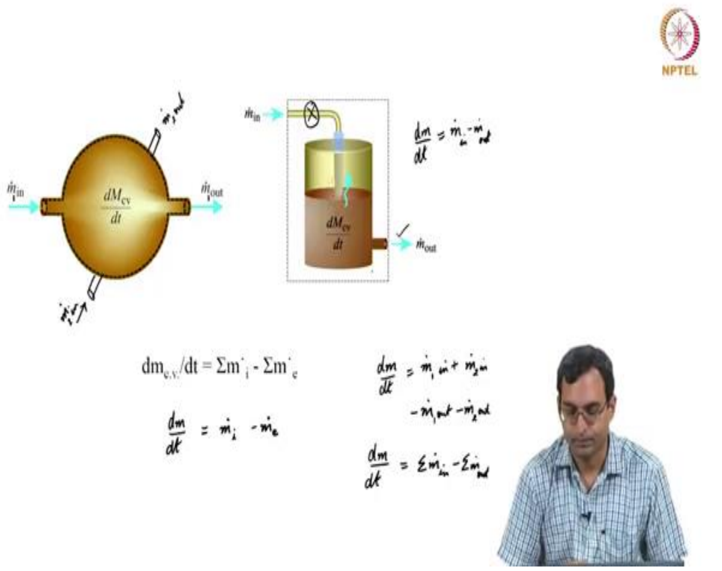

Figure 5.

The rate equation for the mass conservation in control volume is $\frac{dm}{dt}_{cv} = \sum \dot{m}_i - \sum \dot{m}_e$, where $\dot{m}_i$ is the rate at which the mass enters the control volume, $\dot{m}_e$ is the rate at which the mass leaves the control volume (i is for inlet and e is for exit/outlet), $\frac{dm}{dt}_{cv}$ is the rate of change of mass within the control volume. Summation symbol should be used when you have more than one inlet and outlet. For example, for a control volume with 2 inlets and 2 outlets, $\frac{dm}{dt}_{cv} = \dot{m}_{1_i} + \dot{m}_{2_i} - \dot{m}_{1_e} - \dot{m}_{2_e}$. For a control volume with a single inlet and outlet,

$\frac{dm}{dt}_{cv} = \dot{m}_i - \dot{m}_e$. If $\dot{m}_i$ and $\dot{m}_e$ are constant, then the change in mass of the control volume Δm over time interval Δt equals $\dot{m}_i\Delta t - \dot{m}_e\Delta t$. If $\dot{m}_i$ and $\dot{m}_e$ are not constant and we know their variation with time, then also we can calculate Δm by integrating the rate equation for mass conservation.

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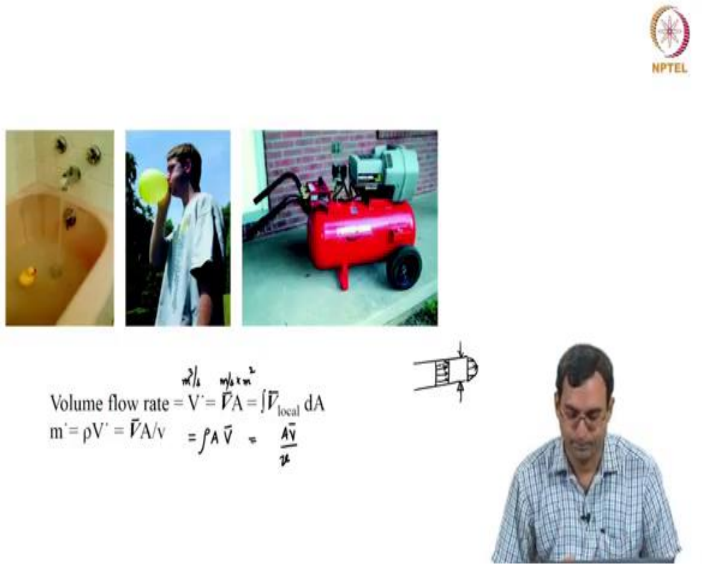

Figure 6.

Similar to the rate equation for mass conservation, we have the rate equation for energy conservation for a control volume. Figure 6 shows three control volumes, a bathtub, a balloon and a compressor across the boundaries of which there is a mass transfer. The system boundary in the case of balloon changes (the balloon expands or contracts), whereas in the other two examples it doesn't change. The volumetric flow rate into and out of the control volume is $\dot{V} = \bar{V}A = \int \overline{V_{local}}dA$, where $\bar{V}$ is the flow velocity (m/s), A is the area (m^2) perpendicular to the flow direction. In the case where the velocity is not uniform over a given cross-sectional area, we have to integrate local velocity over that area to get the volume flow rate, e.g., to get the volumetric flow rate through a pipe where the fully developed velocity profile is not uniform, we need to integrate local velocity over the cross-sectional area of the pipe. If the fluid is incompressible, we can obtain the mass flow rate as $\dot{m} = \rho\dot{V} = \rho A\bar{V} = \frac{A\bar{V}}{v}$, where ρ is the density of the fluid and v is the specific volume of the fluid. In the case the density is varying, we need to integrate to get the mass flow rate.

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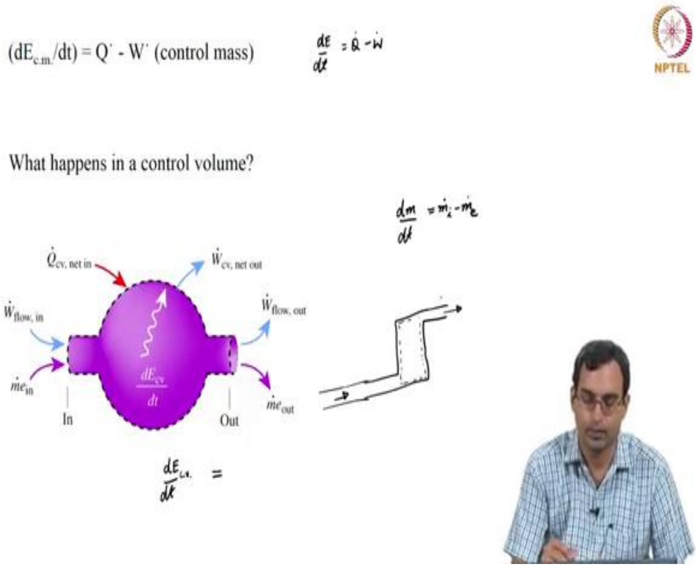

Figure 7.

For a control volume, in addition to energy transfer across its boundaries (in the form of heat and work), there is a transfer of mass also, which does not happen in the case of a control mass. The incoming and outgoing mass brings in and takes out energy with it from the control volume.

Figure 7 shows a control volume with a single inlet and outlet. The mass enters the control volume at the rate $\dot{m}_i$ and exits at the rate $\dot{m}_e$. The rate equation for the conservation of mass is $\frac{dm}{dt} = \dot{m}_i - \dot{m}_e$. The net heat and work transfer across the control volume boundaries are represented as $\dot{Q}_{cv,net\ in}$ and $\dot{W}_{cv,net\ out}$. In addition to $\dot{W}_{cv,net\ out}$, there are two more terms related to work, which are $\dot{W}_{flow,\ in}$ and $\dot{W}_{flow,\ out}$.

How does mass enter and leave the control volume? The fluid/liquid does not enter the control volume on its own. It has to be pushed. Some work has to be done on the control volume to make the fluid enter it (surroundings do work on the control volume by pushing the fluid in). Similarly, this fluid which has entered the control volume has to be pushed out, otherwise it is just going to sit there. Hence, the control volume has to do some work. Here, the control volume does work on the surroundings by pushing the fluid out. These work interactions are related to the flow work, $\dot{W}_{flow}$.

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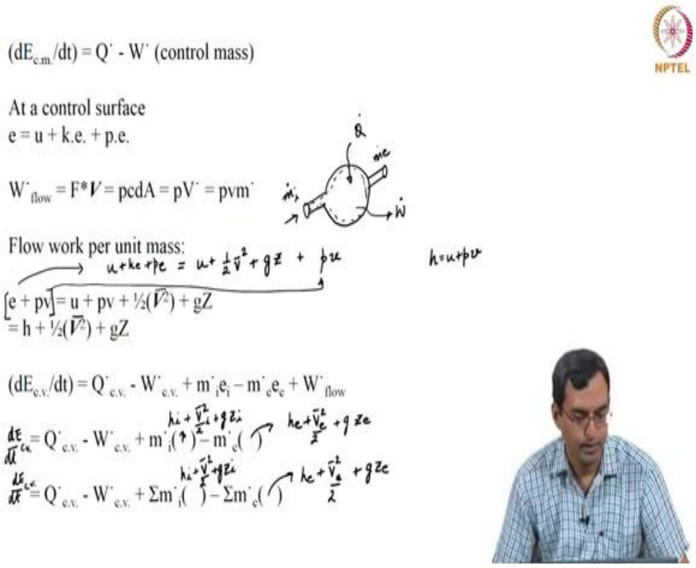

Figure 8.

The fluid entering the control volume brings with it internal energy (u), kinetic energy (k.e.) and potential energy (p.e.). Similarly, the fluid leaving the volume takes out with it the same three forms of energy. The specific energy of the fluid e = u+k.e.+p.e = $u + \frac{1}{2}\bar{V}^2 + gZ$, where Z is the height from some reference.

The rate of flow work is power, which is given as $\dot{W}_{flow} = F\bar{V}$, where F is the force with which the fluid is pushed and $\bar{V}$ is the fluid velocity. Now, $F = pA$, where p is the pressure and A is the area. Hence, $\dot{W}_{flow} = F\bar{V} = pA\bar{V} = p\dot{V} = p\frac{\dot{m}}{\rho} = pv\dot{m}$, where $\dot{V}$ is the volumetric flow rate, $\dot{m}$ is the mass flow rate, ρ is the fluid density and v is the specific volume of the fluid (the fluid is considered incompressible). The rate of flow work at the inlet is $\dot{W}_{flow,in} = pv\dot{m}_i$ and the rate of flow work at the outlet is $\dot{W}_{flow,out} = pv\dot{m}_e$. The flow work per unit mass is $W_{flow} = pv$. The rate of change of energy for the control volume ($\frac{dE}{dt}$) is the rate at which heat is transferred to the control volume ($\dot{Q}_{cv}$) minus the rate at which work is done by the control volume in some form ($\dot{W}_{cv}$), maybe as shaft work or as electrical work, etc. plus the mass flow rate of the fluid coming in ($\dot{m}_i$) into the energy which is being brought in by the incoming mass (e_i) minus the mass flow rate of the fluid going out ($\dot{m}_e$) into the energy which is being taken out by the outgoing mass (e_e) plus any flow work which is coming in and as well as going out ($\dot{W}_{flow}$).

$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i e_i - \dot{m}_e e_e + \dot{W}_{flow} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i e_i - \dot{m}_e e_e + p_i v_i \dot{m}_i - p_e v_e \dot{m}_e$

Substituting the expression for e and h=u+pv (h is enthalpy of the fluid) at the inlet and outlet,

$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\ 2} + gZ_i\right) - \dot{m}_e(h_e + \frac{1}{2}\bar{V}_e^{\ 2} + gZ_e)$

This is the first law for a flow process. For a control volume with multiple inlets and outlets, the above equation becomes,

$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{1}{2}\bar{V}_i^{\ 2} + gZ_i\right) - \sum \dot{m}_e \left(h_e + \frac{1}{2}\bar{V}_e^{\ 2} + gZ_e\right)$