Thermodynamics
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Lecture 52
Tutorial problems (1 numbers)

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Figure 1.

Solution of the problem in Fig. 1:

Compartment A (air): $m_A = 2\ kg, T_{A_1} = 298\ K, p_{A_1} = 1\ bar = 0.1\ MPa$

Compartment B (steam-water mixture): $m_B = 1\ kg, x_1 = 0.5, p_{B_1} = 1\ bar = 0.1\ MPa$

A schematic of the system is drawn in Fig. 1.

The total volume is conserved. Hence, $V_{A_1} + V_{B_1} = V_{A_2} + V_{B_2}$.....(1)

Air is considered as an ideal gas. Hence, $p_{A_1} V_{A_1}^{\gamma} = p_{A_2} V_{A_2}^{\gamma} = constant$......(2)

We can calculate V_{A_1} as $V_{A_1} = \dfrac{m_A R T_{A_1}}{p_{A_1}} = \dfrac{m_A \left(\bar{R}/M_{air}\right) T_{A_1}}{p_{A_1}} = 1.716\ m^3$ $(M_{air} = \sum x_i M_i = x_{N_2} M_{N_2} + x_{O_2} M_{O_2} = 0.79 \times 28 + 0.21 \times 32 = 28.84\ \dfrac{kg}{kmol}\ (it\ is\ an\ approximate\ calculation). Hence, R \approx 288\ \dfrac{J}{kg \cdot K})$

We need to find the initial volume for the compartment B, V_{B_1}.

For $p_{B_1} = 1\ bar$ and $x_1 = 0.5$, $v_{B_1} = v_f + x_1\left(v_g - v_f\right) = 0.8475\ \dfrac{m^3}{kg}$. Now, $V_{B_1} = v_{A_1} m_B = 0.8475\ m^3$.

(1) implies $V_{A_1} + V_{B_1} = V_{A_2} + V_{B_2} = 2.563\ m^3$......(3)

(2) implies $p_{A_1} V_{A_1}^\gamma = p_{A_2} V_{A_2}^\gamma = constant = 2.129 \times 10^5$ ($\gamma = 1.4$ for air assuming it to be a diatomic gas)............. (4)

(3) and (4) are always true.

Heat is added to the steam-water mixture until it reaches saturation. There is no heat transfer between A and B. The air in A undergoes adiabatic compression.

During the process, the pressure in A and B is always equal, $p_A = p_B$.

We don't have enough information to arrive at the answer. Hence, we will do it iteratively.

Assume that the final pressure in B, $p_{B_2} = 2\ bar$. As there is only vapor at state 2 (final state) in B, $V_{B_2} = v_g$ at 2 bar $\times m_B = 0.8857\ m^3$.

(3) implies $V_{A_2} = 1.6773\ m^4$. It should satisfy (4). Now, $p_{A_2} V_{A_2}^\gamma = 4.12 \times 10^5$. Hence, the guess $p_{B_2} = 2\ bar$ is not good. Let's make another guess.

Assume that the final pressure in B, $p_{B_2} = 0.14\ MPa$. Now, $V_{B_2} = v_g$ at 0.14 MPa $\times m_B = 1.237\ m^3$. (3) implies $V_{A_2} = 1.326\ m^3$. Now, $p_{A_2} V_{A_2}^\gamma = 2.07 \times 10^5$ which is close to the actual value 2.129×10^5 from (4).

Hence, $p_{A_2} = p_{B_2} = 0.14\ MPa$.

The temperature in B is the saturation temperature at 0.14 MPa which is $T_{B_2} = 109\ °C$.

The temperature in A can be calculated using the ideal gas relation.

$$T_{A_2} = \frac{p_{A_2} V_{A_2}}{m_A R_{air}} = 322\ K.$$

Students are encouraged to try out more guesses for p_{B_2} so that it satisfies the condition in (4) accurately.