

Thermodynamics
Professor Anand T N C
Department of Mechanical Engineering
Indian Institute of Technology Madras
Lecture 52
Tutorial problems (1 numbers)

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A rigid vessel is divided into two compartments A and B by a freely floating piston. Compartment A contains 2 kg of air at 1 bar, 25 °C, initially. Compartment B contains 1 kg of steam-water mixture at 1 bar and quality of 50%. Now, heat is added to the steam-water mixture until saturation. Assuming that there is no heat transfer between A and B, and that the piston is frictionless and adiabatic, calculate the final states of the air and steam, and the heat transfer. Assume that the air in chamber A undergoes adiabatic compression.

A: air
 $m_a = 2 \text{ kg}$
 $p_a = 1 \text{ bar}$
 $T_a = 25^\circ\text{C} = 298 \text{ K}$
 $V_a = \frac{mRT}{p} = 1.716 \text{ m}^3$
 $V_A + V_B = V_A + V_B = 2.563 \text{ m}^3$
 $\gamma = 1.4$
 $p_a V_a^\gamma = p_{2a} V_{2a}^\gamma = C = 2.129 \times 10^5$

B: steam/water
 $m_b = 1 \text{ kg}$
 $x = 0.5$
 $p_b = 1 \text{ bar}$
 $V_b = 0.8475 \text{ m}^3$
B - finally saturated steam
 $p_A = p_B$

Assume $p_2 = 2 \text{ bar}$
 $V_{2a} = m \times v_g @ 2 \text{ bar} = 0.8857 \text{ m}^3$
 $V_{2a} = 2.563 - 0.8857 = 1.6773 \text{ m}^3$
 $p_{2a} V_{2a}^\gamma = 4.12 \times 10^5$



Figure 1.

Assume $p_2 = 0.14 \text{ MPa}$
 $V_{2a} = m_a \times v_g @ 0.14 \text{ MPa}$
 $V_{2a} = 1 \times 1.237 = 1.237 \text{ m}^3$
 $V_{2a} = 2.563 - 1.237 = 1.326 \text{ m}^3$
 $p V^\gamma = 1.4 \times 10^5 \times (1.326)^\gamma = 2.07 \times 10^5$

Assume $p_2 = 0.16$
 $v_g = 0.00105 + x v_g = 1.091 \text{ m}^3/\text{kg}$
 $V_{2a} = 1 \times 1.091 = 1.091 \text{ m}^3$
 $V_{2a} = V_{2a} = 2.563 - 1.091$
 $V_{2a} = 1.472 \text{ m}^3$
 $p_2 V_{2a}^\gamma = 0.16 \times 10^6 \times (1.472)^\gamma$
 $p V^\gamma = 2.74 \times 10^5$

$p = 0.14 \text{ MPa}$
 $V_A = 1.326 \text{ m}^3$

steam $p = 0.14 \text{ MPa}$
 $V = 1.237 \text{ m}^3$
 $T = 104.3^\circ\text{C}$

Air $R = ?$
 $\frac{8314.5}{\text{MW}}$
 $\text{air} = 79\% \text{ N}_2$
 $21\% \text{ O}_2$
 $M_{\text{air}} = x M_1$
 $= 0.79 \times 28 + 0.21 \times 32$
 $M_{\text{air}} = 28.84$
 $R = \frac{R}{M} = \frac{288.5}{28.84} \text{ J/kgK}$

Steam
 $v_g = 0.001043 \text{ m}^3/\text{kg}$
 $v_f = 1.694 \text{ m}^3/\text{kg}$
 $v = v_f + x(v_g - v_f)$
 $V = v \times m$
 $V = 0.8475 \times 1 \text{ kg}$
 $V = 0.8475 \text{ m}^3$



Properties of Substances - Pressure Units											
Pressure	Temperature	sat. vapor (T _{sat})					sat. liquid (T _{sat})				
		MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa	MPa
0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2
5	5	5	5	5	5	5	5	5	5	5	5
10	10	10	10	10	10	10	10	10	10	10	10
20	20	20	20	20	20	20	20	20	20	20	20
50	50	50	50	50	50	50	50	50	50	50	50
100	100	100	100	100	100	100	100	100	100	100	100
200	200	200	200	200	200	200	200	200	200	200	200
500	500	500	500	500	500	500	500	500	500	500	500
1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000



Solution of the problem in Fig. 1:

Compartment A (air): $m_A = 2 \text{ kg}, T_{A1} = 298 \text{ K}, p_{A1} = 1 \text{ bar} = 0.1 \text{ MPa}$

Compartment B (steam-water mixture): $m_B = 1 \text{ kg}, x_1 = 0.5, p_{B1} = 1 \text{ bar} = 0.1 \text{ MPa}$

A schematic of the system is drawn in Fig. 1.

The total volume is conserved. Hence, $V_{A1} + V_{B1} = V_{A2} + V_{B2} \dots (1)$

Air is considered as an ideal gas. Hence, $p_{A1} V_{A1}^Y = p_{A2} V_{A2}^Y = \text{constant} \dots (2)$

We can calculate V_{A1} as $V_{A1} = \frac{m_A R T_{A1}}{p_{A1}} = \frac{m_A (\bar{R}/M_{air}) T_{A1}}{p_{A1}} = 1.716 \text{ m}^3$ ($M_{air} = \sum x_i M_i =$

$$x_{N_2} M_{N_2} + x_{O_2} M_{O_2} = 0.79 \times 28 + 0.21 \times 32 =$$

$$28.84 \frac{\text{kg}}{\text{kmol}} \text{ (it is an approximate calculation). Hence, } R \approx 288 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

We need to find the initial volume for the compartment B, V_{B1} .

For $p_{B1} = 1 \text{ bar}$ and $x_1 = 0.5$, $v_{B1} = v_f + x_1(v_g - v_f) = 0.8475 \frac{\text{m}^3}{\text{kg}}$. Now, $V_{B1} = v_{A1} m_B = 0.8475 \text{ m}^3$.

$$(1) \text{ implies } V_{A1} + V_{B1} = V_{A2} + V_{B2} = 2.563 \text{ m}^3 \dots (3)$$

(2) implies $p_{A_1} V_{A_1}^\gamma = p_{A_2} V_{A_2}^\gamma = \text{constant} = 2.129 \times 10^5$ ($\gamma = 1.4$ for air assuming it to be a diatomic gas)... .. (4)

(3) and (4) are always true.

Heat is added to the steam-water mixture until it reaches saturation. There is no heat transfer between A and B. The air in A undergoes adiabatic compression.

During the process, the pressure in A and B is always equal, $p_A = p_B$.

We don't have enough information to arrive at the answer. Hence, we will do it iteratively.

Assume that the final pressure in B, $p_{B_2} = 2 \text{ bar}$. As there is only vapor at state 2 (final state) in B, $V_{B_2} = v_g \text{ at } 2 \text{ bar} \times m_B = 0.8857 \text{ m}^3$.

(3) implies $V_{A_2} = 1.6773 \text{ m}^4$. It should satisfy (4). Now, $p_{A_2} V_{A_2}^\gamma = 4.12 \times 10^5$. Hence, the guess $p_{B_2} = 2 \text{ bar}$ is not good. Let's make another guess.

Assume that the final pressure in B, $p_{B_2} = 0.14 \text{ MPa}$. Now, $V_{B_2} = v_g \text{ at } 0.14 \text{ MPa} \times m_B = 1.237 \text{ m}^3$. (3) implies $V_{A_2} = 1.326 \text{ m}^3$. Now, $p_{A_2} V_{A_2}^\gamma = 2.07 \times 10^5$ which is close to the actual value 2.129×10^5 from (4).

Hence, $p_{A_2} = p_{B_2} = 0.14 \text{ MPa}$.

The temperature in B is the saturation temperature at 0.14 MPa which is $T_{B_2} = 109 \text{ }^\circ\text{C}$.

The temperature in A can be calculated using the ideal gas relation.

$$T_{A_2} = \frac{p_{A_2} V_{A_2}}{m_A R_{air}} = 322 \text{ K}.$$

Students are encouraged to try out more guesses for p_{B_2} so that it satisfies the condition in (4) accurately.