

Thermodynamics
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Lecture 41
Tutorial Problem - Part 4

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A piston-cylinder assembly contains a mixture of 1 kg of H₂ (C_v = 10352 J/kgK) and 2 kg of N₂. The mixture is compressed in a polytropic process which follows $pV^{1.2} = C$. The temperature increases from 22 degrees C to 150 degrees C. Calculate heat and work interaction. The piston-cylinder assembly is frictionless.

Handwritten notes:
 $m_{H_2} = 1 \text{ kg}$
 $m_{N_2} = 2 \text{ kg}$
 $T_1 = 22^\circ\text{C} = 22 + 273 = 295 \text{ K}$
 $T_2 = 150^\circ\text{C} = 150 + 273 = 423 \text{ K}$
 $pV^{1.2} = C$
 Q, W
 $dE = \delta Q - \delta W$
 $\Delta U = m_{N_2} - m_{H_2}$



Figure 1.

Solution of the problem in Fig. 1:

$$m_{H_2} = 1 \text{ kg}, m_{N_2} = 2 \text{ kg}, m_{mix} = 3 \text{ kg}, T_1 = 22^\circ\text{C} = 295 \text{ K}, T_2 = 150^\circ\text{C} = 423 \text{ K}$$

The polytropic process follows, $pV^{1.2} = \text{Constant}$.

The system here is the mixture of gases inside the piston-cylinder assembly. We assume that the mixture is an ideal gas.

We know the expression for the work done by the system in a polytropic process.

$$W = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{m_{mix}R_{mix}(T_1 - T_2)}{n-1} \quad (\text{for an ideal gas, } p_1V_1 = mRT_1, p_2V_2 = mRT_2)$$

We need to find R_{mix} .

$$\text{Now, } R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{\bar{R}}{1/\sum y_i/M_i} = \frac{\bar{R}}{1/\left(\frac{y_{H_2}}{M_{H_2}} + \frac{y_{N_2}}{M_{N_2}}\right)} = \frac{8314.5 \frac{J}{kmol \cdot K}}{1/\left(\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{28}\right) \frac{kg}{kmol}} = 1583.7 \frac{J}{kg \cdot K}$$

$$\text{Now, } W = \frac{3 \times 1583.7 \times (295 - 423)}{1.2 - 1} = -3 \times 10^6 J$$

The work interaction is negative. Work is done on the system.

The first law in the integrated form for a process, $\Delta U = Q - W$ (there are no changes in kinetic and potential energy of the system). To calculate Q, we need to calculate ΔU .

$$\Delta U = m_{mix} C_{v,mix} (T_2 - T_1)$$

There are different ways to find out $C_{v,mix}$; $C_{v,mix} = \frac{R_{mix}}{\gamma - 1}$ or $C_{v,mix} = \sum y_i C_{v_i}$.

Since the mixture is made of diatomic gases, $\gamma = 1.4$ for the mixture. Hence, $C_{v,mix} = \frac{R_{mix}}{\gamma - 1} = \frac{1583.7}{0.4} = 3959 \frac{J}{kg \cdot K}$.

$C_{v,mix} = \sum y_i C_{v_i}$ should give the exact value if we consider sufficient digits after the decimal point in all the calculations.

$$\text{Now, } \Delta U = m_{mix} C_{v,mix} (T_2 - T_1) = 3 \times 3959 \times (423 - 295) = 1.52 MJ$$

The first law implies $Q = \Delta U + W = 1.52 \times 10^6 - 3 \times 10^6 = -1.5 \times 10^6 J$.

The heat interaction is also negative. It means heat is leaving the system. However, the change in internal energy is positive. It means the internal energy of the system increased after the process. Since, for an ideal gas, internal energy is only a function of temperature, the temperature has also increased after the process.

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$$pV^\gamma = c$$

$$W_2 = \int p dV = \int \frac{c}{V^{\gamma+1}} dV$$

$$W_2 = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

$$W_2 = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$W_2 = \frac{3 \times 1583.7 \times (295 - 423)}{0.2}$$

$$W_2 = -3 \times 10^6 \text{ J}$$

$$\Delta U = m \times C_{v, \text{mix}} \times (T_2 - T_1)$$

$$\gamma = 1.4$$

$$C_v = \frac{R_{\text{mix}}}{\gamma - 1}$$

$$pV^\gamma = c$$

$$\frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$p_1 V_1 = mR T_1$$

$$p_2 V_2 = mR T_2$$

$$m = 3 \text{ kg}$$

$$R_{\text{mix}} = \frac{R}{M_{\text{mix}}}$$

$$M_{\text{mix}} = \frac{1}{\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{28}}$$

$$M_{\text{mix}} = 5.25 \text{ kg/kmol}$$

$$R_{\text{mix}} = \frac{8.314}{5.25} = 1583.7 \text{ J/kgK}$$




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$$pV^{1.2} = c$$

$$Q, W$$

$$dE = \delta Q - \delta W$$

$$\Delta U = Q_2 - W_2$$




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$$C_v = \frac{R_{min}}{\gamma - 1} = \frac{1588.7}{0.4}$$

$$C_{vmin} = 3959 \text{ J/kgK}$$

$$C_{v,H_2} = 10352 \text{ J/kgK}$$

$$C_{v,N_2} = \frac{R_{N_2}}{\gamma - 1} = \frac{8314.5}{2.8 - 1}$$

$$C_{v,N_2} = 7423.6 \frac{\text{J}}{\text{kgK}}$$

$$C_{vmin} = \sum y_i C_{v_i}$$

$$= \frac{1}{3} \times 10352 + \frac{2}{3} \times 7424$$

$$C_{vmin} = 3946 \frac{\text{J}}{\text{kgK}}$$



$$pV^\gamma = c$$

$$W_2 = \int p dV$$

$$= \int \frac{c}{V^\gamma} dV$$

$$W_2 = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

$$W_2 = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{3 \times 1588.7 \times (295 - 423)}{0.2}$$

$$W_2 = -3 \times 10^6 \text{ J}$$

$$\Delta U = m \times C_{vmin} \times (T_2 - T_1) \quad \gamma = 1.4$$

$$= 3 \times 3959 \times (423 - 295) \frac{\text{J}}{\text{K}}$$

$$\Delta U = 152 \text{ MJ}$$

$$Q_2 = \Delta U + W_2 = -1.5 \times 10^6 \text{ J}$$

$$pV^\gamma = c$$

$$\frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$p_1 V_1 = mRT_1$$

$$p_2 V_2 = mRT_2$$

$$m = 3 \text{ kg}$$

$$R_{min} = \frac{R}{M_{min}}$$

$$M_{min} = \frac{1}{\sum \frac{y_i}{M_i}}$$

$$y_{H_2} = \frac{1}{3}$$

$$y_{N_2} = \frac{2}{3}$$

$$M_{min} = \frac{1}{\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{28}}$$

$$M_{min} = 5.25 \text{ kg/kmol}$$

$$R_{min} = \frac{R}{5.25} = 1588.7 \frac{\text{J}}{\text{kgK}}$$



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 \end{aligned}$$

$$pV^{1.2} = C \quad Q, W$$

$$dE = \delta Q - \delta W$$

$$\Delta U = 1Q_2 - 1W_2$$

