

Thermodynamics
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Lecture 40
Tutorial Problem - Part 3

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A rigid container contains 1.2 kg of Argon (Molar mass = 40, $C_v = 392 \text{ J/kgK}$) at 300 K and 101 kPa. Another similar tank contains 1 kg of O₂ (Molar mass = 32, $C_v = 696 \text{ J/kg K}$) at 400 K, 500 kPa. The tanks are connected by a pipe with a valve. The valve is opened and the gases are allowed to mix until they reach an equilibrium temperature of 360 K. Determine (a) the volume of each tank (b) final pressure and (c) the heat interaction. Use ideal gas model.



Figure 1.

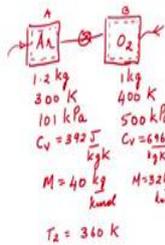


V
 R_2
 Q

$$V_{A1} = \frac{m \bar{R} T}{p M} = \frac{1.2 \times 8314.5 \times 300}{101 \times 10^3 \times 40} = 0.74 \text{ m}^3$$

$$V_{O2} = \frac{1 \times 8314.5 \times 400}{500 \times 10^3 \times 32} = 0.207 \text{ m}^3$$

$$V_B = V_{O2} = 0.207 \text{ m}^3$$



$$m_2 = 1.2 + 1 = 2.2 \text{ kg}$$

$$V_2 = 0.947 \text{ m}^3$$

$$m_2 = 0$$

$$T_2 = 360 \text{ K}$$

$$p_2 = \frac{2.2 \times 8314.5 \times 360}{35.95 \times 0.947} = 193.4 \text{ kPa}$$

$$pV = mRT$$

$$R_2 = \frac{\bar{R}}{M_{\text{mix}}}; M_{\text{mix}} = \frac{1}{\frac{y_1}{M_1} + \frac{y_2}{M_2}}$$

$$y_{O2} = \frac{m_{O2}}{m_T} = \frac{1}{2.2} = 0.454$$

$$y_{Ar} = \frac{m_{Ar}}{m_T} = 1 - y_{O2} = 0.545$$

$$M_{\text{mix}} = \frac{1}{\frac{0.454}{32} + \frac{0.545}{40}} = 35.95 \frac{\text{kg}}{\text{kmol}}$$

First Law

$$dU = \delta Q - \delta W$$

$$\Delta U = Q - W$$

$$\Delta U = Q$$



$$\begin{aligned}
 Q_2 = \Delta U &= m_{Ar} \Delta U_{Ar} + m_{O_2} \Delta U_{O_2} \\
 &= m_{Ar} \times C_{v,Ar} \times (T_2 - T_1)_{Ar} + m_{O_2} \times C_{v,O_2} \times (T_2 - T_1)_{O_2} \\
 &= 1.2 \times 392 \times (360 - 300) + 1 \times 696 \times (400 - 360) \\
 \Delta U &= 56 \text{ kJ}
 \end{aligned}$$



Solution of the problem in Fig. 1:

$m_{Ar} = 1.2 \text{ kg}, m_{O_2} = 1 \text{ kg}, T_{1Ar} = 300 \text{ K}, T_{1O_2} = 400 \text{ K}, p_{1Ar} = 101 \text{ kPa}, p_{1O_2} = 500 \text{ kPa}, C_{vAr} = 392 \frac{\text{J}}{\text{kg}\cdot\text{K}}, C_{vO_2} = 696 \frac{\text{J}}{\text{kg}\cdot\text{K}}, M_{Ar} = 40 \frac{\text{kg}}{\text{kmol}}, M_{O_2} = 32 \frac{\text{kg}}{\text{kmol}}, T_f = T_2 = 360 \text{ K}.$

Both the gases can be treated as ideal gases. Hence, they follow ideal gas relationship. We are asked to calculate volume of each tank.

$$\text{So, } V_{Ar} = \frac{m_{Ar} R T_{1Ar}}{p_{1Ar}} = \frac{m_{Ar} (\bar{R}/M_{Ar}) T_{1Ar}}{p_{1Ar}} = 0.74 \text{ m}^3 = \text{volume of the tank containing Argon}$$

Similarly, $V_{O_2} = 0.207 \text{ m}^3 = \text{volume of the tank containing Oxygen}$

After mixing, $m_{mix} = m_{Ar} + m_{O_2} = 2.2 \text{ kg}, V_{mix} = V_{Ar} + V_{O_2} = 0.947 \text{ m}^3, T_2 = T_{mix} = 360 \text{ K}$

The mixture is also an ideal gas. Hence, $p_{mix} V_{mix} = m_{mix} R_{mix} T_{mix}.$

$$\text{Now, } R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{\bar{R}}{\sum \frac{y_i}{M_i}} = \frac{\bar{R}}{\frac{m_{Ar}}{m_{mix}} + \frac{m_{O_2}}{m_{mix}}} = \frac{8314.5 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{35.95 \frac{\text{kg}}{\text{kmol}}} = 231.3 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad [M_{mix} = 1/\text{summation}(y_i/M_i)]$$

$$\text{Therefore, } p_{mix} = \frac{m_{mix} R_{mix} T_{mix}}{V_{mix}} = 193.4 \text{ kPa}$$

The work interaction for this process of mixing is 0, i.e., $W=0$ (there is no shaft work, electrical work, magnetic work, surface tension work, etc.).

The first law in the integrated form, $\Delta U = Q - W$ (there are no changes in kinetic and potential energy).

$$\text{Since } W=0, \quad Q = \Delta U = m_{Ar}\Delta u_{Ar} + m_{O_2}\Delta u_{O_2} = m_{Ar}C_{vAr}(T_2 - T_{1Ar}) + m_{O_2}C_{vO_2}(T_2 - T_{1O_2}) = 1.2 \times 392 \times (360 - 300) + 1 \times 696 \times (360 - 400) = 384 \text{ J}$$

Q is positive. Hence, there is a flow of heat into the system (both the tanks considered together) during the process of mixing and the internal energy increases.