Thermodynamics
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Lecture 04
Tutorial problems on exact and inexact differential

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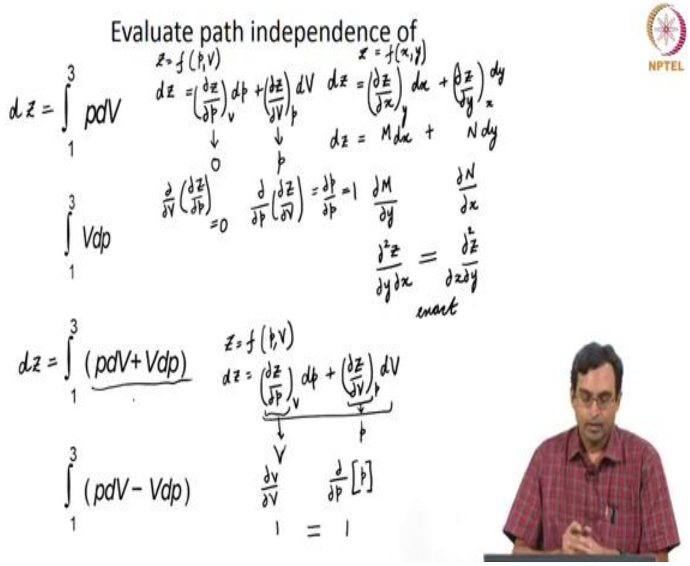

Figure 1

If we have z as a function of x and y, we should be able to write it in the form $dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$. $\left(\frac{\partial z}{\partial x}\right)_y$ represents derivative of z with respect to x keeping y constant (partial derivative of z with respect to x) and $\left(\frac{\partial z}{\partial y}\right)_x$ represents derivative of z with respect to y keeping x constant (partial derivative of z with respect to y). How do we know that dz is an exact differential or inexact differential?

Let's say we have $M = \left(\frac{\partial z}{\partial x}\right)_y$ and $N = \left(\frac{\partial z}{\partial y}\right)_x$. Now, differentiate M with respect to y and N with respect x. Therefore, $\frac{\partial M}{\partial y} = \frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial y} = \frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial N}{\partial x} = \frac{\partial\left(\frac{\partial z}{\partial y}\right)}{\partial x} = \frac{\partial^2 z}{\partial x \partial y}$. Now, compare $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$. If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, dz is an exact differential. However, if they are not equal, dz is an inexact differential.

Physically, exact differentials can be used to represent points on a graph. They represent properties or state functions. Inexact differentials can be used to represent path functions.

We want to know if $\int_1^3 pdV$ is an exact or inexact differential.

Let's write it as $dz = \int_1^3 pdV$. Here, z is a function of two variables p and V.

Hence, $dz = \left(\frac{\partial z}{\partial p}\right)_V dp + \left(\frac{\partial z}{\partial V}\right)_p dV$. Let's write dz as $dz = Mdp + NdV$. Here, M=0 as there is no dp term in the expression and N=p. Apply the same strategy as before.

$\frac{\partial M}{\partial V} = \frac{\partial\left(\frac{\partial z}{\partial p}\right)}{\partial V} = 0$ and $\frac{\partial N}{\partial p} = \frac{\partial\left(\frac{\partial z}{\partial V}\right)}{\partial p} = \frac{\partial p}{\partial p} = 1$.

Hence, $\frac{\partial M}{\partial V} \neq \frac{\partial N}{\partial p}$. So, dz is not exact in this case. Hence, it is a path function.

We encourage you to do a similar exercise with the expression $\int_1^3 Vdp$. This is also an inexact differential.

We will see if the third expression $\int_1^3 pdV + Vdp$ is an exact or inexact differential. Let's follow the same steps. Here again, z is a function of two variable p and V. So, $dz = \left(\frac{\partial z}{\partial p}\right)_V dp + \left(\frac{\partial z}{\partial V}\right)_p dV$.

Therefore, $M = \left(\frac{\partial z}{\partial p}\right)_V = V$ and $N = \left(\frac{\partial z}{\partial V}\right)_p = p$. Now, $\frac{\partial M}{\partial V} = \frac{\partial V}{\partial V} = 1$ and $\frac{\partial N}{\partial p} = \frac{\partial p}{\partial p} = 1$. Hence, $\frac{\partial M}{\partial V} = \frac{\partial N}{\partial p} = 1$. Therefore, dz is an exact differential. z represents a property or a state function. The value of pdV+Vdp does not depend on the path followed by a system. It depends only on the state of the system. In this case, the integral depends only on the values of pdV+Vdp at states 1 and 3. It does not depend on how you go from 1 to 3.

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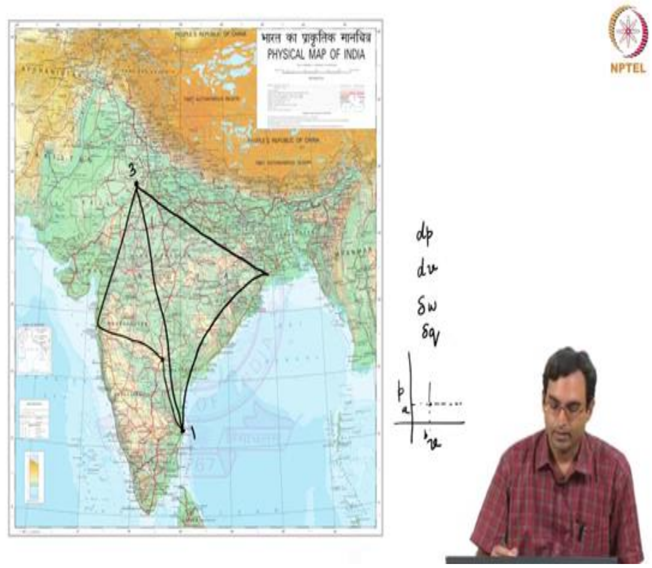

Figure 2: The map of India

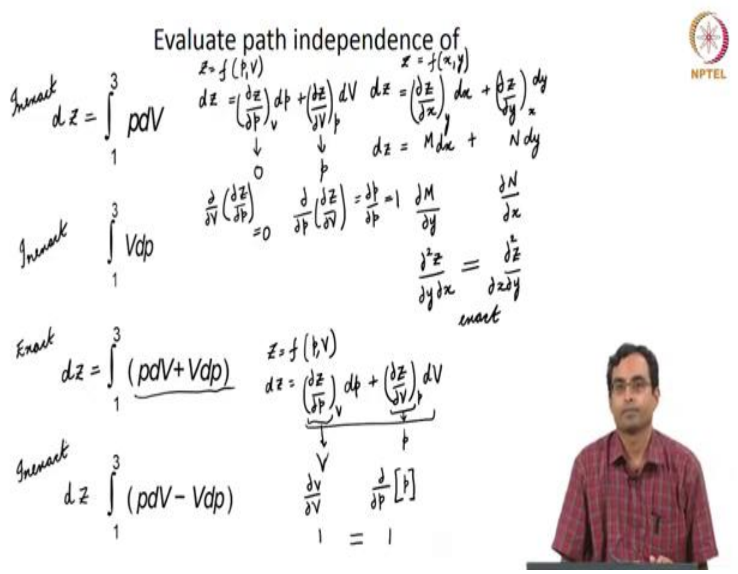

Figure 3

If we were to look at the map of India again (Fig. 2), the difference in pressure and density at two different locations at a particular time does not depend on the path followed in going from one location to another because density and pressure are state functions.

I request you to find out if the expression $\int_1^3 pdV - Vdp$ represent an exact or inexact differential. This differential is inexact.

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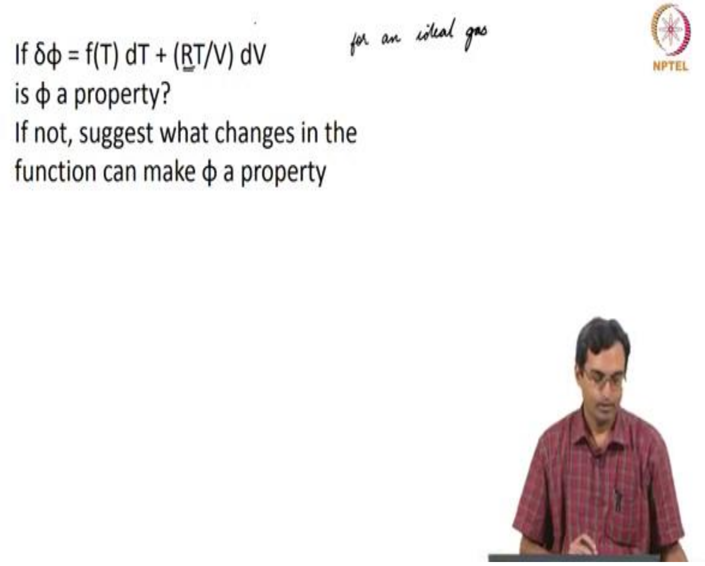

Figure 4

In Fig. 4, we have a function $\delta\phi = f(T)dT + \left(\frac{RT}{V}\right)dV$. Let's say this expression was written for an ideal gas. We want to find out whether ϕ is a property or not. If it is not a property, make changes in the expression such that ϕ becomes a property. Here, R is a constant (gas constant). Apply the same procedure as we applied in the earlier problems. There are many ways ϕ could be made a property.