

Thermodynamics
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Lecture 39
Tutorial Problem – Part 2

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Q1 Consider an ideal gas mixture of 1 kmol of hydrogen and 1 kmol of nitrogen. Find the mass fractions and mole fractions of hydrogen and nitrogen. Also find the molecular weight, specific gas constant R and the specific heats C_p and C_v of the mixture. If this mixture is heated from an initial temperature of 300 K to the final temperature of 400 K at constant pressure, find: Δu , Δh , Q and W for the process.



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Q1 Consider an ideal gas mixture of 1 kmol of hydrogen and 1 kmol of nitrogen. Find the mass fractions and mole fractions of hydrogen and nitrogen. Also find the molecular weight, specific gas constant R and the specific heats C_p and C_v of the mixture. If this mixture is heated from an initial temperature of 300 K to the final temperature of 400 K at constant pressure, find: Δu , Δh , Q and W for the process.



Figure 1.

Solution of the problem in Fig. 1:

$$n_{H_2} = 1 \text{ kmol}, n_{N_2} = 1 \text{ kmol}, M_{H_2} = 2 \frac{\text{kg}}{\text{kmol}}, M_{N_2} = 28 \frac{\text{kg}}{\text{kmol}}, \gamma = 1.4 \text{ for a diatomic gas.}$$

Now, $x_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{N_2}} = \frac{1}{2} = 0.5$ and $x_{N_2} = \frac{n_{N_2}}{n_{H_2} + n_{N_2}} = 0.5$

Molecular weight of the mixture, $M_{mix} = \sum x_i M_i = 0.5 \times 2 + 0.5 \times 28 = 15 \frac{kg}{kmol}$

$n_{H_2} = 1 \text{ kmol}$
 $n_{N_2} = 1 \text{ kmol}$
 x_i, y_i, R_i, C_i, C_v

$$x_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{N_2}} = \frac{1}{1+1} = 0.5$$

$$x_{N_2} = 1 - x_{H_2} = 0.5$$

$$M_{mix} = \sum x_i M_i = 0.5 \times 2 + 0.5 \times 28$$

$$= 1 + 14$$

$$M_{mix} = 15 \text{ kg/kmol}$$

$$y_{H_2} = \frac{x_{H_2} \times M_{H_2}}{M_{mix}} = \frac{0.5 \times 2}{15} = 0.066$$

$$y_{N_2} = 1 - y_{H_2} = 0.934$$

$$R_{mix} = \sum y_i R_i$$

$$R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{8314.5}{15}$$

$$R_{mix} = 554.3 \frac{J}{kg \cdot K}$$

$$R_i = \frac{\bar{R}}{M_i}$$

$$\frac{J}{kg \cdot K}$$



Mass fractions, $y_{H_2} = \frac{x_{H_2} M_{H_2}}{M_{mix}} = 0.066$ and $y_{N_2} = 1 - 0.066 = 0.934$

We can find R_{mix} using different ways, $R_{mix} = \sum y_i R_i$ or $R_{mix} = \frac{\bar{R}}{M_{mix}}$

We use the second option, $R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{8314.5 \frac{kJ}{kmol \cdot K}}{15 \frac{kg}{kmol}} = 554.3 \frac{J}{kg \cdot K}$

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$n_{H_2} = 1 \text{ kmol}$
 $n_{N_2} = 1 \text{ kmol}$
 x_i, y_i, R_i, C_i, C_v

$$x_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{N_2}} = \frac{1}{1+1} = 0.5$$

$$x_{N_2} = 1 - x_{H_2} = 0.5$$

$$M_{mix} = \sum x_i M_i = 0.5 \times 2 + 0.5 \times 28$$

$$= 1 + 14$$

$$M_{mix} = 15 \text{ kg/kmol}$$

$$y_{H_2} = \frac{x_{H_2} \times M_{H_2}}{M_{mix}} = \frac{0.5 \times 2}{15} = 0.066$$

$$y_{N_2} = 1 - y_{H_2} = 0.934$$

$$R_{mix} = \sum y_i R_i$$

$$R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{8314.5}{15}$$

$$R_{mix} = 554.3 \frac{J}{kg \cdot K}$$

$$\bar{R}_{mix} = \sum x_i \bar{R}_i = \bar{R}$$

$$\bar{R}_i = \frac{\bar{R}}{M_i}$$

$$\frac{J}{kg \cdot K}$$



$C_p, C_v = ?$
 $C_p, C_v \rightarrow \frac{J}{kg \cdot K}$ $\bar{C}_p, \bar{C}_v \rightarrow \frac{J}{kmol \cdot K}$
 $C_{p,mix} = \sum x_i C_{p,i} = 0.066 \times 14550 + 0.934 \times 1040$
 $\bar{C}_{p,mix} = \sum x_i \bar{C}_{p,i}$
 $= 0.066 \times 29100 \frac{J}{kmol \cdot K} + 0.934 \times 29100 \frac{J}{kmol \cdot K}$
 $\bar{C}_{p,mix} = 29100 \frac{J}{kmol \cdot K}$
 $C_{v,mix} = \frac{C_{p,mix}}{M_{mix}} = \frac{29100}{15} = 1940 \frac{J}{kg \cdot K}$
 $C_{v,mix} = C_p - R = 1940 - 554.3 = 1385.7$
 $\gamma_{mix} = \frac{C_{p,mix}}{C_{v,mix}} = \frac{29100}{1940} = 1.4949 \approx 1.4$
 $p = C$ $T_1 = 300 \text{ K}$ $T_2 = 400 \text{ K}$
 $Q = m C_p \Delta T = m \Delta h$
 $\Delta U = m \Delta u = m C_v \Delta T$
 $W = p \Delta V = Q - \Delta U$

$C_p = \frac{\gamma R}{\gamma - 1}$; $\bar{C}_p = \frac{\gamma \bar{R}}{\gamma - 1}$
 $C_v = \frac{R}{\gamma - 1}$; $\bar{C}_v = \frac{\bar{R}}{\gamma - 1}$
 $R = \frac{\bar{R}}{M}$
 $C_{p,H_2} = \frac{1.4 \times 8314.5}{2}$
 $R_{H_2} = \frac{14550 (1.4 - 1)}{2}$
 $R_{H_2} = 4157.25 \frac{J}{kg \cdot K}$
 $C_v = C_p - R$
 $= 10393 \frac{J}{kg \cdot K}$
 $R_{N_2} = 297 \frac{J}{kg \cdot K}$
 $C_{v,N_2} = \frac{R}{\gamma - 1} = 742.5 \frac{J}{kg \cdot K}$
 $C_p = C_v + R$
 $= 1040 \frac{J}{kg \cdot K}$

Q1 Consider an ideal gas mixture of 1 kmol of hydrogen and 1 kmol of nitrogen. Find the mass fractions and mole fractions of hydrogen and nitrogen. Also find the molecular weight, specific gas constant R and the specific heats C_p and C_v of the mixture. If this mixture is heated from an initial temperature of 300 K to the final temperature of 400 K at constant pressure, find: Δu , Δh , Q and W for the process.

We need to find C_p and C_v of the mixture. Let's also find \bar{C}_p and \bar{C}_v of the mixture.

$$\text{Now, } C_{p,H_2} = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \left(\frac{\bar{R}}{M_{H_2}} \right)}{1.4 - 1} = \frac{1.4 \times 4157.25}{0.4} = 14550 \frac{J}{kg \cdot K} \text{ and } \bar{C}_{p,H_2} = \frac{\gamma \bar{R}}{\gamma - 1} = \frac{1.4 \times 8314.5}{1.4 - 1} = 29100 \frac{J}{kmol \cdot K}$$

$$\text{Now, } C_{v,H_2} = C_{p,H_2} - R = \frac{R}{\gamma - 1} = 10393 \frac{J}{kg \cdot K} \text{ and } \bar{C}_{v,H_2} = \bar{C}_{p,H_2} - \bar{R} = \frac{\bar{R}}{\gamma - 1} = 20786 \frac{J}{kmol \cdot K}$$

$$\text{In the similar fashion, } C_{p,N_2} = 1040 \frac{J}{kg \cdot K}, C_{v,N_2} = 742.5 \frac{J}{kg \cdot K}, \bar{C}_{p,N_2} = 29100 \frac{J}{kmol \cdot K}, \bar{C}_{v,N_2} = 20786 \frac{J}{kmol \cdot K} \text{ } (\gamma = 1.4 \text{ for both the gases})$$

Now, $C_{p,mix} = \sum y_i C_{p,i} = 0.066 \times 14550 + 0.934 \times 1040 = 1931 \frac{J}{kg \cdot K}$.

$\bar{C}_{p,mix} = \sum x_i \bar{C}_{p,i} = 0.5 \times 29100 + 0.5 \times 29100 = 29100 \frac{J}{kmol \cdot K}$ ($C_{p,mix}$ can also be calculated as $C_{p,mix} = \frac{\bar{C}_{p,mix}}{M_{mix}}$)

Now, $C_{v,mix} = C_{p,mix} - R_{mix} = 1385.7 \frac{J}{kg \cdot K}$

$\bar{C}_{v,mix} = \bar{C}_{p,mix} - \bar{R} = 29100 - 8314.5 = 20786 \frac{J}{kmol \cdot K}$

$\gamma = \frac{C_{p,mix}}{C_{v,mix}} = \frac{\bar{C}_{p,mix}}{\bar{C}_{v,mix}} = 1.4$

There are various ways to calculate $C_{p,mix}$ and $C_{v,mix}$. You get the same values through all the ways if you consider sufficient digits after decimal point.

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The image shows handwritten calculations for the specific heat capacities and gamma of a gas mixture. The calculations are as follows:

- $C_p, C_v = ?$
- $C_p, C_v \rightarrow \frac{J}{kg \cdot K}$
- $\bar{C}_p, \bar{C}_v \rightarrow \frac{J}{kmol \cdot K}$
- $C_{p,mix} = \sum y_i C_{p,i} = 0.066 \times 14550 + 0.934 \times 1040 = 1931 \frac{J}{kg \cdot K}$
- $\bar{C}_{p,mix} = \sum x_i \bar{C}_{p,i} = 0.5 \times 29100 + 0.5 \times 29100 = 29100 \frac{J}{kmol \cdot K}$
- $C_{v,mix} = C_{p,mix} - R_{mix} = 1931 - 554.3 = 1385.7 \frac{J}{kg \cdot K}$
- $\bar{C}_{v,mix} = \bar{C}_{p,mix} - \bar{R} = 29100 - 8314.5 = 20786 \frac{J}{kmol \cdot K}$
- $\gamma_{mix} = \frac{C_{p,mix}}{C_{v,mix}} = \frac{\bar{C}_{p,mix}}{\bar{C}_{v,mix}} = 1.3999 \approx 1.4$
- Process: $p=C$, $T_1=300K$, $T_2=400K$
- $Q = m C_{p,mix} \Delta T = m \Delta h = 30 \times 1931 \times 100 = 5.8 \text{ MJ}$
- $\Delta U = m \Delta u = m C_{v,mix} \Delta T = 30 \times 1385.7 \times 100 = 4.1 \text{ MJ}$
- $W = p \Delta V = Q - \Delta U = 1.7 \text{ MJ}$
- General formulas: $C_p = \frac{\gamma R}{\gamma - 1}$; $\bar{C}_p = \frac{\gamma \bar{R}}{\gamma - 1}$
- $C_v = \frac{R}{\gamma - 1}$; $\bar{C}_v = \frac{\bar{R}}{\gamma - 1}$
- $R = \frac{\bar{R}}{M}$
- $C_{p,H_2} = 1.4 \times 8314.5 = 11640.3 \frac{J}{kg \cdot K}$
- $R_{H_2} = \frac{8314.5}{2} = 4157.25 \frac{J}{kg \cdot K}$
- $C_{v,H_2} = C_{p,H_2} - R_{H_2} = 11640.3 - 4157.25 = 7483.05 \frac{J}{kg \cdot K}$
- $R_{N_2} = \frac{8314.5}{28} = 297 \frac{J}{kg \cdot K}$
- $C_{p,N_2} = \frac{R}{\gamma - 1} = 748.5 \frac{J}{kg \cdot K}$
- $C_{v,N_2} = C_{p,N_2} - R_{N_2} = 748.5 - 297 = 451.5 \frac{J}{kg \cdot K}$





$$n_{H_2} = 1 \text{ kmol}$$
$$n_{N_2} = 1 \text{ kmol}$$
$$m_{\text{mix}} = 30 \text{ kg}$$

$$M = 2 \text{ kg/kmol}$$
$$M = 28 \text{ kg/kmol}$$

$$m_{H_2} = 2 \text{ kg}$$
$$m_{N_2} = 28 \text{ kg}$$



We are also asked to calculate Δu , Δh , Q and W .

It is a constant pressure process. $T_1 = 300 \text{ K}$, $T_2 = 400 \text{ K}$.

$$Q = m\Delta h = mC_p\Delta T = 30 \times 1940 \times (T_2 - T_1) = 5.8 \text{ MJ} \quad (m = n_{H_2}M_{H_2} + n_{N_2}M_{N_2} = 2 + 28 = 30 \text{ kg})$$

$$\Delta U = m\Delta u = mC_v\Delta T = 30 \times 1385 \times 100 = 4.1 \text{ MJ}$$

According to the first law in the integrated form, $W = Q - \Delta U = 1.7 \text{ MJ}$.