Thermodynamics
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Lecture 37
Ideal gas - Part 5

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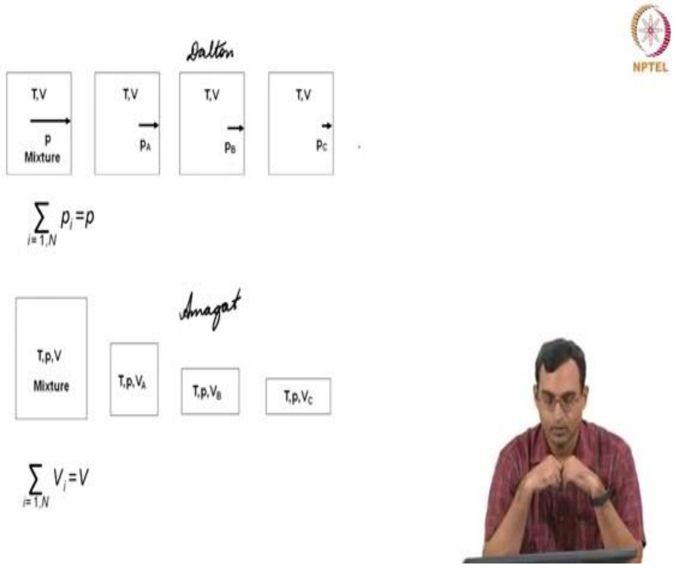

Figure 1.

Let's look at the Dalton's law of partial pressures.

Consider a mixture of gases A, B and C occupying a volume V_m at temperature T_m. According to Dalton's law of partial pressures, the pressure exerted by this mixture equals the sum of partial pressures of the gases A, B and C. Partial pressure of the gas A is the pressure exerted by A when it occupies volume V_m individually at the mixture temperature T_m. In the similar way, partial pressures of B and C are calculated. Hence, according to Dalton's law, the pressure exerted by mixture of A, B and C when it occupies volume V_m is $p = p_A + p_B + p_C$ where, p_A, p_B, p_C represent partial pressures of A, B and C.

Similarly, we have Amagat's law of partial volumes.

According to this law, the total volume of the mixture of gases at constant temperature and pressure is equal to the sum of individual partial volumes of constituent gases. At the mixture pressure and temperature, the constituent gases occupy different volumes individually. Hence, for a mixture of gases A, B and C at constant temperature T_m and pressure p_m, the total volume $V = V_A + V_B + V_C$, where V_A, V_B, V_C are the volumes these gases occupy individually at the mixture temperature T_m and pressure p_m.

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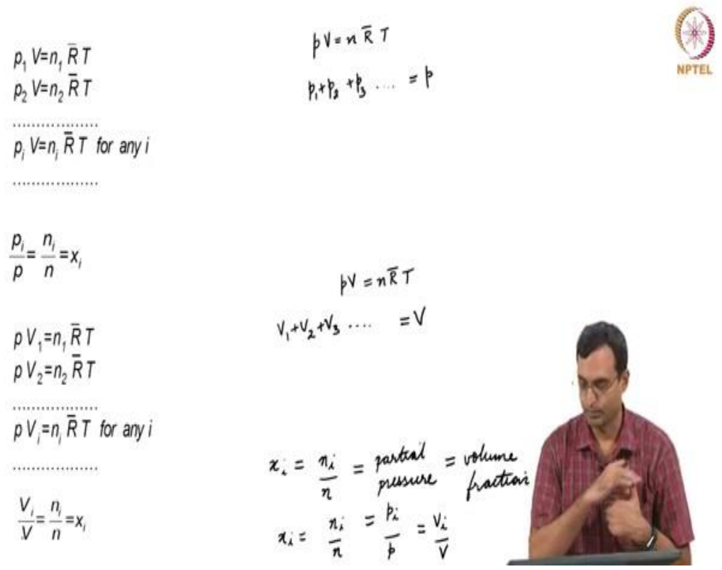

Consider a mixture of N ideal gases which is at temperature T_m and occupies volume V_m. $n_1, n_2, \dots . n_N$ represent the number of moles of those gases and the total number of moles of the mixture $n = n_1 + n_2 + \cdots + n_N$. Since these are ideal gases, they satisfy ideal gas equation $pV = n\bar{R}T$. So, we can write, for individual gases, $p_1V_m = n_1\bar{R}T_m$, $p_2V_m = n_2\bar{R}T_m, \dots \dots p_iV_m = n_i\bar{R}T_m$. According to Dalton's law of partial pressures, the pressure of the mixture $p_m = p_1 + p_2 + \cdots + p_i + \cdots p_N$. The mixture also satisfies the ideal gas equation. Hence, for the mixture, $p_mV_m = n\bar{R}_mT_m$. If we take the ratio of the pressure of the ith gas to the pressure of the mixture, $\frac{p_i}{p_m} = \frac{\frac{n_i\bar{R}T_m}{V_m}}{\frac{n\bar{R}T_m}{V_m}} = \frac{n_i}{n} = x_i$=mole fraction of the ith gas.

Consider again a mixture of N ideal gases which is at pressure p_m and temperature T_m. So, we can write, $p_mV_1 = n_1\bar{R}T_m$, $p_mV_2 = n_2\bar{R}T_m \dots \dots p_mV_i = n_i\bar{R}T_m$. For the mixture, $p_mV = n\bar{R}T_m$, where $V = V_1 + V_2 + \cdots + V_N$ (according to Amagat's law). If we take the ratio of the volume of the ith gas to the volume of the mixture, $\frac{V_i}{V} = \frac{\frac{n_i\bar{R}T_m}{p_m}}{\frac{n\bar{R}T_m}{p_m}} = \frac{n_i}{n} = x_i$= mole fraction of the ith gas.

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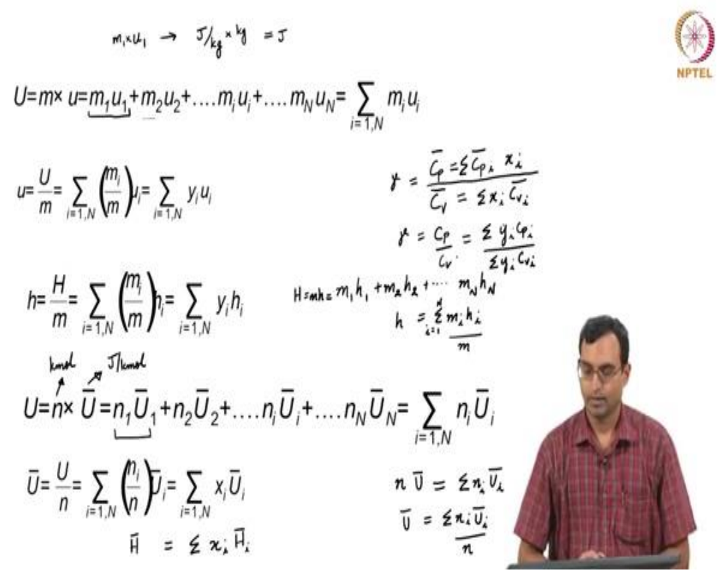

We need to find the properties of a mixture of ideal gases such as energy, enthalpy, etc.

Consider a mixture of N ideal gases. Let $n_1, n_2, \ldots . n_N$ and $m_1, m_2, \ldots . m_N$ represent the number of moles and the masses of individual gases. The mass of the mixture $m = m_1 + m_2 + \cdots + m_N$ and total number of moles of the mixture $n = n_1 + n_2 + \cdots + n_N$. Similarly, $u_1, u_2, \ldots . u_N$ represent specific internal energy of individual gases. Now, the internal energy of the mixture $U = mu = m_1 u_1 + m_2 u_2 + \cdots + m_i u_i + \cdots m_N u_N = \sum_{i=1,N} m_i u_i$. The specific internal energy of the mixture $u = \frac{U}{m} = \frac{\sum_{i=1,N} m_i u_i}{m} = \sum_{i=1,N} y_i u_i$, where y_i is the mass fraction of the ith gas. We can write similar expressions for the enthalpy (H) and the specific enthalpy (h) of the mixture. So, $H = mh = m_1 h_1 + m_2 h_2 + \cdots + m_i h_i + \cdots m_N h_N = \sum_{i=1,N} m_i h_i$ and $h = \frac{H}{m} = \frac{\sum_{i=1,N} m_i h_i}{m} = \sum_{i=1,N} y_i h_i$.

The internal energy of the mixture can also be written as $U = n\bar{U} = n_1 \bar{U}_1 + n_2 \bar{U}_2 + \cdots + n_i \bar{U}_i + \cdots + n_N \bar{U}_N = \sum_{i=1,N} n_i \bar{U}_i$, where $\bar{U}$ represents the molar specific internal energy (its unit is J/kmol) of the mixture and $\bar{U}_1, \bar{U}_2, \ldots, \bar{U}_N$ represent molar specific internal energy of individual gases. Now, $\bar{U} = \frac{U}{n} = \frac{\sum_{i=1,N} n_i \bar{U}_i}{n} = \sum_{i=1,N} x_i \bar{U}_i$, where x_i is the mole fraction of the ith gas. Similarly, molar specific enthalpy of the mixture $\bar{H} = \sum_{i=1,N} x_i \bar{H}_i$.

In the similar fashion, for the mixture, $C_p = \sum_{i=1,N} y_i C_{p_i}$ (the unit of C_p is J/kg·K) and $\bar{C}_p = \sum_{i=1,N} x_i \bar{C}_{p_i}$ (the unit of $\bar{C}_p$ is J/kmol·K). Similarly, we can write the expressions for C_v and $\bar{C}_v$. The ratio of specific heats for a mixture $\gamma = \frac{C_p}{C_v} = \frac{\bar{C}_p}{\bar{C}_v}$.