

Thermodynamics
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Lecture 36
Ideal gas - Part 4

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- Mixtures of ideal gases

Air N_2
 O_2

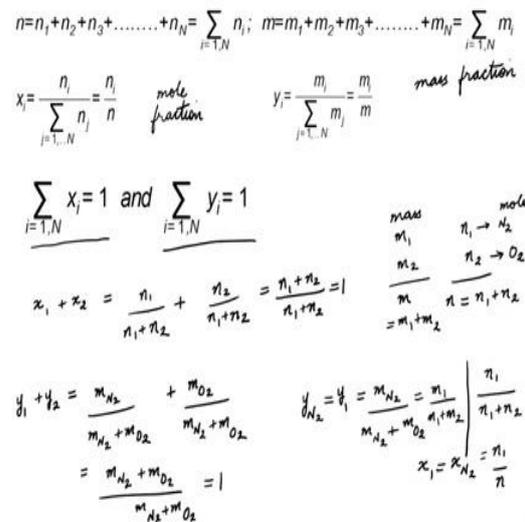
$N_2 + O_2$
 m_1 m_2



Let's look at the mixture of ideal gases.

Take the example of air. It is a mixture of nitrogen, oxygen, and other gases. Under certain conditions, these gases can be treated as ideal gases. Does a mixture of ideal gases behave like an ideal gas? Can we find its properties?

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$n = n_1 + n_2 + n_3 + \dots + n_N = \sum_{i=1, N} n_i$; $m = m_1 + m_2 + m_3 + \dots + m_N = \sum_{i=1, N} m_i$
 $x_i = \frac{n_i}{\sum_{j=1, N} n_j} = \frac{n_i}{n}$ mole fraction; $y_i = \frac{m_i}{\sum_{j=1, N} m_j} = \frac{m_i}{m}$ mass fraction
 $\sum_{i=1, N} x_i = 1$ and $\sum_{i=1, N} y_i = 1$
 $x_1 + x_2 = \frac{n_1}{n_1 + n_2} + \frac{n_2}{n_1 + n_2} = \frac{n_1 + n_2}{n_1 + n_2} = 1$
 $y_1 + y_2 = \frac{m_{N_2}}{m_{N_2} + m_{O_2}} + \frac{m_{O_2}}{m_{N_2} + m_{O_2}} = \frac{m_{N_2} + m_{O_2}}{m_{N_2} + m_{O_2}} = 1$
 $x_1 = \frac{n_1}{n_1 + n_2}$



Consider a system where we have a mixture of N gases. m_1, m_2, \dots, m_N and n_1, n_2, \dots, n_N represent mass and number of moles of each gas.

Now, total mass of the mixture $m = m_1 + m_2 + m_3 + \dots + m_i + \dots + m_N = \sum_{i=1, N} m_i$, where m_i represents the mass of the i^{th} species. Similarly, total number of moles $n = n_1 + n_2 + n_3 + \dots + n_i + \dots + n_N = \sum_{i=1, N} n_i$, where n_i represents the number of moles of the i^{th} species.

Mole fraction of a particular gas in the mixture is the ratio of the number of moles of that particular gas to the total number of moles in the mixture. Mathematically, mole fraction of the i^{th} species, $x_i = \frac{n_i}{n} = \frac{n_i}{\sum_{i=1, N} n_i}$. Similarly, mass fraction of a particular gas in the mixture is the ratio of the mass of that particular gas to the total mass of the mixture. Mathematically, the mass fraction of the i^{th} species, $y_i = \frac{m_i}{m} = \frac{m_i}{\sum_{i=1, N} m_i}$.

Also, $\sum_{i=1, N} x_i = 1$ and $\sum_{i=1, N} y_i = 1$. Check the validity of these two expressions in the case of a mixture of two gases.

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$$m_1 = n_1 \times M_1; m_2 = n_2 \times M_2; \dots \dots m_i = n_i \times M_i; \dots \dots m_N = n_N \times M_N$$

$$m = \sum_{i=1,N} m_i = \sum_{i=1,N} n_i M_i$$

$$y_i = \frac{m_i}{m} = \frac{x_i M_i}{\sum_{i=1,N} x_i M_i} \quad \sum_{i=1,N} x_i M_i = M$$

$M_i = \text{mol. wt. } M_i \text{ (kg/kmol)}$

$m_i = n_i \times M_i \text{ kg}$

$$\frac{m_i}{n} = \frac{n_i M_i}{n} \quad \text{--- (1) } \quad y_i = \frac{m_i}{m} = \frac{x_i M_i}{\sum x_i M_i}$$

$$\frac{m}{n} = \frac{\sum n_i M_i}{n} \quad \text{--- (2)}$$



We can convert number of moles of a gas to its mass by multiplying number of moles by molecular weight (M kg/kmol) of that particular gas. Mathematically, $m_1 = n_1 \times M_1$; $m_2 = n_2 \times M_2$; $m_i = n_i \times M_i$ $m_N = n_N \times M_N$. Now, $m = m_1 + m_2 + m_3 + \dots m_i + \dots m_N = n_1 \times M_1 + n_2 \times M_2 + \dots + n_i \times M_i + \dots n_N \times M_N = \sum_{i=1,N} n_i M_i$.

The mass fraction of the i^{th} species, $y_i = \frac{m_i}{m} = \frac{n_i M_i}{\sum_{i=1,N} n_i M_i}$. Dividing the numerator and the denominator by total number of moles n , $y_i = \frac{m_i}{m} = \frac{n_i M_i}{\sum_{i=1,N} n_i M_i} = \frac{x_i M_i}{\sum_{i=1,N} x_i M_i}$ (as n is constant, it can be taken inside the summation sign in the denominator).

The molecular weight of the mixture $M = \sum_{i=1,N} x_i M_i$.

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$$n_1 = \frac{y_1}{M_1}; n_2 = \frac{y_2}{M_2}; \dots \dots n_i = \frac{y_i}{M_i}; \dots \dots n_N = \frac{y_N}{M_N}$$

$$n = \sum_{i=1,N} n_i = \sum_{i=1,N} \frac{y_i}{M_i}$$

$y \rightarrow \text{mass fraction}$
 $x \rightarrow \text{mole fraction}$

$$x_i = \frac{n_i}{n} = \left(\frac{y_i}{M_i} \right) \left(\sum_{j=1,N} \frac{y_j}{M_j} \right)^{-1}$$



Let's find a way to convert mole fraction into mass fraction.

Consider a mixture of gases which has a mass of 1 kg. In this case, the mass of each gas in the mixture equals its mass fraction, i.e., $m_1 = y_1, m_2 = y_2, \dots, m_N = y_N$. We know that the number of moles of a particular gas equals its mass divided by its molecular weight, i.e., $n_1 = \frac{m_1}{M_1}, n_2 = \frac{m_2}{M_2}, \dots, n_N = \frac{m_N}{M_N}$. In the case of the mixture considered above (whose total

mass is 1 kg), $n_1 = \frac{m_1}{M_1} = \frac{y_1}{M_1}, n_2 = \frac{m_2}{M_2} = \frac{y_2}{M_2}, \dots, n_N = \frac{m_N}{M_N} = \frac{y_N}{M_N}$. Now, the total number of moles of the mixture $n = n_1 + n_2 + n_3 + \dots + n_N = \frac{y_1}{M_1} + \frac{y_2}{M_2} + \dots + \frac{y_N}{M_N} = \sum_{i=1, N} \frac{y_i}{M_i}$. Hence,

the mole fraction of the i^{th} species is $x_i = \frac{n_i}{n} = \frac{\frac{y_i}{M_i}}{\sum_{j=1, N} \frac{y_j}{M_j}}$. Here, the molecular weight of the

mixture would be, $M = \frac{1}{\sum_{i=1, N} \frac{y_i}{M_i}}$ as the total mass of the mixture is 1 kg.