

**Thermodynamics**  
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**Lecture - 30**  
**Tutorial Problem - Part 4**

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There are two tanks containing CO<sub>2</sub>. The tanks are connected by a pipe with a valve. One of the tanks contains 1 kg of CO<sub>2</sub> at 70 °C and 0.5 bar. The other tank contains 4 kg of CO<sub>2</sub> at 20 °C and 1 bar. Tanks and the pipe are not insulated. Now, the valve is opened and the gases are allowed to mix. The final steady state temperature is 40 °C. Calculate the final equilibrium pressure and the heat transfer during the process. Use ideal gas model.  $\gamma=1.3$ .

Figure 1.

$T_{2A} = T_{2B} = 40^\circ\text{C}$   
 $p_{2A} = p_{2B} = \text{---}?$   
 $\gamma = 1.3$   
 $\text{CO}_2 = \text{ideal gas}$      $pV = mRT$   
 $\Delta E = \Delta U$   
 $W = 0$   
 $dE = dU = \delta Q - \delta W$   
 $Q = \Delta U$   
 $dU = m C_v dT$   
 $\Delta U \Big|_2 = [m_A C_v T_2 + m_B C_v T_2] - [m_A C_v T_1 + m_B C_v T_1]$   
 $= 5 \times 630 \times 313 - [1 \times 630 \times 343 + 4 \times 630 \times 293]$   
 $= 630 [5 \times 313 - 343 - 4 \times 293]$   
 $\Delta U = 31500 \text{ J} = 31.5 \text{ kJ}$   
 $Q = \Delta U = 31.5 \text{ kJ}$

$\text{CO}_2$      $1 \text{ kg}$      $4 \text{ kg}$   
 $70^\circ\text{C} = 343 \text{ K}$      $20^\circ\text{C} = 293 \text{ K}$   
 $0.5 \text{ bar}$      $1 \text{ bar}$   
 $R = \text{sp. gas const.} = \frac{\bar{R}}{M} = \frac{8314 \text{ J}}{44 \text{ kg/mol}} = 189 \frac{\text{J}}{\text{kgK}}$   
 $C_v = \frac{R}{\gamma - 1} = \frac{189}{1.3 - 1} = 630 \frac{\text{J}}{\text{kgK}}$   
 $C_p = \gamma R = 1.3 \times 189 = 245.7 \frac{\text{J}}{\text{kgK}}$

Figure 2.

**Solution of the problem given Fig. 1:**

Tank A:

$$m_A = 1 \text{ kg}, T_{1A} = 70 \text{ }^\circ\text{C}, p_{1A} = 0.5 \text{ bar}$$

Tank B:

$$m_B = 4 \text{ kg}, T_{1B} = 20 \text{ }^\circ\text{C}, p_{1B} = 1 \text{ bar}$$

After the mixing is over, the gas achieves an equilibrium state. The final temperature is  $T_f = T_{2A} = T_{2B} = 40^\circ\text{C}$ . Also,  $\gamma = 1.3$ .

We are asked to calculate the heat transfer during the process.

Consider both the tanks as our system. During the process, there is no change in kinetic and potential energy of the system. There is no shaft work, electric work, magnetic work or surface tension work. The transfer of gas within the system does not qualify as work interaction. There is no deformation of volume. Hence, for the process, work interaction  $W = 0$ .

The first law for the system in the integrated form is,

$$\Delta U = Q - W. \text{ Since } W = 0, \Delta U = Q.$$

For an ideal gas,  $Q = \Delta U = mC_v\Delta T = \text{Internal energy at the final state} - \text{internal energy at the initial state} \dots\dots(1)$

We know the masses at the initial state as well as the final state. At the final state, the mass is,  $m_f = m_A + m_B = 1 + 4 = 5 \text{ kg}$ .

To calculate  $C_v$ , we need to know specific gas constant for  $CO_2$  and the specific heat ratio  $\gamma$  as we are using the formula  $C_v = \frac{R}{\gamma-1}$ .

$$R = \frac{\bar{R}}{M} = \frac{8314.5 \frac{J}{\text{kmol}\cdot\text{K}}}{44 \frac{\text{kg}}{\text{kmol}}} = 189 \frac{J}{\text{kg}\cdot\text{K}}.$$

$$\text{Hence, } C_v = 630 \frac{J}{\text{kg}\cdot\text{K}}.$$

We know the temperatures at the initial and final states.

(1) implies,

$$\begin{aligned} Q = \Delta U = mC_v\Delta T &= [(m_A + m_B)C_vT_f] - [m_A C_v T_{1A} + m_B C_v T_{1B}] \\ &= [5 \times 630 \times (40 + 273)] \\ &\quad - [1 \times 630 \times (70 + 273) + 4 \times 630 \times (20 + 273)] = 31.5 \text{ kJ}. \end{aligned}$$

Since  $Q$  is positive, heat is entering the system.

For calculating final equilibrium pressure  $p_f$ , we use the ideal gas equation,  $p_f V = m_f R T_f$ . We need to know the total volume to calculate  $p_f$ .

Total volume  $V$  is the sum of volumes of tank A and B.

$$V_A = \frac{m_A R T_{1A}}{p_{1A}} = \frac{1 \times 189 \times 343}{0.5 \times 10^5} = 1.3 \text{ m}^3$$

Similarly,  $V_B = 2.2 \text{ m}^3$ .

Therefore,  $V = V_A + V_B = 3.5 \text{ m}^3$

$$\text{Now, } p_f = \frac{5 \times 189 \times 313}{3.5} = 84510 \text{ Pa} = 0.84 \text{ bar}$$