

**Thermodynamics**  
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**Lecture - 29**  
**Tutorial Problem - Part 3**

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Q5 A piston-cylinder system contains one kg of air. It is expanded from a specific volume  $v = 0.2 \text{ m}^3/\text{kg}$  and a temperature of 580 K to a specific volume  $v = 0.8 \text{ m}^3/\text{kg}$  and a temperature of 290 K. The expansion process follows  $pv^{1.5}=0.75$  (with  $p$  in bar, and  $v$  in  $\text{m}^3/\text{kg}$ ). Determine the work and heat interaction. The specific heat at constant volume is 0.718 kJ/kg.K.

*Initial state -1*  
 $m = 1 \text{ kg}$   
 $v_1 = 0.2 \text{ m}^3/\text{kg}$   
 $T_1 = 580 \text{ K}$

*final state 2*  
 $v_2 = 0.8 \text{ m}^3/\text{kg}$   
 $T_2 = 290 \text{ K}$

*process*  
 $pv^{1.5} = 0.75$   
 $C_v = 0.718 \text{ kJ/kgK}$

Figure 1.

$pv^{1.5} = c$   
 $pv^{1.5} = 0.75$

$W = \int p dV$   
 $= \int p d(mv)$   
 $W = m \int p dv$   
 $= m \int_1^2 \frac{0.75}{v^{1.5}} dv$

$W = 0.75 \times m \times \left[ \frac{p_1 v_1 - p_2 v_2}{1.5 - 1} \right]$

$p_1 v_1^{1.5} = 0.75 = p_2 v_2^{1.5}$   
 $p_1 = \frac{0.75}{v_1^{1.5}} = \frac{0.75}{(0.2)^{1.5}} = 8.38 \text{ bar}$

$p_2 = \frac{0.75}{v_2^{1.5}} = \frac{0.75}{(0.8)^{1.5}} = 1.04 \text{ bar}$

$W = 0.75 \times 1 \times \left[ \frac{8.38 \times 10^5 \text{ Pa} \times 0.2 \frac{\text{m}^3}{\text{kg}} - 1.04 \times 10^5 \times 0.8}{0.5} \right]$   
 $W = 0.422 \times 10^5 \times 0.75 = 31.65 \times 10^3 \text{ J}$

$V = m \times v$   
 $dV = m dv$

$m C_v \frac{dv}{v} = m C_v \frac{dv}{-n+1}$

**Solution of the problem given in Fig. 1:**

Initial state:

$$m = 1 \text{ kg}, v_1 = 0.2 \frac{\text{m}^3}{\text{kg}}, T_1 = 580 \text{ K}$$

Final state:

$$v_2 = 0.8 \frac{\text{m}^3}{\text{kg}}, T_2 = 290 \text{ K}$$

It is also given that  $pv^{1.5} = 0.75$  and  $C_v$  is  $0.718 \text{ kJ/kg}\cdot\text{K}$ .

The system is undergoing a polytropic process. Assuming the system to be a simple compressible system, the work done by the system is,

$$W = \int pdV = \int pd(mv) = m \int pdv = m \int \frac{0.75}{v^{1.5}} dv \quad [d(mv) = vdm + m dv, \quad m \text{ is constant, i.e., } dm=0, \text{ hence, } d(mv) = m dv]$$

We have integrated this before.

$$W = 0.75m \left( \frac{p_1 v_1 - p_2 v_2}{1.5-1} \right) \dots\dots\dots(1)$$

We need  $p_1$  and  $p_2$ .

$$p_1 v_1^{1.5} = 0.75 = p_2 v_2^{1.5}$$

$$\text{Hence, } p_1 = \frac{0.75}{v_1^{1.5}} = 8.38 \text{ bar.}$$

$$\text{Similarly, } p_2 = 1.04 \text{ bar.}$$

Substituting the values of  $p_1$ ,  $p_2$  and  $m$  in (1) and using SI units for all the quantities,

$$W = 31.65 \times 10^3 \text{ J}$$

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**First law of thermodynamics**

$dE = \delta Q - \delta W$   
 $dU = \delta Q - \delta W$   
 $mC_v dT = \delta Q - \delta W$   
 $\delta Q = mC_v dT + \delta W$   
 ${}_{1R_2} = mC_v \Delta T + {}_{1N_2}$   
 $= 1 \times 718 \frac{J}{kg \cdot K} \times (290 - 580) + 3165 \int_1^2 dT$   
 ${}_{1R_2} = -208 \text{ kJ} + 3165 \text{ kJ}$   
 ${}_{1R_2} = 1976.35 \text{ kJ}$   
 Heat lost = 1976.3 kJ

$E = U + KE + PE$   
 $dE = dU + dKE + dPE$

${}_{1R_2} +ve$   
 ${}_{1N_2} +ve$

**Q5** A piston-cylinder system contains one kg of air. It is expanded from a specific volume  $v = 0.2 \text{ m}^3/\text{kg}$  and a temperature of 580 K to a specific volume  $v = 0.8 \text{ m}^3/\text{kg}$  and a temperature of 290 K. The expansion process follows  $pv^{1.5} = 0.75$  (with  $p$  in bar, and  $v$  in  $\text{m}^3/\text{kg}$ ). Determine the work and heat interaction. The specific heat at constant volume is  $0.718 \text{ kJ/kg}\cdot\text{K}$ .

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final state 2  
 $v_2 = 0.8 \text{ m}^3/\text{kg}$   
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process  
 $pv^{1.5} = 0.75$   
 $C_v = 0.718 \text{ kJ/kg}\cdot\text{K}$

We are also asked to calculate the heat interaction.

The first law of thermodynamics for the given system is,

$dU = \delta Q - \delta W$  (There are no changes in kinetic and potential energy. Hence,  $dE$  can be replaced by  $dU$ )

Hence,  $mC_v dT = \delta Q - \delta W \rightarrow \delta Q = mC_v dT + \delta W$

Integrating,

$$Q = mC_v\Delta T + W = 1 \times 718 \times (290 - 580) + 31.65 \times 10^3 = -176.35 \text{ kJ}$$

The heat interaction is negative for the system. Hence, the system is losing heat.

The work interaction is positive for the system. Hence, the system is doing work.

Here, the system is losing energy by giving out heat and doing work.

$p v^n = c$   
 $p v^{1.5} = 0.75$   
 $W = \int p dV$   
 $= \int p d(mv)$   
 $W_2 = m \int_1^2 p dv$   
 $= m \int_1^2 \frac{0.75}{v^{1.5}} dv$   
 $W_2 = 0.75 \times m \times \left[ \frac{p_1 v_1 - p_2 v_2}{1.5 - 1} \right]$   
 $p_1 v_1^{1.5} = 0.75 = p_2 v_2^{1.5}$   
 $p_1 = \frac{0.75}{v_1^{1.5}} = \frac{0.75}{(0.2)^{1.5}} = 8.38 \text{ bar}$   
 $p_2 = \frac{0.75}{v_2^{1.5}} = \frac{0.75}{(0.8)^{1.5}} = 1.04 \text{ bar}$   
 $W_2 = 0.75 \times m \times \left[ \frac{8.38 \times 10^5 \text{ Pa} \times 0.2 \frac{\text{m}^3}{\text{kg}} - 1.04 \times 10^5}{0.5} \right]$   
 $W_2 = 0.422 \times 10^3 \times 0.75 = 31.65 \times 10^3 \text{ J}$