

Thermodynamics
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Lecture - 22
Tutorial problem – Part 2

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Q2 A frictionless vertical piston-cylinder assembly fitted with an electrical resistor contains air. The mass and the face area of the piston are 50 kg and 0.1 m². The atmospheric pressure is 100 kPa. Electric current is passed through the resistor causing an increase of 0.05 m³ in the volume of air. The mass of the air is 0.4 kg. Because of the heating, its specific energy increases by 43.3 kJ/kg. Assume the assembly (including the piston) is insulated and $g = 9.8 \text{ m/s}^2$. Determine the heat transfer from the resistor to the system, for a system consisting of a) the air alone, b) the air and the piston.

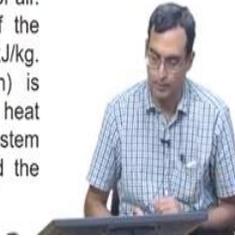


Figure 1.

$m_p = 50 \text{ kg}$
 $A_p = 0.1 \text{ m}^2$
 $\Delta V = 0.05 \text{ m}^3$
 $m_{\text{air}} = 0.4 \text{ kg}$
 $\Delta u = \Delta e = 43.3 \text{ kJ/kg}$
 $g = 9.8 \text{ m/s}^2$

$F_p = m_p g$

Isobaric process
 $p = c$

a) \underline{Q}
 $dE = \delta Q - \delta W$
 $\Delta U = \Delta E = Q - W$
 $\Delta U = m \Delta u = 0.4 \times 43.3 = 17.32 \text{ kJ}$
 $W = \int p dV = p \Delta V = p(V_2 - V_1) = 104.9 \times 0.05 = 5.24 \text{ kJ}$
 $Q = \Delta E + W = 22.56 \text{ kJ}$

b) $\underline{Q} = 22.56 \text{ kJ}$
 $\Delta PE_{\text{piston}} = m_p g \Delta h$
 $\Delta u = 43.3 \text{ kJ/kg}$

$p_{\text{atm}} = 100 \text{ kPa}$

$\frac{m_p g}{A} + p_{\text{atm}} = p_{\text{air}}$
 $\frac{50 \times 9.8}{0.1} + 100 \text{ kPa} = p_{\text{air}}$
 $p_{\text{air}} = 104.9 \text{ kPa}$

$\Delta E =$



Figure 2

Solution of the problem in Fig. 1:

$m_p = 50 \text{ kg}$, $A_p = 0.1 \text{ m}^2$, $\Delta e = \Delta u = 43.3 \frac{\text{kJ}}{\text{kg}}$ (ignoring the potential and kinetic energy changes in air), $p_{atm} = 100 \text{ kPa}$, $g = 9.8 \frac{\text{m}}{\text{s}^2}$, $\Delta V = 0.05 \text{ m}^3$

Case a: the system consists of the air only.

The schematic of the system is drawn in top right hand corner of Fig. 2.

The system is expanding against the constant weight of the piston and constant atmospheric pressure quasi-statically (assume that the process is quasi-static). Hence, it is a constant pressure process. The piston is frictionless.

Hence, pressure of the air inside the system, $p_{air} = \frac{m_p g}{A_p} + p_{atm} = 104.9 \text{ kPa}$

Work done, $W = \int p_{air} dV = p_{air} \int dV = p_{air} \Delta V = 5.24 \text{ kJ}$

$\Delta U = m_{air} \Delta u = 17.32 \text{ kJ}$

First law for a process, $dE = \delta Q - \delta W$. Here, $dU = \delta Q - \delta W$.

Integrating, $Q = \Delta U + W = 22.56 \text{ kJ}$

Case b: the system consists of the air and the piston

Now, the piston is internal to the system. Since the entire assembly including the piston is insulated, there is no heat loss to the surroundings. Hence, in this case also, the heat transfer from the resistor to the system would be $Q = 22.56 \text{ kJ}$.

Let's check if it comes out to be the same in this case.

In this case, work done would be against the constant atmospheric pressure.

Hence, $W = p_{atm} \int dV = 5 \text{ kJ}$

The first law for a process in the integrated form, $\Delta U + \Delta PE = Q - W$. Here, the potential energy of the system changes as the piston is part of the system and it is raised against gravity.

Hence, $\Delta PE = m_p g \Delta h$.

We don't know Δh . But we know change in volume of air, ΔV .

Now, $\Delta V = A_p \Delta h \rightarrow \Delta h = 0.5 \text{ m} \rightarrow \Delta PE = 245 \text{ J}$

Hence, $Q = W + \Delta U + \Delta PE = 5 + 17.32 + 0.245 = 22.57 \text{ kJ}$

Based on the choice of the system, heat transferred, work done and change in energy can change.

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$$\begin{aligned} 0.05 \text{ m}^3 &= \frac{\pi d^2}{4} \times h \\ 0.05 &= A_p \times h \\ 0.05 &= 0.1 \times h \\ h &= \frac{0.05}{0.1} = 0.5 \text{ m} \\ \Delta PE &= m_p g \Delta h = 50 \text{ kg} \times 9.8 \text{ m/s}^2 \times 0.5 \text{ m} \\ &= 245 \text{ J} \\ \Delta U_{\text{air}} &= 17.32 \text{ kJ} \\ W &= 5 \text{ kJ} \\ Q &= \Delta E + W \\ Q &= 22.5 \text{ kJ} \end{aligned}$$



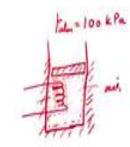


$$m_p = 50 \text{ kg} \quad F_p = m_p g$$

$$A_p = 0.1 \text{ m}^2$$

$$\Delta V = 0.05 \text{ m}^3$$

$$m_{\text{air}} = 0.4 \text{ kg}$$



$$\Delta u = \Delta e = 43.3 \text{ kJ/kg}$$

$$g = 9.8 \text{ m/s}^2$$

Isobaric process
 $p = c$

a) $Q =$

$$dE = \delta Q - \delta W$$

$$\Delta U = \Delta E = Q - W$$

$$\Delta U = m \Delta u = 0.4 \times 43.3 = 17.32 \text{ kJ}$$

$$W = \int p dV = p \Delta V = p(V_2 - V_1) = 104.9 \times 0.05 = 5.24 \text{ kJ}$$

$$Q = \Delta E + W = 22.56 \text{ kJ}$$

b) $Q = 22.56 \text{ kJ}$

$$\Delta PE_{\text{pot}} = m g \Delta h \quad \Delta U = 43.3 \text{ kJ/kg}$$

$$W = p_{\text{atm}} \Delta V = 5 \text{ kJ}$$

$$\frac{m_p g}{A} + p_{\text{atm}} = p_{\text{air}}$$

$$\frac{50 \times 9.8}{0.1} + 100 \text{ kPa} = p_{\text{air}}$$

$$p_{\text{air}} = 104.9 \text{ kPa}$$

