

Thermodynamics
Professor. Anand T N C
Department of Mechanical Engineering
Indian Institute of Technology, Madras
Lecture 14
Tutorial Problem on 'Work' – Part 4

(Refer Slide Time: 00:18)



Q4 A spherical water droplet of diameter 1 mm is subjected to a constant aerodynamic force for a short time interval during which the droplet deforms into a shape which looks like a disk of diameter 2 mm. The droplet just changes the shape but does not move. Once the force is cut off, the droplet regains the spherical shape. Calculate the work done by the aerodynamic force. Neglect viscous forces.



Figure 1.

Solution of the problem in Fig. 1:

A spherical droplet changes shape from a sphere to a disklike structure because of the external aerodynamics force.

Volume of the spherical droplet, $V_{sphere} = \frac{\pi}{6} d_1^3 = \frac{\pi}{6} (1 \times 10^{-3})^3 = 0.523 \times 10^{-9} m^3$

Volume is conserved (liquids are incompressible).

$V_{disk} = \frac{\pi}{4} d_2^2 h = V_{sphere}$ (h is thickness of the disk)

Hence, $h = 0.16 \text{ mm}$

Here, the surface area of the droplet increased against the force of surface tension. We need to calculate the change in surface area as the expression for work interaction due to surface tension is $\sigma \Delta A$, where σ is surface tension and ΔA is the change in surface area.

Surface area of sphere, $A_{sphere} = 4\pi r_1^2 = \pi d_1^2 = 3.14 \times 10^{-6} m^2$

Surface area of disk-shaped droplet, $A_{disk} = 2\pi r_2^2 + \pi dh = 7.28 \times 10^{-6} m^2$

$$\Delta A = A_{disk} - A_{sphere} = 4.14 \times 10^{-6} m^2$$

Work interaction for the aerodynamic force is positive, whereas it is negative for the droplet.

$$W_{aerodynamic\ force} = \sigma \Delta A = 0.298 \mu J = -W_{droplet} \quad (\sigma \text{ for water} = 0.072 \text{ N/m})$$

(Refer Slide Time: 06:32)

$A_1 = 4\pi r_1^2 = \pi d_1^2 = \pi \times 1 \times 10^{-6}$
 $A_1 = 3.14 \times 10^{-6} m^2$

$A_2 = 2\pi r_2^2 + \pi dh$
 $= 2\pi (1 \times 10^{-3})^2 + \pi \times 2 \times 10^{-3} \times 0.16 \times 10^{-3}$
 $d_2 = 2 \text{ mm}$
 $r_2 = \frac{d_2}{2} = 1 \text{ mm}$

$= [2\pi + 2 \times 0.16\pi] \times 10^{-6}$
 $A_2 = 7.28 \times 10^{-6} m^2$
 $(A_2 - A_1) = 4.14 \times 10^{-6} m^2$

Surface tension = $\sigma \Delta A$
 $W_{aero} = \sigma \Delta A$
 $= 0.072 \text{ N/m} \times 4.14 \times 10^{-6} m^2$
 $= 0.298 \times 10^{-6} \text{ Nm}$
 $= 0.298 \mu J$

$W_{drop} = -\sigma \Delta A = -0.298 \mu J$

Initial final
 aero dynamic force \rightarrow 
 $\sigma_{water} = 72 \text{ mN/m}$

