

**Thermodynamics**  
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**Tutorial Problem on 'Work' - Part 1**

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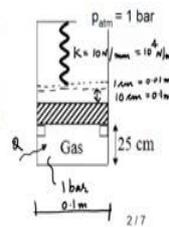
## Tutorial on Work

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A gas is contained in a friction-less piston-cylinder mechanism at an initial pressure of 1 bar. The diameter of the piston is 10 cm and it rests on stops initially as shown in the figure. The piston weight is such that it requires a pressure of 1.5 bar to be lifted from the stops.

The gas is now heated which causes the piston to rise slowly. After a rise of 10 cm, the piston touches a linear spring with a stiffness value of 10 N/mm. The heating of the gas is finally stopped when the spring is compressed by 1 cm. Determine the work interaction for the gas, piston, spring and the atmosphere. Also sketch the process undergone by the gas on a p-V diagram and show the four work interactions mentioned above by hatching the appropriate areas.



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### Solution:

We have a frictionless piston-cylinder mechanism containing gas.

$$\text{Initial volume of the gas, } V_1 = \frac{\pi}{4} d_{cyl}^2 h = 1.96 \times 10^{-3} m^3$$

$$p_1 = 1 \text{ bar} = 10^5 Pa$$

The piston lifts up only when the pressure inside becomes 1.5 bar. Till then, the process is constant volume process. Let's call this state at the end of the constant volume process as state 2.

$$p_2 = 1.5 \text{ bar}$$

$$V_2 = V_1$$

After this, the piston moves 10 cm and hits the spring. It moves against a constant atmospheric pressure of 1 bar and constant weight of piston. Hence, this expansion process is a constant pressure process. Let's call the state after this constant pressure process as state 3.

$$p_3 = p_2 = 1.5 \text{ bar}$$

$$V_3 = V_2 + \text{Volume swept during the movement of the piston by 10 cm}$$

$$= V_2 + \frac{\pi}{4} d_{cyl}^2 (0.1) = 2.74 \times 10^{-3} \text{ m}^3$$

Heating of the gas is still continued till the spring is compressed by 1 cm. This is state 4.

$$V_4 = V_3 + \frac{\pi}{4} d_{cyl}^2 (0.01) = 2.82 \times 10^{-3} \text{ m}^3$$

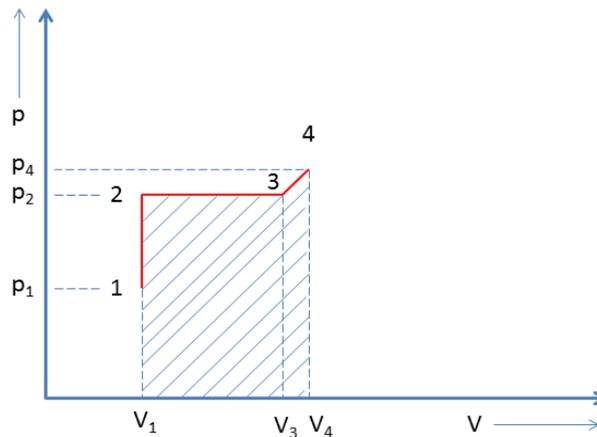
The spring is linear. As it gets compressed, it applies force on the piston which is proportional to its movement. Hence, the pressure of the gas also varies linearly.

$$\text{Pressure of the gas at state 4} = p_3 + \frac{\text{Force on the piston due to spring at state 4}}{\text{Area of the piston}}$$

$$p_4 = p_3 + \frac{0.5kx^2}{\frac{\pi}{4} d_{cyl}^2} = p_3 + \frac{0.5 \times 10 \times 10^3 \times 0.01^2}{7.85 \times 10^{-3}} = 1.5 \times 10^5 + 63.69$$

$$= 150063.69 \text{ Pa}$$

The total work interaction for the gas is the hatched area shown in the following figure:



Total work interaction = Area of the rectangle  $V_1 23 V_3$  + Area of the trapezium  $V_3 34 V_4$

$$= p_2(V_3 - V_2) + 0.5 \times (p_2 + p_4) \times (V_4 - V_3) = 1.5 \times 10^5 \times 0.78 \times 0.001 + 0.5 \times (150000 + 150063.69) \times (0.08 \times 0.001) = 117 + 12 = 129 \text{ J}$$

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$V_1 = \frac{\pi d^2}{4} \times h_1$

$V_1 = \frac{\pi \times 0.01^2 \times 0.25}{4} = \dots \text{ m}^3$

$p_1 = 1 \text{ bar}$   
 $p_2 = 1.5 \text{ bar}$  } volume is constant ( $V_2 = V_1$ )

against  $p_{\text{atm}}$  and  $mg$

$p_3 = p_2$

$V_3 = V_2 + \left[ \frac{\pi d^2}{4} \times 0.01 \right] = \dots \text{ m}^3$

$V_4 = V_3 + \frac{\pi d^4 \times 0.01}{4} = \dots \text{ m}^3$

$p = \frac{F}{A} = \frac{Kx}{A} \mid p + 1.5 \text{ bar} = p_4 = \dots \text{ bar}$

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against  $p_{\text{atm}}$   
= const. pt.

$p_{\text{atm}} (V_f - V_i)$