Fuzzy Logic and Neural Networks
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Lecture – 09
Applications to Fuzzy Sets (Contd.)

We have already discussed the working principle of Mamdani approach of fuzzy reasoning tool in the form of fuzzy logic controller. Now, today, we are going to discuss one numerical example like how to solve a numerical example using the Mamdani approach.

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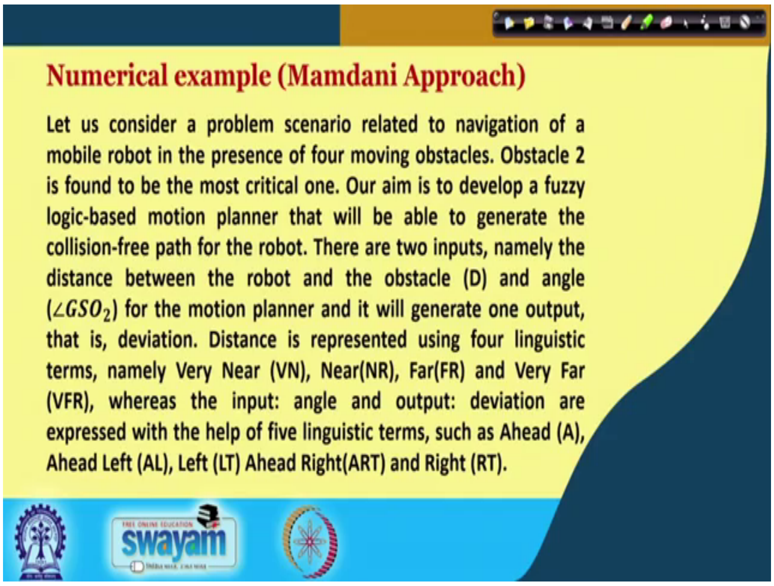

Now, the statement of the problem is as follows; now let us suppose a problem scenario related to navigation of a mobile robot in the presence of four moving obstacles.

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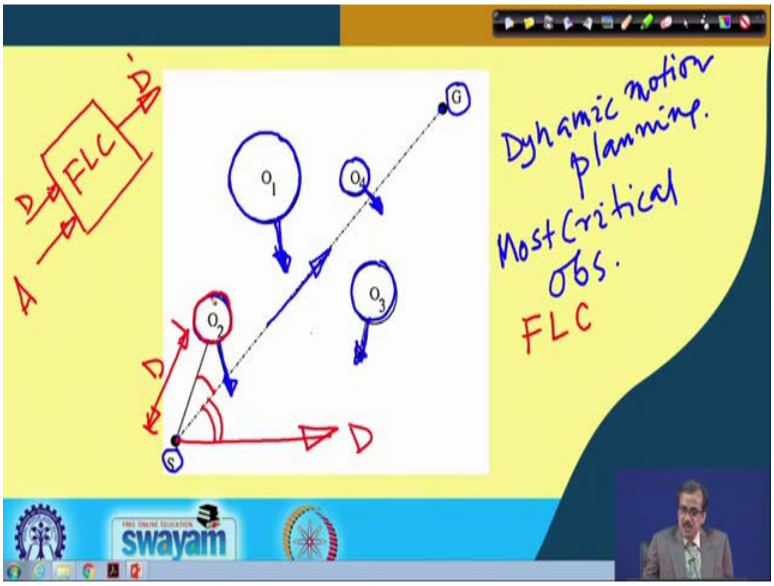

Now, here we are going to solve one navigation problem of mobile robots and this particular problem is known as actually the dynamic motion planning problem. So, this is the dynamic motion planning problem and this is nothing, but a navigation problem of a mobile robot in the presence of some moving obstacles. Now, here, I am just going to explain the problem scenario. Now, for simplicity, the robot the physical robot has been represented by a point. So, this is nothing, but the point robot and the starting position of the point robot is denoted by S and supposing that, the goal of this robot is indicated by the point G. So, G is the goal and S is the starting point.

Now, let us consider one hypothetical situation that there is no, such moving obstacles here. Now, if there is no such moving obstacle and this is the starting point and this is the goal, and if I say to find out one collision-free time-optimal path, the robot will try to find out. This particular path and this is actually the optimal path, if there is no such moving obstacles in the workspace. But, here, the physical problem is slightly different, different in the sense, we have got a few moving obstacles for example, say we have got obstacle 1, that is denoted by O_1 then comes we have got obstacle O_2, then we have got obstacle O_3 and obstacle O_4.

Now, what is our aim? Our aim is to find out the collision-free and time-optimal path for this particular robot starting from point S to reach that point G. Now, to solve this particular problem, the first thing we will have to do is you will have to find out the most critical obstacle for the robot. Now this is actually the direction of your speed. So, this are the velocity directions for the different obstacles. Now, here, to determine the critical obstacle like your most critical obstacle so, what I will have to do is, I will have to see, at least, your two things: one is the distance between the present position of the robot and that of the obstacle, and another is the direction of movement.

Now, here, if you see the obstacle O_1 is moving in this particular direction, obstacle O_2 is moving in this particular direction, O_3 is in this direction and O_4 is in this particular direction. Now, if I consider both the distance as well as the moving direction. So, very easily, we can find out the most critical obstacle, the most critical obstacle is nothing, but is your O_2. So, O_2 is the most critical obstacle; that means, to avoid collision with the most critical obstacle. So, I will have to make some planning with the help of your fuzzy reasoning tool.

So, I am just going to develop one fuzzy logic controller or the fuzzy reasoning tool so, that the robot can avoid collision with this particular, the most critical obstacle. Now, to solve this particular problem actually, what we do is, we take the help of one fuzzy reasoning tool or fuzzy logic controller. So, let me write here FLC; and for this FLC, there should be a few inputs and output also. Now, what we do is, the distance between the robot and the most critical obstacle and we will have to consider its boundary. So, this particular distance is nothing, but one input for the fuzzy reasoning tool and another input could be the included angle, that is your G S and O_2.

So, this particular angle could be another input. So, let me write here, angle is another input say denoted by A and we will have to find out one output of the fuzzy reasoning tool, that output could be the angle of deviation to avoid the collision with a the most critical obstacle. Now supposing that the angle of deviation to avoid the collision, it could be something like this. So, this could be the direction of movement of the robot just to avoid collision with the most critical obstacle. Now, if this is the scenario so, this particular angle; that means, if I just give one point here, say D,the angle GSD is noth ing, but actually the deviation; now this deviation so, I can write down that is by say D'. So, D' is nothing, but the angle of deviation to avoid collision with this particular, the most critical obstacle. Now, let us see, how to solve this particular problem using the principle of the Mamdani approach of fuzzy reasoning tool.

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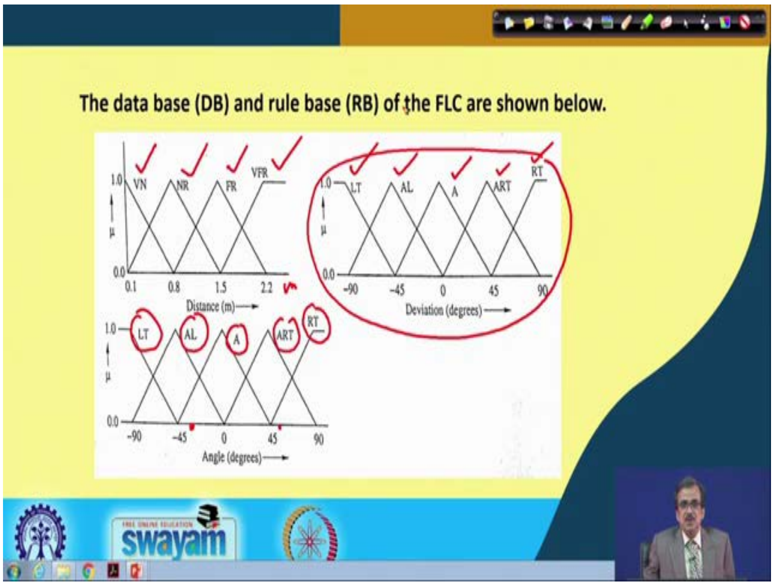

Now, here, the membership function distributions for this distance and the input angle. So, we have considered something like this and this is nothing, but the membership function distribution for the output, that is, the deviation. Now, if you see, if you concentrate on the database or the membership function distribution for the distance. So, the range for the distance has been consider as 0.1 to 2.2 meter and the whole range that is actually distributed or that is represented using four linguistic terms.

For example, we have got very near, then comes near far and very far and for simplicity the membership function distribution has been consider to be triangle. Now, I could have consider some sort of non-linear distribution also like Gaussian, but here, for simplicity, we have considered the triangular membership function distribution. And, you can see that there is a overlapping, for example, this near distance and far distance, there will be some sort of overlapping and that is actually, according to the definition of the fuzzy sets.

Now, the angle input that is divided into the range is divided into 4 or 5 linguistic terms, for example, say. So, if the angle is between minus 45 (minus mean say if I consider this is clockwise and plus if I consider it is anti-clockwise) and plus 45; that means, if I consider my left side, left hand side is negative and my right hand side is positive that is also possible. So, this A stands for the ahead, the angle is ahead; so, this angle ahead that is defined in the range of minus 45 degree to plus 45 degree.

Similarly, we have got ahead right ART and that is defined in the range of 0 to 90 degree, then we have got right, that is, RT plus 45 to 90, then we have got ahead left. So, from minus 90 to 0 and then, comes your left that is denoted by LT; so, minus 90 to minus 45. So, there are 5 such linguistic term to represent the angle input. Now, similarly, to represent the deviation that is the output of the fuzzy logic controller, once again, we are using five linguistic terms like your ahead right, then comes ahead left and left. So, this is the way actually we will have to manually design, the data base, that is the membership function distributions for the two inputs and one output.

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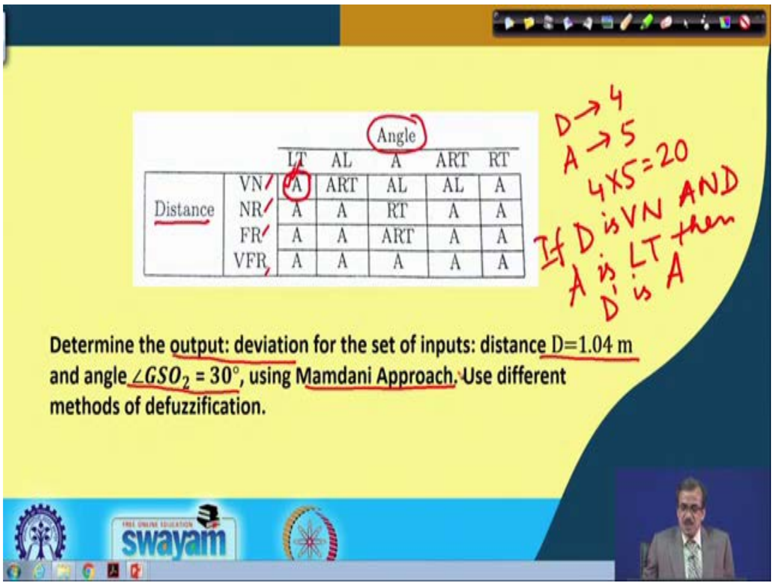

And, once they have got this particular membership function distribution now, we are in a position to design this particular rule base, that is the manually constructed rule base; that means, the based on the information of this particular problem of the designer, the designer is going to design this particular rule base.

Now, we have seen that for the distance input we have got like four linguistic terms and for the angle input, we have got five linguistic terms. So, we have got 4 multiplied by 5 like 20 possible combinations of the input variables and we have got a maximum of 20 rules. Now, this table shows all such 20 rules. Now, here, I have put distance and distance is expressed in terms of the linguistic terms like very near, far and very far and this particular angle input is represented as left, ahead left, ahead right and right.

Now, the first entry that is your first row first column entry it indicates A; A means your ahead. So, this is nothing, but the deviation. So, corresponding to this particular entry, if I write down the rules, it looks like this. So, if the distance D is very near and the angle is your the left then the output that is nothing, but the deviation, that is, D'. So, this is nothing, but D' is nothing, but is your ahead. So, this is nothing, but a rule and similarly, we have got 20 such rules, here.

Now, this particular rule base has to be manually designed, but it may not be the optimal one and we will be discussing one method, how to optimize this particular rule base after sometime. Now, here, in this numerical example, the statement is not yet complete. Now, here, what I will have to do is, we will have to find out the output, that is, the deviation for the set of inputs like distance is 1.04 meter. And, angle like GSO 2 is nothing, but 30 degrees and we will have to use Mamdani approach and we are going to use the different methods of defuzzification. Now, let us see how to proceed with this particular numerical example and how to solve it.

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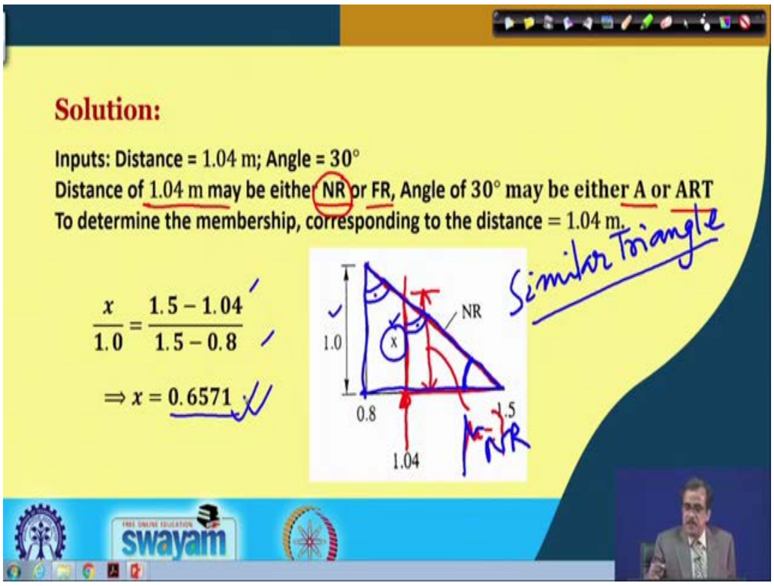

Now, before I just go for that, let me once again try to concentrate here. So, the distance is 1.04; that means, your, I might be here. So, let me just draw here.

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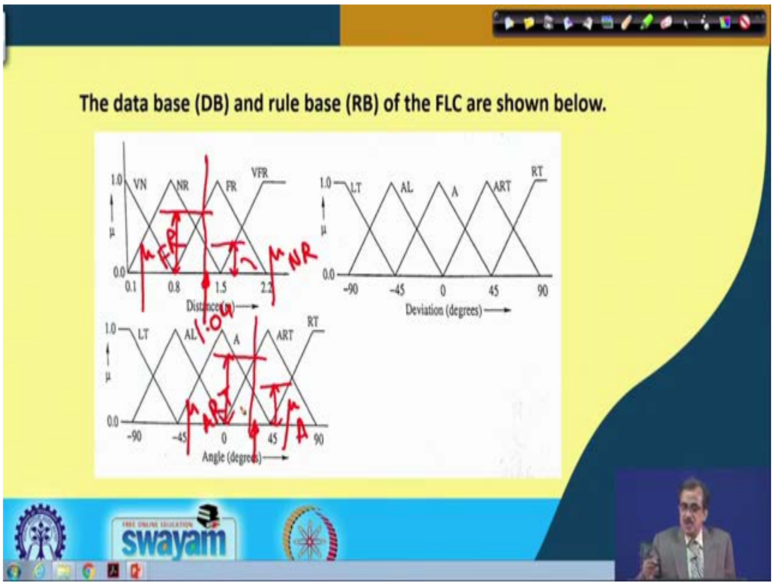

So, this is my 1.04 distance, say it is 1.04 meter and the angle input it is 30. So, might be might be I am here. So, if I am here, now corresponding to this particular 1.04, this particular distance can be called near with this much of membership function value, and this can also be called far with this much of membership function value.

Now, similarly, this angle 30 degree can be called ahead with this much of membership function value and it can also be called ahead right with this much of membership function value. Now, that means, if we just write it down here. So, corresponding to this, this is nothing, but is your μ_{NR} and this is nothing, but is your μ_{FR} and here. So, this is nothing, but is your μ_{A} and this is your μ_{ART}.

So, this is the way, actually we can find out the membership function distribution for the set of inputs. Now, with this particular information, let me start with finding the solution. Now, as I told, that the distance of say 1.04 meter can be called either near or far, similarly the angle of 30 degree may be called either ahead or ahead right. Now, let us try to find out your the membership function value.

Now, how to determine the membership function value, it is very simple, now let me first concentrate on the distance, near. So, if you see the membership function distribution for the near distance, you will be getting this type of distribution for the near lying in the range of 0.8 to 1.5, and here, the input is your 1.04. Now, corresponding to 1.04, my aim is to find out, what should be this particular μ.

So, this μ actually, I will have to find out. Now, what we do is, we use the principle of similar triangle. Now, if I consider say this is one triangle, and another triangle, if I consider something like this, if I consider another triangle is something like this. So, this is another triangle. So, these two triangles are actually similar. Now, if these two triangles are similar, then we can say that this angle is actually common to both the triangles, then this particular angle is equal to that particular angle.

So, I can use the principle of similar triangle. So, similar triangle, if I use, then very easily you can find out what should be the μ value. Now, here, I have written. So, x divided by your 1.0. So, x divided by 1.0 and opposite to that is this particular angle and that is actually common for both the triangles, then you concentrate on this particular angle.

So, now opposite to this is nothing, but 1.5 minus 0.4. So, 1.5 minus 1.04 and opposite to this is nothing, but 1.5 minus 0.8. So, 1.5 minus 0.8, and if you solve it, you will be getting x equals to 0.6571. So, this is the way, by using the principle of similar triangle, very easily, I can find out what should be the value of this particular x and that is nothing, but your μ_{NR}. So, this is nothing, but is your μ_{NR}, ok. So, μ_{NR} is nothing, but 0.6571, now once you have got this particular μ_{NR}. So, very easily you can find out, what is μ_{FR}.

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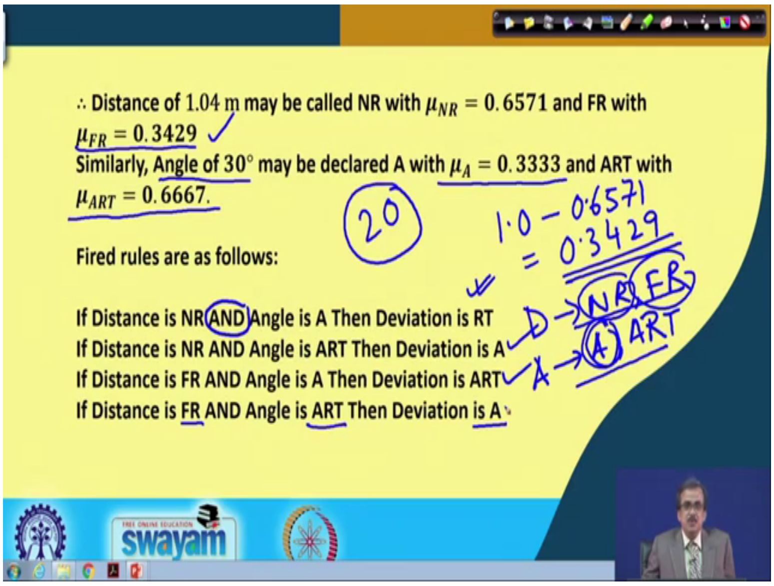

Now, this μ_{FR} is nothing, but is your 1 minus 0.6571 and that is nothing, but is your 0.3429. So, this 0.3429 is nothing, but is your μ_{FR}.

Now, by following the similar procedure, the angle input of 30 degree can be called either ahead or ahead right. So, this can be called ahead with the membership function value of 0.3333 and this can also be called ahead right with the membership function value of 0.6667, and once you have got these membership function values, we are in a position just to find out what should be the fired rules. Now, here, we have seen that the distance input could be either near or it could be far, similarly the angle input, it could be either the ahead or we have got ahead right.

Now, using this particular information, I can write down the four fired rules, there could be a maximum of four fired rules. Because here we have got two possibilities for the distance and we have got two possibilities for the angle. So, 2 multiplied by 2, there could be a maximum of 4 fired rules, but what is the total number of rules we have in the rule base? That is nothing, but 20, then out of 20, a maximum of four can be fired. Now, what are the fired rules, if distance is near, the first fired rule is if distance is near (now, as I have already mentioned that this particular AND is actually AND operator not the conjunction and that is why you have to write in capital AND) AND angle A is then deviation is RT.

So, this is actually the first fired rule. So, I am using NR and A combination, now, I will have to use NR and ART combination; that means, if distance is NR and angle is ART then deviation is A. So, this is nothing, but the second fired rule, then comes your the third fired rule. So, I will have to use this particular FR along with A. So, if distance is far, that is, FR and angle is A, that is, ahead, then deviation is ahead right. So, this is the third fired rule, then comes the fourth fired rule, if distance is FR and angle is ART then the deviation is A. So, as I mentioned there could be a maximum of four fired rules and out of these of 20 rules, these 4 rules could be fired.

Now, let us see how to determine the output corresponding to each of the fired rules, now, to determine the output of each of the fired rules the first thing we will have to do is, we will have to find out the firing strength of each of the fired rules.

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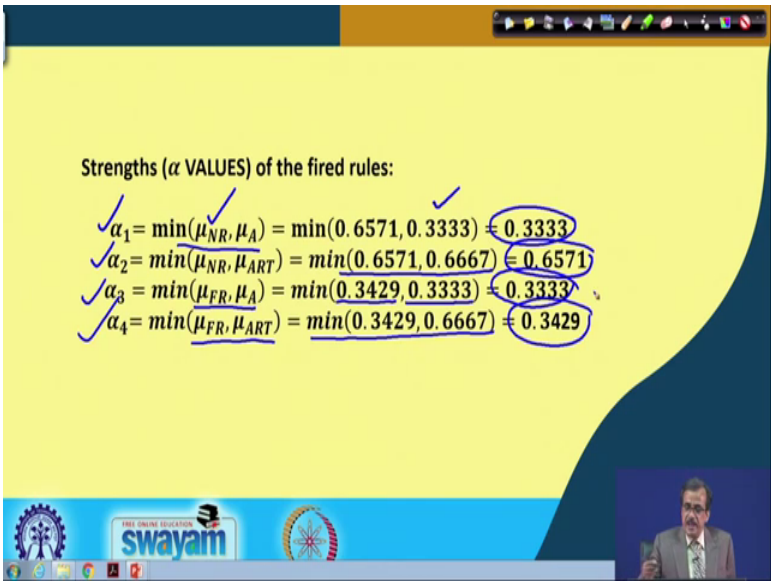

Now, the firing strength for the first rule that is denoted by is your α_1 is nothing, but the minimum between μ_{NR} and μ_A. Now, minimum between 0.6571, that is your μ_{NR} and μ_A is nothing, but 0.3333 and we will have to find out the minimum.

So, this is nothing, but the minimum. Similarly, the firing strength for the second rule that is α_2 is a minimum between μ_{NR} and μ_{ART} and that is nothing, but the minimum between 0.6571 and 0.6667, and the minimum is your 0.6571, then comes your third fired rule. And, its firing strength, that is, α_3 is nothing, but the minimum between μ_{FR} and μ_A that is the minimum between 0.3429 and 0.3333 and the minimum is your this.

Now, we will have to concentrate on the firing strength of the fourth rule that is your α_4 and that is nothing, but the minimum between μ_{FR} and μ_{ART} and that is nothing, but the minimum between 0.3429 and 0.6667 and actually, the answer is 0.3429. So, this is the way actually, we can determine the firing strength of each of these particular rules. And, once you have got the firing strength, now we are in a position to determine what should be the fuzzified output corresponding to actually each of the fired rules.

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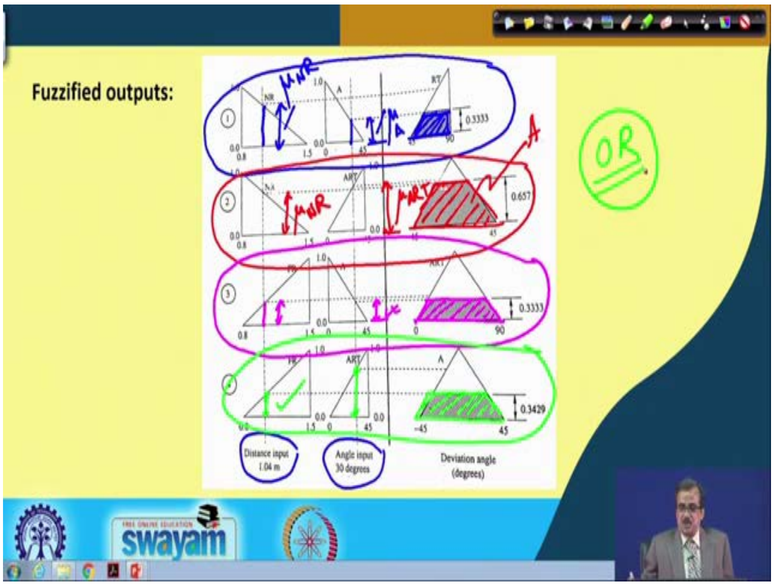

Now, here, if you see the first fired rule, that is represented by actually this; this is nothing, but the first fired rule, and if you see this particular rule, if distance is near and angle is ahead then the deviation is right. So, this is actually the first fired rule and what are the inputs? The inputs are: distance is your 1.04 meter and the angle is your 30 degrees. So, these two inputs, we are passing and corresponding to the distance equals to 1.04, I will be getting actually μ and this is nothing, but is your μ_{FR}.

So, this is your μ sorry μ_{NR}. So, this is your μ_{NR} and similarly, corresponding μ_{NR} to the angle of 30 degree, I will be getting one μ here. So, this particular μ and this μ is nothing, but is your μ_A, now we will compare. So, μ_{NR} and your μ_A and there is an

AND operator, this is a minimum operator. So, we will try to find out the minimum between this μ_{NR} and μ_A and if you see the minimum is nothing, but your μ_A.

So, I will be getting actually the output of the first fired rule is something like this. So, this is actually the shaded area, is actually the output corresponding to the first fired rule. Now, by following the similar procedure; I can also find out your what should be the output corresponding to the second fired rule. So, this is the second fire rule and here corresponding to the distance equals to 1.04, I will be getting μ_{NR} is nothing, but this. So, this is your μ_{NR} and corresponding to this particular angle. So, I can also find out the μ and that is nothing, but is your μ_{ART}.

And, if I just compare these two μ values. So, the minimum will be your μ_{NR} and corresponding to that, actually I can find out, what could be the fuzzified output. So, this is nothing, but the fuzzified output and truly speaking, this is actually the membership function distribution for the ART direction, ok. So, this is this shaded portion is nothing, but the fuzzified output corresponding to the second fired rule.

Now, by following the similar procedure; so, I can also find out what should be the your the fuzzified output corresponding to your third fired rule. So, if I just concentrate on the third fired rule, this is the third fired rule and corresponding to the distance. So, this is one μ so, I will be getting one μ that and I will be getting another μ here corresponding to the second input, and if I just compare these two μs. So, this is the minimum. So, I will be getting the fuzzified output is nothing, but this shaded area.

Now, corresponding to the fired rule; now corresponding to the fourth fired rule, what you can do is, so, we can find out what should be the fuzzified output, now here corresponding to the distance input, this is nothing, but your μ and here, corresponding to the angle, this is nothing, but is your μ and if I compare these two μs, this is the minimum. So, the fuzzified output corresponding to the fourth fired rule will be something like this. So, this is the way actually, we can find out what should be the fuzzified output for each of the fired rules, and after that actually we will have to use the OR operator just to combine all such outputs into one, and as I told this OR operator is the max operator.

So, we will have to superimpose all such fuzzified outputs, the truncated area whatever we have got and then, we will have to use this OR operator or the max operator. So, just to find out what should be the fuzzified output considering to all four fired rules. Now, if you see this particular fuzzified output corresponding to your all the fired rules, now you will be getting actually this type of area.

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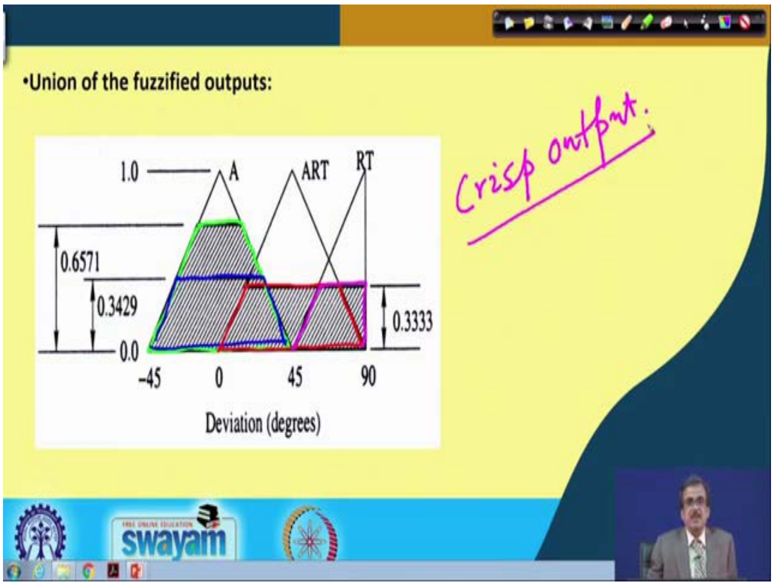

Now, if you just see so, corresponding to one of the fired rules, you will be getting these types of your output. So, this output is corresponding to one fired rule. So, this is the output. Now, corresponding to another rule the output is something like this. So, this is corresponding to another rule and then, comes your another fired rule. So, the output is something like this. So, this is actually another fuzzified output. So, this is the fuzzified output and corresponding to another, you can find out.

So, this is nothing, but the fuzzified output. So, all such things actually, you will have to superimpose to find out what should be the fuzzified output considering all the fired rules. Now, these are all fuzzified outputs, that means, this is actually the fuzzified output now, which cannot be directly used for controlling a particular process or to take any decision. So, what you will have to do is, from this fuzzified output actually we will have to find out the crisp output. So, how to find out the crisp output that I will be discussing.

Thank you.